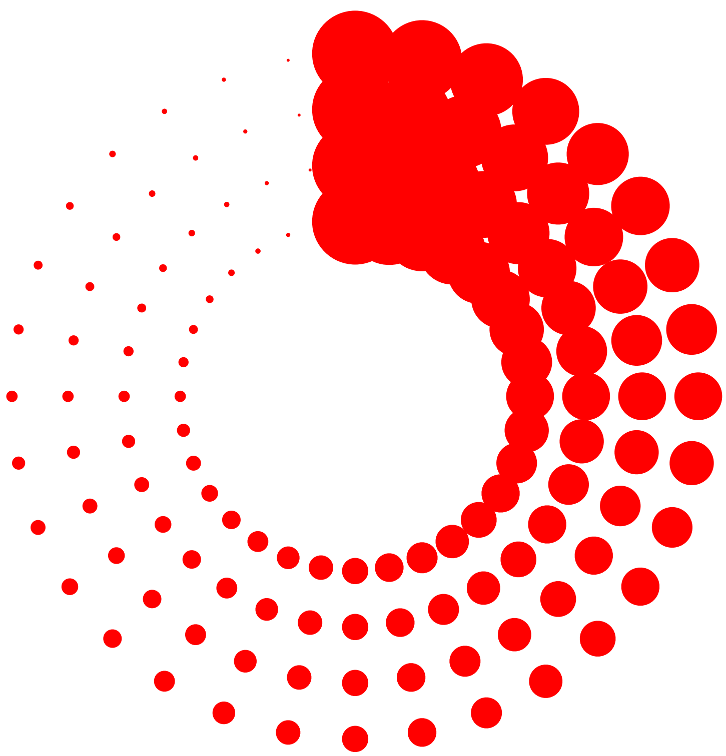
IMPRECISION IN ENGINEERING DESIGN



IMPRECISION IN ENGINEERING DESIGN

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Preface

It must be considered that there is nothing more difficult to carry out, nor more dangerous to conduct, nor more doubtful in its success, than an attempt to introduce changes. For the innovator will have for his enemies all those who are well off under the existing order of things, and only lukewarm supporters in those who might be better off under the new.

-- Niccolò Machiavelli
The Prince and The Discourses, 1513
Chapter 6

“Scientists investigate that which already is;
Engineers create that which has never been.”

Albert Einstein

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Introduction

Erik Antonsson

This book is a collection of publications produced from research conducted in the Engineering Design Research Laboratory at the California Institute of Technology. The research thread, to which all of these papers are related, is the notion of Imprecision in Engineering Design.

Research over the past 17 years has demonstrated that information with a range of precisions is an essential component of engineering design, and that formal methods can be developed to represent and manipulate this imprecise information. The goal of this volume is to collect into one place a set of relevant publications describing the results of research in this area.

The Chapters

This volume begins with a chapter introducing the notion of imprecision in engineering design, and motivating the research to follow. Chapter 2 introduces strategies for trade-offs among multiple incommensurate attributes of engineering designs. Chapter 3 introduces a method for incorporating uncontrolled variations (noise) into design decision-making. Chapter 4 extends the notion of noise to include *tuning parameters*: those aspects of a design that are adjusted to compensate for uncontrolled variations. Chapter 5 provides an overview of the Method of Imprecision, and a review of and comparison with other methods. Chapter 6 develops the mathematics of aggregation of incommensurate attributes for design decisions. Chapter 7 extends the aggregation methods to negotiation among multiple people or groups involved in an engineering design. Chapter 8 demonstrates the Method of Imprecision on an automobile structure design problem. Chapter 9 shows that methods for economic decision-making or social choice do not necessarily apply to engineering design. Finally, Chapter 10 presents a method for determining how to aggregate multiple incommensurate attributes of an engineering design.

References

[1] WOOD, K. L., AND ANTONSSON, E. K. Computations with Imprecise Parameters in Engineering Design: Background and Theory. *ASME Journal of Mechanisms, Transmissions, and Automation in Design 111*, 4 (Dec. 1989), 616–625.

[2] OTTO, K. N., AND ANTONSSON, E. K. Trade-Off Strategies in Engineering Design. *Research in Engineering Design 3*, 2 (1991), 87–104.

[3] OTTO, K. N., AND ANTONSSON, E. K. Design Parameter Selection in the Presence of Noise. *Research in Engineering Design 6*, 4 (1994), 234–246.

[4] OTTO, K. N., AND ANTONSSON, E. K. Tuning Parameters in Engineering Design. *ASME Journal of Mechanical Design 115*, 1 (Mar. 1993), 14–19.

[5] ANTONSSON, E. K., AND OTTO, K. N. Imprecision in Engineering Design. *ASME Journal of Mechanical Design 117(B) (Special Combined Issue of the Transactions of the ASME commemorating the 50th anniversary of the Design Engineering Division of the ASME.)* (June 1995), 25–32. Invited paper.

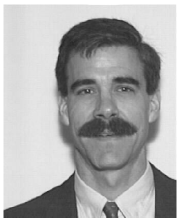
[6] SCOTT, M. J., AND ANTONSSON, E. K. Aggregation Functions for Engineering Design Trade-offs. *Fuzzy Sets and Systems 99*, 3 (1998), 253–264.

[7] SCOTT, M. J., AND ANTONSSON, E. K. Formalisms for Negotiation in Engineering Design. In *8th International Conference on Design Theory and Methodology* (Aug. 1996), ASME.

[8] SCOTT, M. J., AND ANTONSSON, E. K. Preliminary Vehicle Structure Design: An Industrial Application of Imprecision in Engineering Design. In *10th International Conference on Design Theory and Methodology* (Sept. 1998), ASME.

[9] SCOTT, M. J., AND ANTONSSON, E. K. Arrow's Theorem and Engineering Design Decision Making. *Research in Engineering Design 11*, 4 (2000), 218–228.

[10] SCOTT, M. J., AND ANTONSSON, E. K. Using Indifference Points in Engineering Decisions. In *11th International Conference on Design Theory and Methodology* (Sept. 2000), ASME.



Erik K. Antonsson is Executive Officer (Dept. Chair) and Professor of Mechanical Engineering at the California Institute of Technology in Pasadena, CA, U.S.A., where he organized the Engineering Design Research Laboratory and has conducted research and taught since 1984.

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In 1983 he joined the Mechanical Engineering faculty at the University of Utah, as an Assistant Professor. In 1984 he became the Technical Director of the Pediatric Mobility and Gait Laboratory, and an Assistant in Bioengineering (Orthopaedic Surgery), at the Massachusetts General Hospital. He also simultaneously joined the faculty of the Harvard University Medical School as an Assistant Professor of Orthopaedics (Bioengineering).

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Dr. Antonsson is currently on the editorial board of the International Journals: *Research in Engineering Design*, and *Fuzzy Sets and Systems*, and from 1989 to 1993 served as an Associate Technical Editor of the ASME Journal of Mechanical Design, (formerly the *Journal of Mechanisms, Transmissions and Automation in Design*), with responsibility for the Design Research and the Design Theory and Methodology area. He serves as a member of the Engineering and Applied Science Division Advisory Group, and as Chairman of the Engineering Computing Facility at Caltech. He was a member of the Caltech Faculty Committee on Patents and Relations with Industry from 1992 to 1999, and since 1990 has been a member of the CALSTART Technical Advisory Committee. He has published over 95 scholarly papers in the engineering design research literature, and holds 4 U.S. Patents. He is a Registered Professional Engineer in California, and serves as an engineering design consultant to industry, research laboratories (including NASA's Jet Propulsion Laboratory and the 10 meter W. M. Keck Telescope), and the Intellectual Property bar.

Chapter 1

COMPUTATIONS WITH IMPRECISE PARAMETERS IN ENGINEERING DESIGN: BACKGROUND AND THEORY

Kristin L. Wood and Erik K. Antonsson

ASME *Journal of Mechanisms, Transmissions, and Automation in Design*
Volume 111, Number 4 (December 1989), pages 616-625.

Abstract

A technique to perform design calculations on imprecise representations of parameters has been developed and is presented. The level of imprecision in the description of design elements is typically high in the preliminary phase of engineering design. This imprecision is represented using the fuzzy calculus. Calculations can be performed using this method, to produce (imprecise) performance parameters from imprecise (input) design parameters. The Fuzzy Weighted Average technique is used to perform these calculations. A new metric, called the γ-level measure, is introduced to determine the relative coupling between imprecise inputs and outputs. The background and theory supporting this approach are presented, along with one example.

1. Introduction

Engineering design, both in practice and research, is evolving rapidly, especially in the development of computer-based tools. Emphasis is moving from the later stages of design, to computational tools for preliminary design. In an earlier paper [33], a general approach to computational tools in preliminary engineering design and a model of the design process was described. The primary aim of this model is to provide a structure for the development of tools to assist the designer in: managing the large amount of information encountered in the design process; determining a design's functional requirements and constraints; evaluating the coupling between the design parameters; and carrying out the process of choosing between alternative design concepts.

We are particularly interested in developing tools to assist the designer in the *preliminary* phase of engineering design, by making more information available on the performance of design alternatives than is available using conventional design techniques. The most important design decisions (and potentially the most costly, if wrong) are made at the preliminary stage. Our hypothesis is that increased information, over what is available by traditional design methods, will enable these decisions to be made with greater confidence and reduced risk. The effect will be greater, the earlier in the design cycle additional information can be made available.

The preliminary phase of the engineering design process is one that embodies many functions: concept generation; evaluation of imprecise descriptions of simplified versions of the design; judgement of design feasibility; etc. [26, 15, 28]. The concept generation and simplification processes will not be addressed by the research reported here, rather our aim is to provide a technique for representing, manipulating, and evaluating the approximate, or imprecise, parametric descriptions of the (preliminary) design artifact.

Typical examples of imprecise descriptions in preliminary design include: an irregular cross-section structural member may be represented by a rectangular section for the purposes of initial evaluation; a gear set may be represented by a pair of circles rolling on each other (without slip), and an approximate speed ratio; a length of shaft may be represented as "about 25 cm"; etc. These are approximate, or *imprecise*, descriptions of the design artifact, not incomplete descriptions. The gear set, imprecisely represented above, has all of the *functional* attributes of a gear set, but none of the detail.

As the design process proceeds from the preliminary stage to more detailed design and analysis, the level of imprecision in the description of the design artifact is reduced. Naturally at the end of the design cycle, the level of imprecision is very small, although uncertainties (e.g., tolerances) remain. It is this spectrum of levels of precision that characterizes progress through the design process, from a description of a need, to a (precise) description of a device to fulfill that need.

Unfortunately it has been difficult to provide computational tools for the preliminary phase of the design process, largely because of the relative paucity of algorithms and techniques that can operate on imprecise data. Solid modeling, optimization, mechanism analysis, and other CAD methods all require a highly precise representation of the objects being designed. This paper presents a novel (to the engineering design process) application of a method for representing and manipulating imprecision.[1] This technique calculates the approximate output quantities from the imprecise input parameters for each of the design alternatives, and determines the qualitative relations between the input parameters and the performance parameters (outputs). The designer is able to rank the input parameters according to their impact on the performance parameters, and to rate a design alternative according to its merit in relation to the others under consideration.

These computational tools are for use in the preliminary and conceptual synthesis stages of design, but do not attempt to supplant the designer. The idea is not to fully automate the design process, nor to automatically generate design alternatives, rather it is to make it easier for the designer to evaluate more alternatives in less time, and to provide more information on the performance of each of those alternatives. These developments form a *semi-automated* approach to design.

Terminology used to describe the design process will be defined first, background and theory as it is applied to engineering design will be presented next, followed by an example.

2. Terminology

2.1 Design Definitions

Parameter: A variable or quantity used in the design process.

Design Parameter [DP]: Any free or independent parameter whose value is determined during the design process.
(synonyms: Design Variable, Input Parameter).

Performance Parameter [PP]: Any parameter used in the design process that has a specified value [FR] determined independent of (and usually in advance of) the design process. The performance parameters [PPs] are usually dependent on the design parameters [DPs], and possibly some other PPs.

Output Parameter [OP]: Any parameter used in the design process that is dependent on the design parameters [DPs], and possibly some performance parameters [PPs], but has no specified functional requirement [FR] value.

Functional Requirement [FR]: A value, or range of values, or fuzzy number that is the specified value for a Performance Parameter [PP].

[1] Fuzzy sets have been applied to other domains including: seismic risk analysis [8, 6, 7, 9, 18], optimization [13, 29], reliability [24, 37], expert systems [25], logic and decision support [1, 2, 3, 4, 17, 19, 36, 41, 39], language and grammar [17, 21, 42], and others.

This value is determined independent of (and usually in advance of) the design process. Each Performance Parameter has a FR.
(synonyms: Performance Specification, Constraint).
(Note that this distinction between the Performance Parameter and its specified value [Functional Requirement] is to permit a Performance Parameter not to be identically equal to its specified Functional Requirement value at all times during the design process.)

Performance Parameter Expression [PPE]: An expression, relationship, or equation relating some or all of the Design Parameters to a Performance Parameter. Each PP has a PPE.

2.2 Fuzzy Set Implementation Definitions

Support: A crisp set of all values of a fuzzy set where the membership is greater than zero. Alternatively: The range of parameter values over which the fuzzy set membership is greater than zero.

Imprecision: The range (support) or spread of values about the peak [preference of one (1)] of a parameter's fuzzy set. The greater the imprecision, the greater the spread on the left or right (or both) sides of the preference function. This is loosely analogous to variance in the stochastic sense.[2] The interpretation of Imprecision, as used in design, will be discussed in the next section.

3. Background

Most of engineering, particularly design, can best be represented with some level of imprecision or approximation. According to Goguen [20]:

> *"Fuzziness is more than the exception in engineering design problems: usually there is no well-defined best solution or design."*

The imprecision that is being represented and manipulated by the technique reported here is meant to capture the approximations made during the early phases of engineering design.

---

[2]The mathematics of fuzzy sets are different from the mathematics of probability, and we find fuzziness more well suited to solving *imprecise* (i.e., "uncertainty in choosing among alternatives") problems in the preliminary phase of design. Probability continues to be most appropriate for representing and manipulating the *uncertainty in truth* aspects of design problems. A comparison of probabilistic and fuzzy methods in design will be the subject of a later publication. Many design problems will require both methods.

3.1 Representation and Interpretation of Imprecision

A simple range might be used to represent the imprecision for a parameter. This is the technique used in interval analysis [27]. Instead of a range, we represent the imprecise parameter by a range and a preference function to describe the desirability of using that particular value within the range. This preference function is similar to the notion of a *fuzzy set*, or more specifically a fuzzy number which is restricted to the set of real numbers.

A *fuzzy set* (as developed by Zadeh [38]) is a set with boundaries which are not sharply defined. Membership in the set is not the customary 0 or 1, but can be described by a continuum of grades of membership. In the approach described here we use preference values, analogous to membership, to represent imprecision or approximation of engineering design parameters. For example, a designer may want to represent a dimension of "about 25 cm". He or she would do so by specifying a preference function to represent that approximate parameter.

The first step is to decide the range of values that the parameter may assume. Values less than the low end of the range, and greater than the high end of the range will have membership of zero (0) in the fuzzy representation. For example, there may be a restriction on the dimension to be being greater than 20 cm, and the designer may wish to keep it shorter than 30 cm. The value, or values, that the designer feels the greatest confidence in using, or desire to use, are assigned a preference of one (1). Certainly 25 cm will have a preference of 1 (one) in the fuzzy set: "about 25 cm", and values away from 25 will have lower preference, as shown in Figure 1.1. Preference is assigned depending on the designer's desires to use those parameter values. The more confident, or the more the designer desires to use an input value, the higher its preference in the parameter's set. The resulting function of this process is a quantification of design preference, and not the customary notion of membership in a symbolically labeled fuzzy set, which usually denotes vagueness in meaning. In this way parameters whose values are not known precisely can be represented (and manipulated), and the designer's experience and judgement can be represented and incorporated into the design evaluation.

Therefore we interpret imprecision as representing the designer's desire to use a particular value for a design parameter. Naturally these desires may change as the design proceeds, and this is easily accomplished using fuzzy sets. This evolution of knowledge, preference, and emphasis is a common element of the design process, and the technique reported here permits their representation and manipulation.

In the example given above, Figure 1.1, the input preference function depends solely on the subjectivity of the designer. Preference functions need not

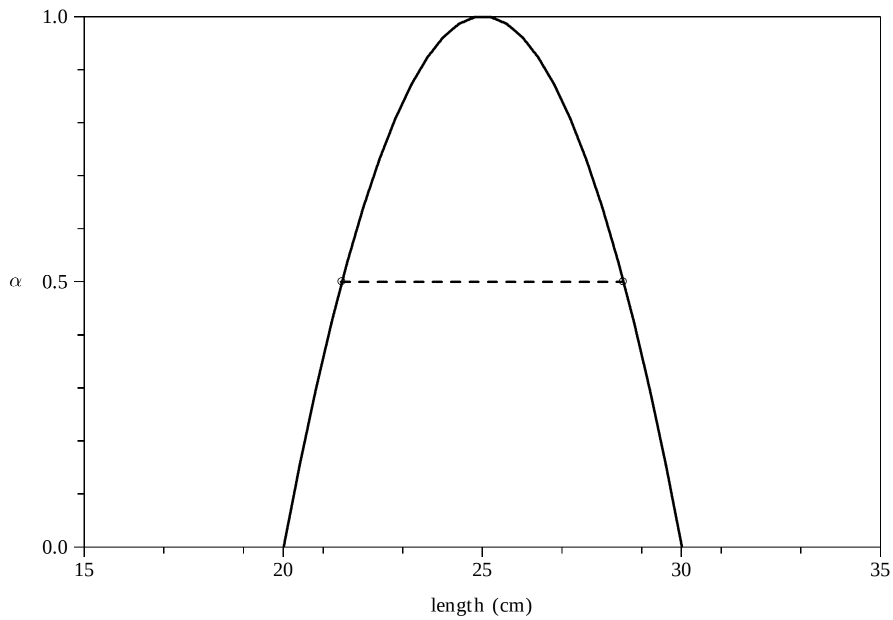

Figure 1.1 Imprecise Representation of “About 25 cm”, α-cut at 0.5.

always be dependent in this way, engineering data may also be used in certain situations. For example, a variety of materials might be used, and the preference of the designer is to minimize cost, solubility, or some other measurable material property, (or any combination of these). If the cost or other material data are available, the preference function can be constructed by normalizing the data between zero and one, and interpolating a curve between the data points (a method for handling discrete data is presented in [35]). Figure 1.2 is an example preference function constructed from the cost data for certain steel alloys, where the designer has specified a preference of minimum cost.

The *desirability* interpretation, as discussed above, applies to input DPs (those parameters whose value the designer is free to choose). Target values for Performance Parameters are specified by Functional Requirements, not directly by the designer's desires. Performance Parameters, resulting from calculations with imprecise input Design Parameters (using the authors' implementation of the Fuzzy Weighted Average algorithm [16] described below), will also be represented by fuzzy preference functions. These output preference functions also represent the designer's desires, but in a slightly different way from the inputs. The output parameter value with a preference of 1 (one) corresponds to the input values with preference of 1. This is a natural consequence of calculations using the fuzzy calculus [17, 23, 43]. This implies that

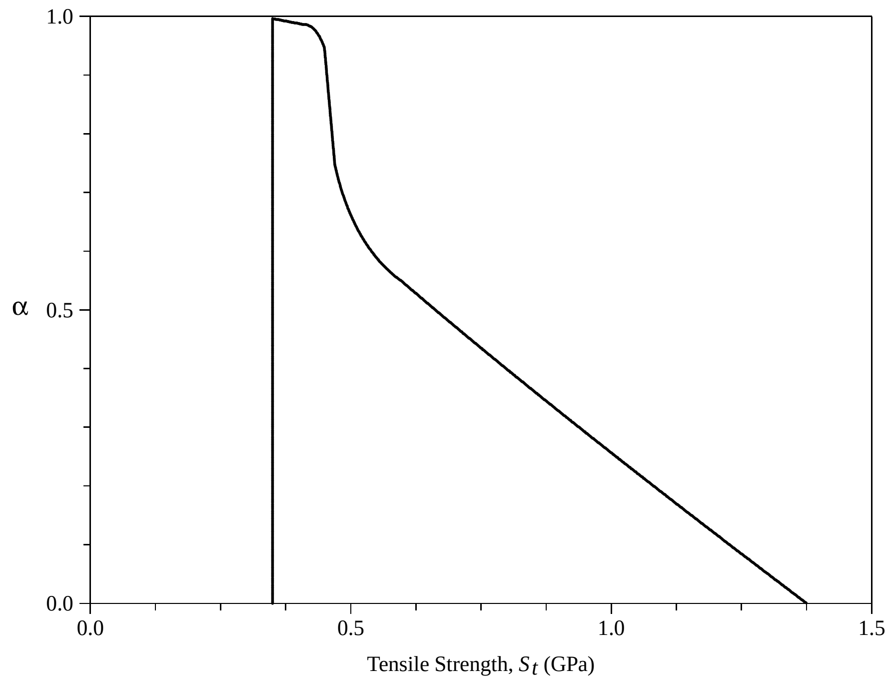

Figure 1.2 Preference Function: Steel Alloy Data

if the designer's desires are met (inputs with preference of 1), then the performance will be the output value with preference of 1. Correspondingly, if the performance parameter output value with preference of 1 satisfies the Functional Requirement(s), then the designer can use the input Design Parameter values with preference of 1. If it is required to use an off-peak value for the performance (to satisfy a Functional Requirement), then either the designer's desires must be adjusted, or input values other than the most desirable must be used. This will be discussed in detail below.

3.2 Existing Techniques

There exists a variety of means by which imprecise parameters can be represented and manipulated in engineering design calculations. The most basic approach is to choose single (crisp, non-fuzzy) values for each of the parameters, substitute these into the governing equations, and record the crisp single-valued output. This method benefits from simplicity, but suffers from the time required to "explore" any real design space.

Optimization schemes potentially provide a means for handling imprecise parameters. These methods include direct search methods such as Simplex and three-point equal-interval search, gradient methods such as Newton's and the Conjugate Gradient search [30]. However, conventional optimization methods require precise representations and analyses, and are therefore most useful in the latter stages of design. A. Diaz [13, 14] is developing an optimization technique using imprecise (fuzzy) constraints. This method will be useful for solving imprecise optimization problems, but will not provide as much information on the performance of a design operating over a range of design parameters as the method reported here.

Interval analysis [27] is another method for carrying out computations with imprecise parameters. In this technique an interval (a range of numbers represented by its boundaries) is used to represent a DP in the design calculations. The output (PP) is similarly represented by the two numbers at the end points of an interval. This method has some similarity to the method developed by the authors in that it indicates ranges of possible values for inputs and outputs. Interval analysis, however, provides no information on the performance of a design *within* the interval. All that can be said, when interpreting a Performance Parameter output, is that the design will perform somewhere between the boundaries of the interval. Furthermore, the input values which contributed to any one particular value of the output cannot be directly determined (except at the boundaries). As the number of intervals used to represent DPs increases (e.g. a succession of decreasing interval sizes may be used to cause a PP to approach a desired value), interval analysis approaches the method reported here.

G. Taguchi [10, 32] has developed a technique for evaluating the "quality" of a design based on his loss function. This function is essentially a preference function for a fuzzy representation.[3] Taguchi does not apply the mathematics of fuzzy sets to the evaluation or comparison of designs. Instead, his method employs the principles of "experimental design" which "explores" the design space one crisp design parameter value at a time. Taguchi suggests that the Parameter Design phase will have the most impact on quality. In this phase the values for DPs can be selected to create a design that will be as insensitive as possible to manufacturing errors, environmental conditions, variability in use, etc. The design technique presented here will be a useful extension to Taguchi's method in the Parameter Design phase (by permitting a more thorough evaluation of the performance parameters over ranges of the design parameters), as well as performing its intended purpose in the preliminary design phase.

Sensitivity analysis permits the evaluation of the rate of change of an output PP as input DPs change. This relies on the evaluation of partial derivatives or Lagrange multipliers of system equations.[4] Sensitivity analysis is a powerful design tool, but provides information only at a single operating point each time it is evaluated, and will provide no information when only discrete values of input design parameters are available. Furthermore, the change in desirability of inputs and outputs is not included in the calculation. For example: one input may have a narrow range of acceptable values, and a different input may have a much wider range of desirability. Even if the numerical sensitivity of one output is the same with respect to these 2 inputs, different design decisions should be reached regarding the effect of altering them. When a preference function is used instead of a range (to represent the designer's desires) even more information in the form of the rate of change of desirability of an output with respect to an input's desirability can be found. We introduce the γ-level measure later to evaluate this effect. Sensitivity analysis, as it is usually applied, does not include the effects of imprecision, or the designer's desires.

If a multi-valued logic form of probability analysis is used (instead of the more common event-frequency form), imprecision of input DPs may be represented, and imprecise output PPs can be calculated [5, 11, 22, 31]. However, the calculus of probabilities does not permit the relationships between inputs and outputs to be found. If, for example, a probability calculation shows that the desired performance has a low likelyhood, determining which DPs to change, and how to change them is not possible from the probability calculations alone. Only the expectation of the outputs is available. Furthermore,

[3] See particularly the Quadratic Loss Function shown in Figure 3 of [10].

[4] Reference [30] pages 168 and 609.

some probability calculations (on imprecise parameters rather than uncertain parameters) can produce unexpected results.[5]

The method presented here, based on a fuzzy representation of imprecision, extends the capabilities of the methods described above by permitting: representation of imprecise input Design Parameters; calculation of resulting Performance Parameters (with corresponding levels of imprecision); evaluation of Design Parameters to attain a desired Performance Parameter; and estimates the relationship between DPs and PPs over a wide range of values.

4. Approach

As described in the previous section, we have adopted the fuzzy calculus as a mathematical representation of imprecision in engineering design. The arithmetic and calculus of fuzzy sets and fuzzy numbers provides us with a method for manipulating these imprecise representations.

Fuzzy numbers and their associated arithmetic and calculus are the subject of many publications and several textbooks [17, 23, 43] and will not be presented here.

In brief, fuzzy arithmetic is based on Zadeh's extension principle [40]. Kaufman and Gupta [23] have shown analytically that this is equivalent to an α-cut form of the mathematics [23]. The α-cut form of some simple mathematical operations are shown in the Appendix.[6] Finally, a discrete version of the mathematics utilizing interval analysis at discrete α-cuts has been developed by Wong and Dong [16] utilizing their Fuzzy Weighted Average (FWA) algorithm.[7]

Figure 1.1 shows an α-cut at preference 0.5. The discrete FWA algorithm treats each α-cut as an interval, and performs interval analysis to calculate each output preference interval [16]. The important addition to interval analysis, however, is the preference value associated with each value in the fuzzy number. It can be seen that as successively smaller intervals are used in a calculation, interval analysis approaches fuzzy set mathematics.

One important ramification of fuzzy mathematics is that once a forward calculation is made (operating on inputs to determine an output fuzzy function),

[5] For example: $y = mx + b$ where m, x, and b have probabilistic representations centered at a value of 3.0, produces an output with a peak likelyhood at $y = 11.6$ rather than the value 12. A detailed comparison of probability analysis and the authors' technique is the subject of a later publication.

[6] A discussion of the Extension principle and α-cuts can be found in Dubois and Prade [17], Chapter 2, pages 36-67 and page 19 respectively.

[7] The analytical method of calculating a fuzzy output from imprecise inputs is infeasible for computer-assisted design applications. Wong and Dong's FWA algorithm is used here, in a computationally efficient implementation developed by the authors. This algorithm has computational complexity of order: $M \cdot 2^{(N-1)} \cdot \kappa$, where $M =$ the number of α-cuts for each parameter, $N =$ the number of parameters, and $\kappa =$ the number of (combinatorial interval-analysis) operations for one α-cut. Example fuzzy-set calculations are shown in the Appendix.

then *backward* calculations can be obtained with no further computation. The peak of a fuzzy output corresponds to the peak value for each of the inputs, off-peak output values correspond to off-peak inputs with the same preference value. For example, if a designer performed a fuzzy calculation, and the output parameter's peak value (preference of one (1)) was not acceptable, then he or she could select a different output value and determine its preference value. The designer then knows that the inputs required to produce that output have the same preference or less. If the designer wishes to use an output parameter value with preference of 0.7, then he or she knows that at least one input must also have a preference of 0.7 or less, the other inputs having preference distributed about 0.7. In this way the relationship between inputs and outputs is readily observed. The backward path through the calculations is a natural consequence of the fuzzy arithmetic implementation developed by the authors, and requires no further calculations once the forward path has been calculated.

4.1 Preference Function Shapes for Design Parameters

A simple form of the preference functions described above is triangular (single most desired/confident value with linear interpolation to the zero confidence values) or trapezoidal (interval of most desired/confident values at preference of one (1)). For preliminary design, the experiments conducted to-date indicate that these two classes of preference function shape will adequately approximate input DPs imprecise representation. These types of functions also satisfy the normality and convexity conditions required of fuzzy numbers. If it becomes necessary to use higher-order functions, they can be included without modification to the technique or implementation described here. For example, to bias a preference around the most preferred input, a quadratic function can be used. Likewise, to bias the preference in the opposite sense, an inverse quadratic function, which approaches a Dirac delta function in the extreme case, may be applicable. Furthermore, if multiple peaks are found to be required, then the convexity condition may be relaxed slightly such that the preference functions are treated as multiple locally convex functions.

Besides triangular and trapezoidal functions, preference functions can be constructed exactly from engineering data (Figure 1.2), if the data and interpretation are available. For an incomplete set of data, a preference function may be approximated by curve fitting (analogous to the construction of subjective probability density functions) to certain points of preference in a design parameter's input range.

For triangular inputs, the outputs of design performance analysis functions may not always be linear functions, as shown by the example in the Appendix. A fuzzy multiplication with triangular input functions does not result in a

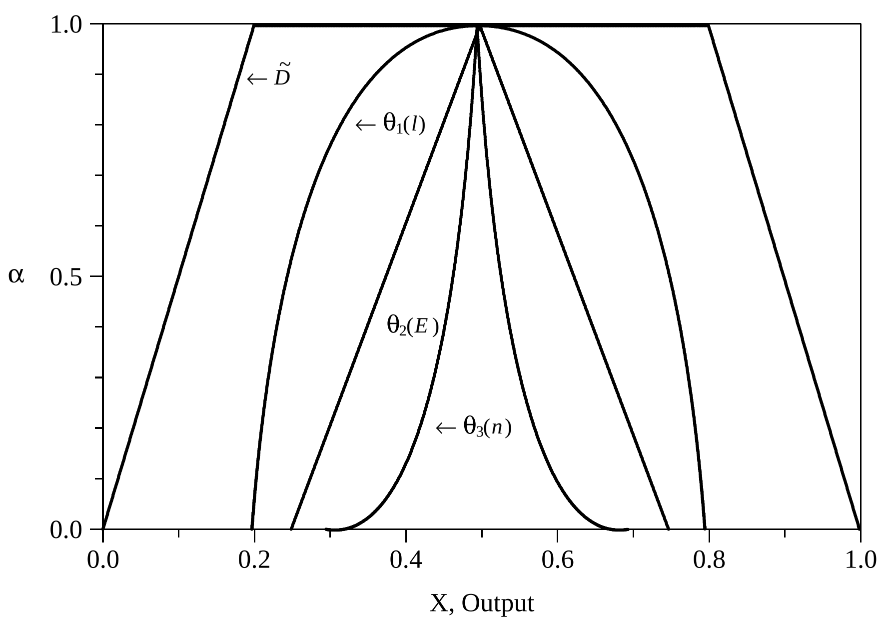

Figure 1.3 Measure of Fuzziness Example

triangular output function, but instead two combined functions raised to the one-half power. Addition and subtraction will preserve the shape of the input function, but the multiplication and division operators both produce nonlinear results. In general, curves of different shape than the input may be expected for the results of fuzzy engineering design computations, however, the result of a fuzzy calculation may be interpreted as previously discussed, whatever its shape.

4.2 A Design Measure

In any design calculation, some input parameters are very strongly coupled to the outputs, and others are nearly independent. A means of determining the relative coupling between imprecise (fuzzy) inputs and outputs can be used to determine which parameters the designer can change and produce little effect on the performance, and which parameters will have the most profound effect on the output. A new measure developed for this purpose, called the γ-level measure, is presented below, along with a well known Measure of Fuzziness.

4.2.1 Measure of Fuzziness. The *Measure of Fuzziness* expresses "the difficulty of deciding which elements belong and which do not belong to a given fuzzy set" [17]. The following entropy function satisfies the conditions required of a measure of fuzziness [12]:

$$d(\tilde{C}) = K \sum_{i=1}^{|X|} \Psi(\alpha_{\tilde{C}}(x_i)), \tag{1.1}$$

where:

$$\Psi(y) = -y \ln(y) - (1 - y) \ln(1 - y),$$

$\alpha_{\tilde{C}}$ is the membership function of the fuzzy set $\tilde{C}$, $\mid X \mid$ is the length of the discretized support (region of non-zero membership) of $\tilde{C}$, and K is an integer.

Unfortunately the entropy function as defined in Equation 1.1 measures values centered on $\alpha_{\tilde{C}} = \frac{1}{2}$. A membership value of one-half has the highest degree of “difficulty of deciding” whether it is a member of the set or not. Memberships close to one (1) are closer to being in the set, memberships close to zero (0) are closer to being out of the set. Thus this measure indicates how much of the membership function is close to one-half. In design, the engineer needs a measure of the values centered on $\alpha_{\tilde{C}} = 1$, indicating the “spread” of the preference function (near 1), not the steepness of the bounding curves (for membership functions). Figure 1.3 illustrates the difference. The *Measure of Fuzziness* will have the same value for membership functions $\tilde{C}_1$ and $\tilde{C}_2$ since these two curves have the same amount of x near $\alpha = 0.5$, however, $\tilde{C}_1$ has much greater imprecision (in the preference function interpretation) than $\tilde{C}_2$ (a much larger amount of x near $\alpha = 1.0$). To avoid this difficulty, a new measure has been developed by the authors.

4.2.2 The γ-Level Measure. We have developed a new measure which we will call the γ-*level measure*. We define this measure in the following manner:

$$D(\tilde{C}) = \sum_{i=1}^{|X|} (e^{\beta(x_i)} - 1)^m, \tag{1.2}$$

where

$$\beta(x_i) = \begin{cases} \frac{\alpha_{\tilde{C}}(x_i)}{\gamma} & \text{if } \alpha_{\tilde{C}} \leq \gamma \\ \frac{2\gamma - \alpha_{\tilde{C}}(x_i)}{\gamma} & \text{if } \alpha_{\tilde{C}} \geq \gamma, \end{cases}$$

$$0 < \gamma \leq 1,$$

and m is an integer such that as m increases, the measure becomes more concentrated for values about $\alpha_{\tilde{C}} = \gamma$. The value of γ may be set so that $D(\tilde{C})$ measures values in the support centered about it. For $\gamma = \frac{1}{2}$ the γ-*level* measure satisfies the conditions for the *Measure of Fuzziness* [12]. We will use $\gamma = 1.0$ and $m = 1.0$ in Equation 1.2.

An outline of the process by which this γ-level measure can be used as a qualitative measure of the relationship between input design parameters and

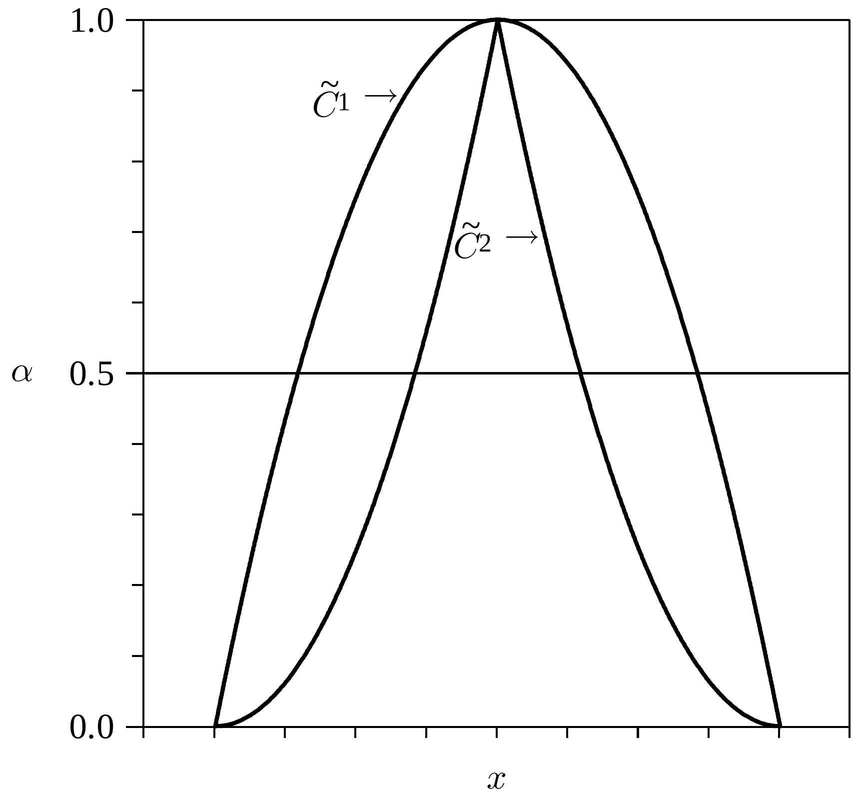

Figure 1.4 γ-Level Measure Application

output performance parameters is shown below. Let $\tilde{C}_1, \ldots, \tilde{C}_N$ be N input, imprecise inputs (Design Parameters), and let $\tilde{D}$ be the output (Performance Parameter) of the computation $y = f(x_1, \ldots, x_N)$.

1. Determine $\tilde{D}$ using the FWA algorithm [16].

2. Let λ_1 and λ_2 be equal to the two x values for which $\alpha_{\tilde{D}} =$ *minimum* on both the left and right extremes of $\tilde{D}$. $\Lambda = [\lambda_1, \lambda_2]$ makes up an interval of the support of $\tilde{D}$.

3. Discretize the interval Λ into n equally spaced steps, such that $| X |= n$ in Equation 1.2.

4. For each input parameter, $\tilde{C}_i,\ i = 1, \ldots, N$, set all other $\tilde{C}_j, i \neq j$, to their nominal crisp value (where $\alpha_{\tilde{C}} = 1$). For $i = 1, \ldots, N$, use the FWA to calculate the output, Θ_i, where the ith fuzzy input remains fuzzy in the calculation, and all others are made crisp as above.

5. Calculate the γ-*level* measure ($\gamma = 1$) for $\tilde{D}$ and all Θ_i.

6. Normalize the $D(\Theta_i)$'s with respect to $D(\tilde{D})$. The result is an ordering of the inputs according to importance (relative measure), giving a qualitative relationship of inputs to the output.

For the engineer who utilizes fuzzy preference functions in the description of design and performance parameters, this new measure provides the ability to determine some information on the coupling between the inputs and outputs of design calculations. The measure can also be used to determine which parameters the designer can change and produce little or no effect on the performance, and which parameters will alter the output the most. Those parameters with small influence may be fixed to the most-desired value by the engineer, resulting in a simplification of the design problem. The coupling information not only includes the rate of change of an output with respect to an input (over the range of acceptable values), but also includes the change in desirability of the parameters. If a small change of an input produces a large change in an output, but a small change in the desirability of the output, the γ-level measure will be small (even though the *sensitivity* of the output to that input is large). Similarly, if a large change of an input produces a small change in an output, but a large change in the desirability of the output, the γ-level measure will be large.

Figure 1.4 illustrates an example application of the γ-*level* measure. $\tilde{D}$ is the output fuzzy set of some performance parameter which is functionally related through a PPE to three imprecise input parameters E, n, and l. The Θ_i sets make up fuzzy outputs for only one fuzzy input parameter (and the other inputs held at their crisp value). After applying the γ-level measure to each of these output sets, the results may be ordered from largest to smallest. In this case, the ordering consists of the following: $D(\tilde{D})$, $D(\Theta_1)$, $D(\Theta_2)$, $D(\Theta_3)$. Normalizing the output measures $D(\Theta_i)$ with respect to $D(\tilde{D})$ shows that $D(\Theta_1)$ is much greater than for $D(\Theta_3)$. The parameter for Θ_3 (n) contributes very little to the preliminary design analysis when compared to the parameter for Θ_1 (l). Thus, the input parameter n might be fixed to its crisp value (where its preference equals one (1)).

5. Example

A simple mechanical design example using the approach described in the previous section is presented here. The problem is to design a mechanical structure, attached to a wall at one end, which will support an overhanging vertical point load. Constraints on the problem include: the distance the load is from the wall; the total width of the supporting structure; and the materials used for the structural elements. One possible configuration, shown in

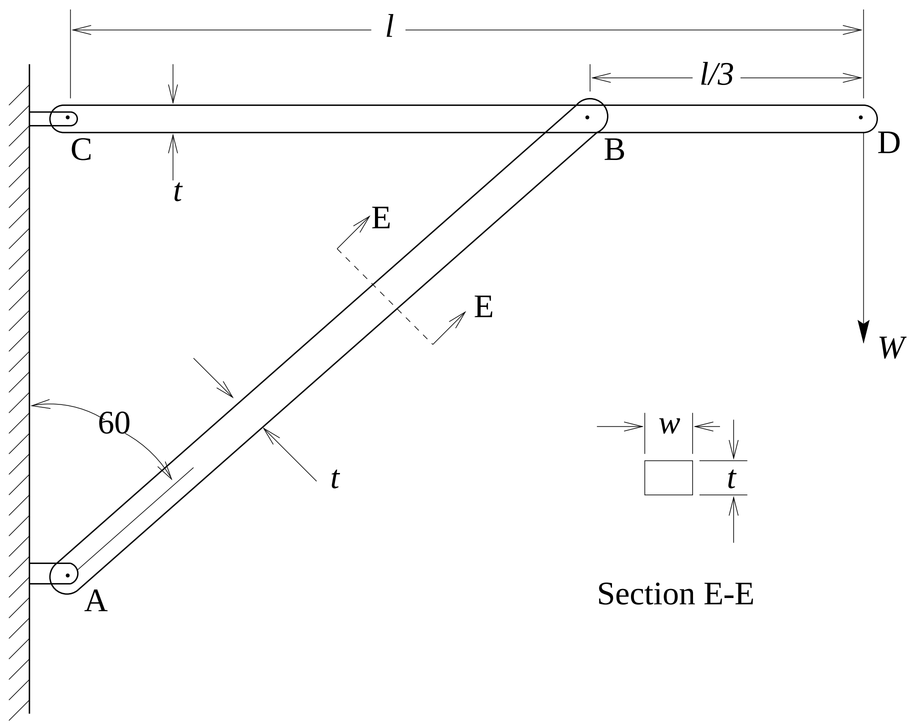

Figure 1.5 Design Problem: Frame Configuration

Figure 1.5, consists of a two-member frame, where the compression member (AB) is attached to the wall at an angle of sixty degrees ($60°$) and both members have rectangular cross-sections. The global design objective is to avoid failure in either component of the frame. Performance expressions may be obtained for the two Functional Requirements by considering beam bending theory[8] for the horizontal member (CD), and buckling for the compression member (AB). The resulting Performance Parameters for the design are the maximum bending stress σ in CD and the column load F_B on AB:

$$\sigma = \frac{2l(W + \frac{W_{CD}}{6})}{w_{CD}t^2}, \tag{1.3}$$

$$F_B = \sqrt{\{\frac{9}{2\sqrt{3}}(W + \frac{W_{CD}}{2} + \frac{W_{AB}}{3})\}^2 + \{\frac{3}{2}(W + \frac{W_{CD}}{2})\}^2}. \tag{1.4}$$

[8]Shear stress in the horizontal member and elastic deformation of the entire frame do not contribute significantly to the problem.

The design parameters for this example are as follows: the applied load W; the length of member CD l; the width of the compression member w_{AB}; and the thickness t. If a different material is used, or a range of material properties are available, E and ρ may also be included as imprecise DPs. The relationships for the weight of the two members, and a constraint on width (w) are:

$$W_{CD} = \rho g w_{CD} t l, \tag{1.5}$$

$$W_{AB} = \rho g w_{AB} t (\frac{4\sqrt{3}l}{9}), \tag{1.6}$$

$$w_{CD} = w_{AB} - 2.5 \text{ cm}. \tag{1.7}$$

5.1 Performance Specifications

In this design, σ must be less than the maximum bending stress before yield. This example assumes that the material has been specified to be steel. Thus, the functional requirement for maximum bending stress is:

$$\sigma \leq \quad \sigma^r \quad = 225 \quad \text{MPa},$$

where the superscript r denotes "requirement."

For simplicity, we will only consider the Functional Requirement on bending stress σ in member CD in the example shown here. Future publications will demonstrate the technique with examples containing more realistic design complexities, and comparisons of design alternatives. In the actual design of a frame, such as the one used in this example, buckling of member AB would need to be included in the analysis.

5.2 Input Design Parameters

The designer specifies the input parameters as preference functions according to the approach outlined previously. Here the parameters that need to be selected as part of the design process are: W, w_{AB}, l, and t. In this example, the subjective knowledge, experience, and desires of the engineer are used to imprecisely determine these input parameters. For example, the applied vertical load W is constrained by a maximum load that a proposed configuration is expected to withstand without failure. There also exists some latitude (due to other design considerations) by which this design load may be decreased such that the design is still *satisfactory*, but less *desirable* due to the decrease. Thus, the input parameter W is imprecisely defined in a range of possible values where the desirability decreases from the maximum value in the range to the

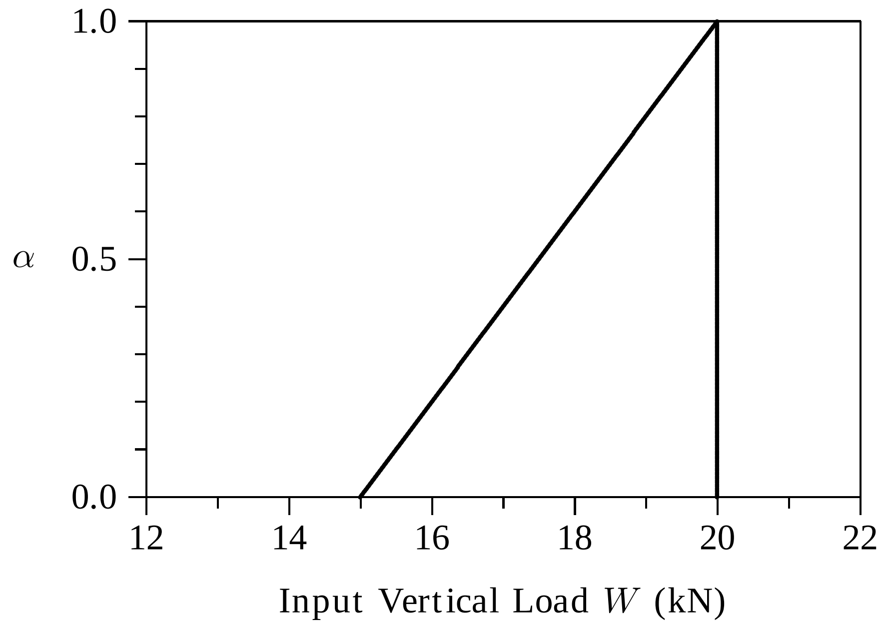

Figure 1.6 Input Parameter: W

Table 1.1 Example Problem: Fuzzy Design Parameter Data.

DPs (units)	$\alpha = 0$	$\alpha = 1$	$\alpha = 0$
W (kN)	15.0	20.0	20.0
w_{AB} (m)	0.04	0.07	0.13
l (m)	3.0	4.0	4.0
t (m)	0.04	0.06	0.10

minimum value shown in Figure 1.6. For this design problem, the maximum design load is 20 kN, which corresponds to the upper endpoint of the range. W may not be less than 15 kN, corresponding to the lower endpoint.

The remaining design parameters may be specified in a similar manner. Because each input set for this problem is in the form of a triangular function (naturally more complex functions could have been used), the fuzzy DPs can be represented by three-values: left-extreme value for preference of zero, peak value for preference of one, and right-extreme value for preference of zero. Table 1.1 provides the necessary data for constructing the preference functions for the entire set of design parameters, and Table 1.2 lists other constant data used in this example design problem.

Table 1.2 Design Example: “Constant” Data.

Constant (units)	Value
E (GPa)	207.0
ρ (kg/m^3)	7830.0
g (m/sec^2)	9.81

5.3 Output Performance Parameters

The fuzzy output for the performance parameter σ may be obtained by use of Equation 1.3 and the application of the FWA algorithm described earlier. The results are shown in Figure 1.7.

After the calculations have been performed to produce the output, the next step is to compare the output set with the performance criterion. Figure 1.7 shows the imprecise performance parameter results for the maximum bending stress of member CD (Equation 1.3). The output at the peak of $\tilde{\sigma}_{(\,at\,\alpha=1)}$ is equal to 994 MPa. This peak output does not satisfy the functional requirement $\sigma^r = 225$ MPa. To satisfy the requirement σ^r, the input parameters must deviate from the peak (most desired) values. At least one design parameter must decrease in preference, to the left of the peak, by between 0.5 and 0.6 ($\tilde{\sigma}_{(\,at\,\alpha=0.5)} = 259$ MPa and $\tilde{\sigma}_{(\,at\,\alpha=0.4)} = 206$ MPa), in order to meet the requirement on σ. (If a factor of safety is desired, a further decrease in preference will be required.)

The backward path of the imprecise calculation may be applied at this point to determine the effect of changing the preference of any one input design parameter. Data from the solution for $\tilde{\sigma}$ shows that the input parameters of W and l could be decreased to the left of their peak values (at $\alpha = 1$) so that σ will meet its Functional Requirement, whereas the inputs w_{AB} and t must be decreased to the right of their peak values. This result cannot easily be obtained from inspection of the governing equation since w_{AB} and t appear in the denominator *and* the numerator of Equation 1.3 when combined with Equation 1.6. While this same result could be obtained through calculation of partial derivatives of the output with respect to each of the inputs, it was instead found by use of stored values calculated during the solution of the (imprecise) performance parameter by use of the authors’ implementation of the FWA algorithm. No additional calculations were required. These results show that σ^r may be satisfied by the frame configuration, but only with a large change in preference of the DPs from the most desired input peak values. If other PPs were part of this design analysis (in addition to σ), care must be taken when adjusting the DPs (which are coupled to σ) to obtain acceptable performance values in those other PPs. A small adjustment of one DP to obtain

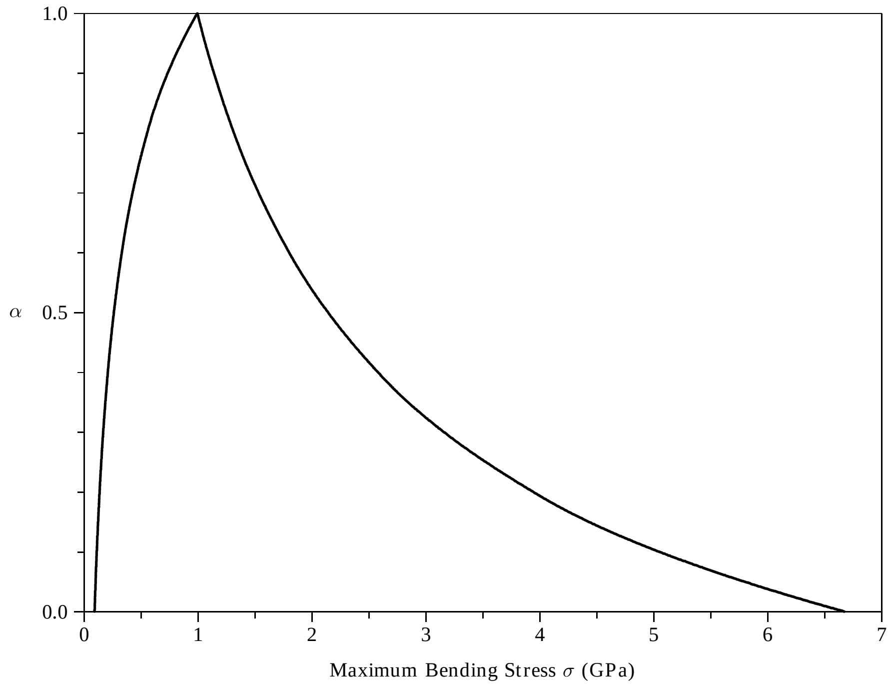

Figure 1.7 Output Parameter: Maximum Bending Stress σ

a satisfactory performance value for one PP may adversely affect a different PP. The γ-level measure may be used to determine the magnitude of the coupling between parameters, and permit the designer to minimize the adverse effect of DP adjustment.

5.4 Applying the γ-Level Measure

The γ-*level* measure, as described earlier, may be used to provide the engineer with qualitative information on the relationship between input parameters in the design. When a design parameter has the greatest qualitative importance for a given performance parameter, the numerical measure produces a normalized value of one (1). As the measure decreases in value, the corresponding input has little affect in determining the performance, meaning that even a large change in the design parameter (decrease in preference/desirability) produces a small change in output. The output of the γ-level measure is loosely analogous to sensitivity, but applies to the imprecise parameters, and represents the entire range of the parameters, not a single operating point. Moreover, this sensitivity is weighted by the designer's desires, as identified in the input parameters' preference functions.

Chapter 2

TRADE-OFF STRATEGIES IN ENGINEERING DESIGN

Kevin N. Otto and Erik K. Antonsson

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Abstract

A formal method to allow designers to explicitly make trade-off decisions is presented. The methodology can be used when an engineer wishes to rate the design by the weakest aspect, or by cooperatively considering the overall performance, or a combination of these *strategies*. The design problem is formulated with preference rankings, similar to a utility theory or fuzzy sets approach. This approach separates the design *trade-off strategy* from the performance expressions. The details of the mathematical formulation are presented and discussed, along with two design examples: one from the preliminary design domain, and one from the parameter design domain.

1. Introduction

For a robust automation, design decision making methods need to be advanced to represent and manipulate a design's different concerns and uncertainties. This development is crucial, since the preliminary decision making process of any design cycle has the greatest effect on overall cost [3, 6, 14]. In a design decision making process, engineers must trade-off widely differing concepts to realize a result which maximizes their *overall* preference for a design. These concepts are usually incommensurate: for example, they could be as different as cost, degree of safety, degree of manufacturability, or amount of various performance indicators: stress, heat dissipation, etc. This paper presents *design metrics* to represent and manipulate these concerns. These metrics take the form of formal design strategies to permit the designer to trade-off one (or more) parameter(s) against others, and to implement an overall approach to

Table 1.3 γ-*Level* Measure Results: Frame Configuration.

Performance Parameter: σ	
DPs	γ-Level Measure
W	0.129
w_{AB}	1.000
l	0.129
t	0.910

Table 1.3 lists that γ-level results for the frame configuration. Analyzing the γ-level measures for σ, the input parameters t and w_{AB} are obviously the most important parameters which must be changed from their peak preference values in order to meet the Functional Requirement. W and l contribute very little when compared with t and w_{AB}. Thus, W and l may be set to their representative or desired values, resulting in a simplification of the frame design.

When more than one performance parameter is used to describe a design, and when there exist imprecise performance specifications, the γ-level measure may also be used to determine the coupling of the design parameters, as well as the importance of the DPs for maximization or minimization. A subsequent publication [34] demonstrates the use of the γ-level measure for these purposes.

5.5 Discussion

This example shows how imprecision in the design parameters can be handled, how the designer can move forward and backward through the design calculations to determine interactions of the DPs for the performance parameters, and how the γ-level measure may be used to determine information relative to the importance of the design parameters. Conclusions may be drawn from the results as to the ability of the configuration to satisfactorily meet the performance criteria (including consideration of the designer's desires), and if the configuration should be carried on to the next stage in the design process.

This design problem has been a simple example, with none of the complications that normally beset engineering designers, such as alternative configurations or technologies to compare; simultaneous analysis of multiple performance parameters; poor knowledge of the relationships between functional requirements and design parameters; and intangible requirements and specifications, such as aesthetics. The example does, however, demonstrate an enhanced capability for the designer to determine acceptable DP values, or ranges, simply and quickly by use of imprecise computations. Examples, which are considerably more complex in terms of comparing different design alternatives and in terms of including uncertainty effects, in addition to imprecision, will be presented in later publications.

6. Conclusions

One of the goals of the research reported here is to increase the amount of information available to engineering designers regarding the performance of design alternatives, over that available with conventional design analyses. The effect will be greater, the earlier in the design process the information is made available. Ultimately the most important (and costly) decisions in the design cycle are made in the very early stages. Engineering designs are typically represented imprecisely at the early, conceptual (preliminary) stage of design. Computational tools for this area of the design process are rare, largely because of the scarcity of techniques capable of handling imprecise data. One of the central hypotheses of the research reported here is that representing and manipulating imprecise descriptions of design artifacts during the preliminary phase (and hence increasing the information available to the designer) will enable design decisions to be made with greater confidence and reduced risk, and that this will ultimately result in better designs.

The technique and implementation reported here represents a new application (to the engineering design process) of a powerful approach to represent and manipulate imprecise engineering design data. The example shown here demonstrates that it can be applied to engineering design problems, and provide the ability to perform design calculations on a variety of imprecise parameters. The (correspondingly) imprecise-calculation results provide more information to the designer than conventional single-valued design analyses. The technique used is a modified implementation of the Fuzzy Weighted Average operating on fuzzy representations of design parameters. Preference functions are used here to represent the designer's desire to use particular values for these parameters.

Additional useful information that this method can provide, through the use of the γ-*level measure*, is the coupling between imprecise representations of design parameters (inputs) and the performance parameter results. This coupling information can be used to focus the engineer's resources on those aspects of the design problem with the largest effect on the resulting performance.

Subsequent publications will compare this method with probability analysis, as well as develop additional examples.

7. Acknowledgements

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A Appendix: Fuzzy Arithmetic

A.1 Operations for Fuzzy Numbers

Zadeh [40] introduced the extension principle as one of the fundamental ideas of fuzzy set theory. Using this idea, classical mathematics may be extended to the fuzzy domain. Specifically, let the fuzzy sets (or fuzzy numbers) $\tilde{C}_1, \tilde{C}_2, \ldots, \tilde{C}_N$ be defined in the universes $X_1, X_2, \ldots, X_N$, respectively. The mapping from $X_1 \times \cdots \times X_N$ to a universe Y may be defined as a function f such that $y = f(x_1, \ldots, x_N)$. The extension principle then gives that a fuzzy set (or number) $\tilde{D}$ on Y may be induced from $\tilde{C}_1, \tilde{C}_2, \ldots, \tilde{C}_N$ through f such that the resulting membership function is:

$$\mu_{\tilde{D}}(y) = sup_{x_1,\cdots,x_N} min(\mu_{\tilde{C}_1}, \cdots, \mu_{\tilde{C}_N})$$

where $y = f(x_1, \ldots, x_N)$. The ordinary binary operations may then become extended operations in the fuzzy domain (extended addition, extended multiplication, etc.).

Even though the development of these extended operations may be completed rigorously using the extension principle approach, interval operations for α-level sets will be presented instead, as this method is used in the computer implementation.

DEFINITIONS. Some aspects of Fuzzy arithmetic are presented below based on the material in Kaufmann and Gupta [23].

a. α **- Level-Set** The discussion of a fuzzy number with respect to its membership values leads directly to the idea of defining crisp sets (or intervals of confidence) for each level, α. Specifically, an α *-level-set,* C_α, is a crisp set taken from the fuzzy set $\tilde{C}$ such that

$$C_\alpha = \{x | \mu_{\tilde{C}}(x) \geq \alpha\}, \alpha \in [0,1]. \tag{A.1}$$

b. Addition and Subtraction Two fuzzy numbers, $\tilde{E}$ and $\tilde{F}$, may be summed or subtracted level by level ($\alpha \in [0,1]$) according to the following formulas:[9]

$$\begin{aligned} E_\alpha \oplus F_\alpha &= [e_l^\alpha + f_l^\alpha, e_r^\alpha + f_r^\alpha], \\ E_\alpha \ominus F_\alpha &= [e_l^\alpha - f_r^\alpha, e_r^\alpha - f_l^\alpha] \end{aligned}$$

where

$$E_\alpha = [e_l^\alpha, e_r^\alpha], F_\alpha = [f_l^\alpha, f_r^\alpha].$$

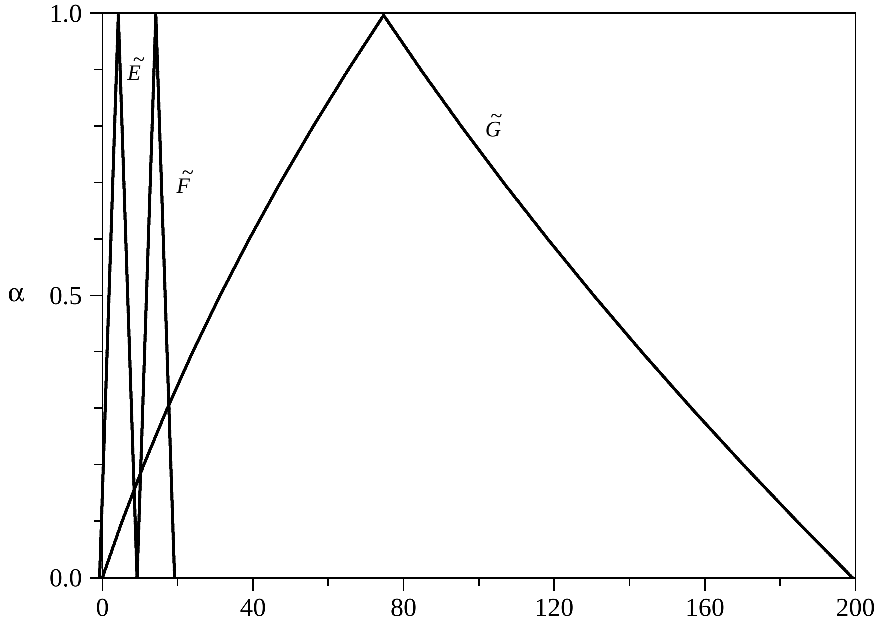

Figure A.1 Multiplication of $\tilde{E} \times \tilde{F} = \tilde{G}$.

c. Multiplication and Division Similarly, two fuzzy numbers, $\tilde{E}$ and $\tilde{F}$, may be multiplied or divided [9] (considering $\Re^+$ only here)

$$\begin{aligned} E_\alpha \odot F_\alpha &= [e_l^\alpha \cdot f_l^\alpha, e_r^\alpha \cdot f_r^\alpha], \\ E_\alpha \oslash F_\alpha &= [e_l^\alpha / f_r^\alpha, e_r^\alpha / f_l^\alpha]. \end{aligned}$$

d. Example of Fuzzy Multiplication For simplicity, consider the fuzzy numbers, $\tilde{E}$ and $\tilde{F}$ as shown in Figure A.1.

The membership functions are given by

$$\begin{aligned} \mu_{\tilde{E}} &= \frac{1}{5}x, \ 0 \le x \le 5, \\ &= -\frac{1}{5}x + 2, \ 5 \le x \le 10, \\ &= 0, \ otherwise, \end{aligned} \tag{A.2}$$

$$\mu_{\tilde{F}} = \frac{1}{5}x - 2, \ 10 \le x \le 15,$$

[9]Although nonfuzzy operations are easily extended to their fuzzy counterparts, it must be noted that certain properties of the classical binary operations are lost in the process [23].

$$\begin{aligned} &= -\frac{1}{5}x + 4,\ 15 \leq x \leq 20, \\ &= 0,\ otherwise. \end{aligned} \tag{A.3}$$

In terms of the levels of presumption, α, Equation A.2 becomes

$$\alpha = \frac{e_l^\alpha}{5} \tag{A.4}$$

and

$$\alpha = -\frac{e_r^\alpha}{5} + 2. \tag{A.5}$$

Similarly, equation A.3 leads to

$$\alpha = \frac{f_l^\alpha}{5} - 2 \tag{A.6}$$

and

$$\alpha = -\frac{f_r^\alpha}{5} + 4. \tag{A.7}$$

Combining the results, we end up with expressions for E_α and F_α:

$$E_\alpha = [5\alpha, -5\alpha + 10]$$

and

$$F_\alpha = [5\alpha + 10, -5\alpha + 20].$$

Multiplying leads to

$$\begin{aligned} G_\alpha &= [(5\alpha)(5\alpha + 10), \\ &\quad\ (-5\alpha + 10)(-5\alpha + 20)] \\ &= [25\alpha^2 + 50\alpha, 25\alpha^2 - 150\alpha + 200], \end{aligned} \tag{A.8}$$

where $G_\alpha = E_\alpha \odot F_\alpha$. Solving the quadratics on each end of the interval and retaining only two roots for α, we have that

$$x_l = 25\alpha^2 + 50\alpha \implies \alpha_l = -1 + \sqrt{\frac{x_l}{25} + 1},$$
$$x_r = 25\alpha^2 - 150\alpha + 200 \implies \alpha_r = 3 - \sqrt{\frac{x_r}{25} + 1},$$

where

$$\begin{aligned} 0 \leq\ & x_l \leq 75, \\ 75 \leq\ & x_r \leq 200. \end{aligned}$$

References

[1] BALDWIN, J. F. A model of fuzzy reasoning and fuzzy logic. Research em/fs10, University of Bristol, Eng. Math. Dept., 1978.

[2] BALDWIN, J. F., AND GUILD, N. C. F. Feasible algorithms for approximate reasoning using a fuzzy logic. Research em/fs8, University of Bristol, Eng. Math. Dept., 1978.

[3] BALDWIN, J. F., AND GUILD, N. C. F. A model of multicriteria decision making using fuzzy logic. Research em/fs12, University of Bristol, Eng. Math. Dept., 1978.

[4] BALDWIN, J. F., AND PILSWORTH, B. W. Axiomatic approach to implication for approximate reasoning using fuzzy logic. Research em/fs6, University of Bristol, Eng. Math. Dept., 1978.

[5] BECK, J. V., AND ARNOLD, K. J. *Parameter Estimation in Engineering and Science*. John Wiley and Sons, New York, 1977.

[6] BLOCKLEY, D. I. Predicting the likelihood of structural accidents. In *Proceedings Institute Civil Engineers* (Dec. 1975), pp. 659–668.

[7] BLOCKLEY, D. I. The role of fuzzy sets in civil engineering. *Fuzzy Sets and Systems 2* (1979), 267–278.

[8] BLOCKLEY, D. I. *The Nature of Structural Design and Safety*. Ellis Norwood Limited, Chichester, 1980.

[9] BROWN, C. B. A fuzzy safety measure. *ASCE Journal of the Engineering Mechanics Division* (Oct. 1980), 855–872.

[10] BYRNE, D. M., AND TAGUCHI, S. The Taguchi approach to parameter design. In *Quality Congress Transaction – Anaheim* (May 1986), ASQC, pp. 168–177.

[11] COX, R. T. *The Algebra of Probable Inference*. Johns-Hopkins University Press, Baltimore, MD, 1961.

[12] DELUCA, A., AND TERMINI, S. A definition of a nonprobabilistic entropy in the setting of fuzzy set theory. *Information and Control 20* (1972).

[13] DIAZ, A. R. Fuzzy set based models in design optimization. In *Advances in Design Automation - 1988* (New York, NY, Sept. 1988), S. S. Rao, Ed., vol. DE-14, ASME, pp. 477–485.

[14] DIAZ, A. R. Modeling of computer-based decisions and optimization in the design process. In *Design Theory '88* (RPI, Troy, NY, June 1988), S. L. Newsome and W. R. Spillers, Eds., NSF, pp. 286–295. 1988 NSF Grantee Workshop on Design Theory and Methodology.

[15] DIXON, J. R., DUFFEY, M. R., IRANI, R. K., MEUNIER, K. L., AND ORELUP, M. F. A proposed taxonomy of mechanical design problems.

In *Computers in Engineering 1988* (New York, NY, June 1988), V. A. Tipnis and E. M. Patton, Eds., ASME, pp. 41–46.

[16] DONG, W. M., AND WONG, F. S. Fuzzy weighted averages and implementation of the extension principle. *Fuzzy Sets and Systems 21*, 2 (Feb. 1987), 183–199.

[17] DUBOIS, D., AND PRADE, H. *Fuzzy Sets and Systems: Theory and Applications*. Academic Press, New York, 1980.

[18] FUTURA, H., ET AL. Optimum aseismic design using fuzzy sets and empirical connectives. Communication with Dr. Yao Purdue, 1986.

[19] FUTURA, H., AND NARUHITO, S. Fuzzy importance in fault tree analysis. *Fuzzy Sets and Systems 12* (1984), 205–213.

[20] GOGUEN, J. A. L-fuzzy sets. *Journal Mathematical Analysis and Applications 18* (1967), 145–174.

[21] GUPTA, M. M., SARIDIS, G. N., AND GAINES, B. R., Eds. *Fuzzy Automata and Decision Processes*. North-Holland Publishers, Amsterdam, 1977.

[22] HAUGEN, E. B. *Probabilistic Mechanical Design*. John Wiley and Sons, New York, NY, 1980.

[23] KAUFMANN, A., AND GUPTA, M. M. *Introduction to Fuzzy Arithmetic: Theory and Applications*. Electrical/Computer Science and Engineering Series. Van Nostrand Reinhold Company, New York, 1985.

[24] KUBIC, W. L., AND STEIN, F. P. A theory of design reliability using probability and fuzzy sets. *AIChE Journal 34*, 4 (Apr. 1988), 583–601.

[25] LEUNG, K. S., AND LAM, W. Fuzzy concepts in expert systems. *Computer 21*, 9 (Sept. 1988), 43–56.

[26] MEUNIER, K. L., AND DIXON, J. R. Iterative respecification: A computational model for hierarchical mechanical system design. In *Computers in Engineering 1988* (New York, NY, June 1988), V. A. Tipnis and E. M. Patton, Eds., ASME, pp. 25–32.

[27] MOORE, R. E. *Methods and Applications of Interval Analysis*. Society for Industrial and Applied Mathematics, Philadelphia, PA, 1979.

[28] PAHL, G., AND BEITZ, W. *Engineering Design*. The Design Council, Springer-Verlag, New York, NY, 1984.

[29] RAO, S. S. Description and optimum design of fuzzy mechanical systems. *ASME Journal of Mechanisms, Transmissions, and Automation in Design* (1986), 1–7.

[30] REKLAITIS, G. V., RAVINDRAN, A., AND RAGSDELL, K. M. *Engineering Optimization, Methods and Applications*. Wiley, New York, NY, 1983.

[31] SIDDALL, J. N. *Probabilistic Engineering Design; Principles and Applications*. Marcel Dekker, New York, NY, 1983.

[32] TAGUCHI, G. *Introduction to Quality Engineering*. Asian Productivity Organization, Unipub, White Plains, NY, 1986.

[33] WOOD, K. L., AND ANTONSSON, E. K. A Fuzzy Sets Approach to Computational Tools for Preliminary Engineering Design. In *Advances in Design Automation - 1987, Volume One: Design Methods, Computer Graphics, and Expert Systems – DE-Vol. 10-1* (New York, Sept. 1987), S. S. Rao, Ed., ASME, pp. 263–271.

[34] WOOD, K. L., AND ANTONSSON, E. K. Computations with Imprecise Parameters in Engineering Design: Application and Example. *ASME Journal of Mechanisms, Transmissions, and Automation in Design* (June 1988).

[35] WOOD, K. L., ANTONSSON, E. K., AND BECK, J. L. Comparing Fuzzy and Probability Calculus for Representing Imprecision in Preliminary Engineering Design. In *Design Theory and Methodology - 1989* (New York, Sept. 1989), W. P. Seering, Ed., ASME, pp. 99–105. Presented at the First International ASME *Design Theory and Methodology Conference* (DTM '89), September 18-19, 1989, Montréal, Canada.

[36] YAGER, R. Satisfaction and fuzzy decision functions. In *Theory and Applications to Policy Analysis and Information Systems*, P. P. Wang and S. K. Chang, Eds. Plenum Press, New York, 1980.

[37] YAO, J. T. P. Application of fuzzy sets in fatigue and fracture reliability. Communication Dr. Housner Caltech, 1979.

[38] ZADEH, L. A. Fuzzy sets. *Information and Control 8* (1965), 338–353.

[39] ZADEH, L. A. Outline of a new approach to the analysis of complex systems and decision processes. *IEEE Transactions on Systems, Man, and Cybernetics 3*, 1 (1973), 29–44.

[40] ZADEH, L. A. The concept of a linguistic variable and its application to approximate reasoning – I. *Information Sciences 8* (1975), 199–249.

[41] ZADEH, L. A. Fuzzy logic and approximate reasoning. *Synthese 30* (1975), 407–428.

[42] ZADEH, L. A., FU, K. S., TANAKA, K., AND SHIMURA, M., Eds. *Fuzzy Sets and their Applications to Cognitive and Decision Processes*. Academic Press, New York, NY, 1975.

[43] ZIMMERMANN, H.-J. *Fuzzy Set Theory - and Its Applications*. Management Science/Operations Research. Kluwer-Nijhoff Publishing, Boston, MA, 1985.

design trade-offs: either conservative, aggressive, or a combination of the two. The terms: conservative and aggressive are defined in the next section. A formal mathematics will be presented to allow designers to explicitly make such trade-off decisions.

This approach can help designers observe, justify, and direct their decision making processes. In any design scenario, there are multiple goals which need to be achieved [32]. Designers restrict and choose parameter values based on a combination of these concerns. This work will permit designers to directly specify a design goal *trade-off strategy* to specify how to trade-off different design goals, and thus allow observation, justification, and recording of decisions made.

1.1 Design Trade-Off Strategies

Design trade-off strategies are always present in the design process. For example, in the design of a spacecraft solar power cell, a strategy might be to trade-off the performance gains of some goals (like available power output) to increase the level of other aspects deemed marginal (like stress), to ensure the cell will always function. We will use the term "conservative" design strategy, or also "non-cooperating" or "non-compensatory" strategy, to describe a design strategy of trading off to improve the lower performing goals. A design's overall preference will be based on the attribute with the lowest preference. Other attributes with higher preference do not compensate for the attribute(s) with lower preference.

On the other hand, a designer may wish to slightly reduce some of the weaker goals in a design if large gains can be made in the other goals, which would more than compensate for the slight loss. For example, in the design of a sports car, the designer might reduce the safety margin of some variables (like stress) to gain in performance of other variables (like horsepower), even though the stress may already be quite high. We will use the term "aggressive" design strategy, or "cooperating" or "compensatory" strategy, to describe a design strategy of always cooperatively trading off the goals to improve the design. Obviously, hybrid forms of these approaches exist and are used, where some portions of a device are designed conservatively, and other portions aggressively. We will use the terms: conservative and aggressive design strategies throughout this paper.

1.2 Designer Preferences

A method for representing and manipulating uncertainties in preliminary design, to formalize the process of making these trade-off decisions, has been introduced and developed by Wood and Antonsson, [34, 35, 36], called the *method of imprecision*. It is used to compare and contrast different objectives within a proposed design and among different design alternatives. The intent is to determine proposed candidates' feasibility and limitations, even with uncertainty in the variables used.

This paper will incorporate the concepts of imprecision, and a brief review will be presented here. Imprecision indicates a designer's uncertainty in selecting a value for a parameter, in the form of a zero to one rank. If a designer prefers to use a value for a parameter, it will be ranked high, near one. On the other hand, if a designer does not prefer a parameter's value, it will be ranked low, near zero. Depending on the domain, the preferences are specified on actual physical variables (such as model dimensions, or calculable quantities such as stress), or, in the preliminary design domain (for example), on features in the design. The parameters (on which the preferences are placed) depend on the domain of the design. The discussion presented in the paper will be appropriate for any stage of the design process, from the early planning stages through production. Examples from both the preliminary and a latter stage of the design process will be presented. In all cases, the scheme is to place preferences on the candidate model features, with the aim of determining an overall preference for each candidate model to determine which to pursue. The reader is referred to [20, 21, 22, 33, 34, 35, 36] for a discussion on how to specify preferences; this paper will not discuss this aspect of the problem. Rather, this paper will focus on the task of combining these individual preferences (of the different parameters' values) to obtain a preference rank for the vector of design parameter values.

The method of imprecision as developed to date used the mathematics of fuzzy sets to perform this combination [34]. The primary objective of this paper is to introduce different methods for combining these preferences, and to show that these different methods effectively represent different design strategies the designer may adopt.

1.3 Related Work

There has been some progress in the development of optimization methods with preference functions. Diaz [7, 8, 9], Rao [20, 21, 22], and Sakawa and Yano [24, 25, 26] have advanced the use of "fuzzy goals" where the objective functions and constraints consist of fuzzy preference functions on different performance parameters. This paper will illustrate the implications on the choice of the form of the fuzzy mathematics used. That is, if conventional fuzzy mathematics is used, it will be demonstrated that this combination corresponds to using a conservative design strategy.

Parallel work by Dubois and Prade considers trading-off multiple goals with preference functions [11]. In [11], they review connectives which could be used to combine goals. However, they provide no compelling reason for selecting any of the possible candidates (as is done here: by specifying a design trade-off strategy). Also, the candidates they propose for combining preferences on incommensurate goals have too many restrictions for the purposes of engineering design. Their developments are derived from the realm of uncertain logic. As such, they are primarily interested in uncertain versions of conjunction and disjunction. In classical logic, these operations are commutative and associative, which Dubois and Prade assume for all of their developments. In general, engineering design connectives should not be commutative. It makes no sense, for example, to require one goal's weighting to be applicable to a different goal, which is as commutativity requires. Yager, in [37], introduces some of the mathematics that we present here, in the context of selecting from a finite set of alternatives. The relationship of his developments to ours will be discussed below.

An alternative to the use of imprecision is utility theory [15, 29]. Utility theory trades off goals by specifying utility curves on each goal, and then maximizes overall utility by aggressively combining the goals. Doing so eliminates the conservative design strategy from consideration, which could be the design strategy of choice in some cases.

In domains involving goals with explicit expressions, one could formulate the design problem using an optimization methodology [17]. Such single objective formulations have been argued to be constraining for actual design problems [32]. Instead, multi-objective function formulations could be used [12, 28, 32]. The methodology presented here is compatible with these multi-objective function algorithms, in that one can use them to solve the formulations presented here, when the domain has sufficient formalization (performance parameter equations) [28]. The focus of this paper is on formally specifying the multi-criteria objective function, not methods for finding its global peak.

In the preliminary design domain, the degree of specification of candidate models is usually incomplete. The method of imprecision can still be used, however, to determine which candidate models offer the most promise. Traditional methods used in this stage of design are matrix methods [2]. Current advanced versions are QFD [1, 13] and Pugh's method [18]. The basis for the combination procedure of such matrix methods will be discussed in an example below.

2. Design Imprecision

In the method of imprecision, designer preferences (μ) are represented on a scale from zero to one, with preferences placed individually on each parameter. We shall denote *design parameters* (DP) as those parameters whose values are to be determined as the objective of the current design process, *e.g.*, lengths, materials, etc. We shall denote *performance parameters* (PP) as those parameters whose values depend on the design parameter values, and which give indications of performance, *e.g.*, stress, horsepower, etc. This paper presents a method to determine an overall rank for a *vector* of design parameters ($\overrightarrow{DP}$) given the individual preference information. The preference information is specified both on the design parameters and on the various performance parameters in the form of specifications or requirements.

Section 2.1 will present axioms governing any preference combination metric. Sections 2.2 through 2.5 will discuss different functions to use as global design metrics, and their relation to design strategies. The discussion will be in the context of parametric design; however, the developments apply to any design stage, as the examples will demonstrate.

2.1 Global Preferences

The objective of this paper is to formalize an approach to define and identify a "best" engineering design. However, the various parameters in the design usually reflect incommensurate concepts, and therefore should be combined using a construct that they share: designer preference. This means that preference information on the design parameters (DPs) and requirement preferences on the the performance parameters (PPs) must be combined into an overall preference rating ($\mathcal{P}$) for that design parameter set ($\overrightarrow{DP}$). Therefore, to combine designer preferences, a global design connective, or metric, is defined, expressed as a function of the known preferences of the goals:

$$\mu(\overrightarrow{DP}) = \mathcal{P}\left[\mu(DP_1), ..., \mu(DP_n), \mu(PP_1(\overrightarrow{DP})), ..., \mu(PP_q(\overrightarrow{DP}))\right] \quad (2.1)$$

This statement implies that the choice of a design parameter set is based on combining (in a yet to be determined fashion) the preferences of the design parameters and performance parameters. It is a formalization of the idea that designers combine incommensurate parameters based on how much each parameter satisfies them.

The design problem is then to find the design parameter set which maximizes the overall preference:[1]

$$\mu(\overrightarrow{DP}^*) = \max_{DPS}\left[\mathcal{P}\left[\mu(DP_1), ..., \mu(DP_n), \mu(PP_1(\overrightarrow{DP})), ..., \mu(PP_q(\overrightarrow{DP}))\right]\right] \quad (2.2)$$

The most preferred design parameter set $\overrightarrow{DP}^*$ is the one which maximizes $\mathcal{P}$ across the design parameter space (DPS), which is the set of all design

[1]Throughout the paper *max* is used to to mean *sup* or *least upper bound*, and *min* is used to mean *inf* or *greatest lower bound*.

Table 2.1 Overall Preference Resolution Axioms.

$\mathcal{P}(0, \mu_{j_1}, \ldots, \mu_{j_{n+q-1}}) = 0 \quad \mathcal{P}(1, ..., 1) = 1$	(boundary conditions)
$\forall j, \mathcal{P}(\mu_1, ..., \mu_j, ...) \leq \mathcal{P}(\mu_1, ..., \mu'_j, ...)$ iff $\mu_j \leq \mu'_j$	(monotonicity)
$\forall j, \mathcal{P}(\mu_1, ..., \mu_j, ...) = \lim_{\mu'_j \to \mu_j} \mathcal{P}(\mu_1, ..., \mu'_j, ...)$	(continuity)
$\mathcal{P}(\mu, ..., \mu) = \mu$	(idempotentcy)

parameter combinations. This is a formalization of the idea that designers choose the design parameter set which maximizes their overall preference.

The choice of method to combine preferences ($\mathcal{P}$) is determined by the design strategy. For this reason, the minimum function ($\mathcal{P} = \mathit{min}$) applied to all of the preferences is not automatically acceptable, as pointed out in [36] (the *min* is commonly used in fuzzy mathematics to combine information). This development shall discuss when different functions are appropriate to use as $\mathcal{P}$. First, a set of axioms with which all proposed resolving functions (to use as connectives, or metrics) must be consistent (at least those which operate with preferences) is introduced in Table 2.1. Then example functions will be given, and it will be shown when each is appropriate for different problems.

The first axiom in Table 2.1 is a boundary condition requirement. It states that if the designer prefers absolutely all of the goals (preference $\mu = 1$), then the design will also be preferred absolutely. Similarly, if the designer has no preference for the value of any one of the goals (preference $\mu = 0$), then the overall design (as a set of goals) will also not be preferred. Weighted sum multi-criteria objective formulations do not conform to this axiom, as discussed by Biegel and Pecht in [5]. Vincent [32] also presents this argument in the case of (non-preference) multi-objective function optimization.

The second axiom is a monotonicity requirement. It states that if an individual goal's preference is raised or lowered, then the design's overall preference is raised and lowered in the same direction, if it changes at all. Hence, in a multi-component design, if one component's preference is increased with the other components' preference remaining the same, then the design's overall preference does not go down. The axiom does not mean the preferences or the performance parameters must be monotonic. If either the preferences specified or the performance parameters used are non-monotonic, then this axiom ensures that $\mathcal{P}$ will monotonically propagate the non-monotonicities.

The third axiom is a continuity requirement. It states that as an individual goal's preference is changed slightly, then the overall preference for the design will change at most slightly. It does not mean the preference for any goal

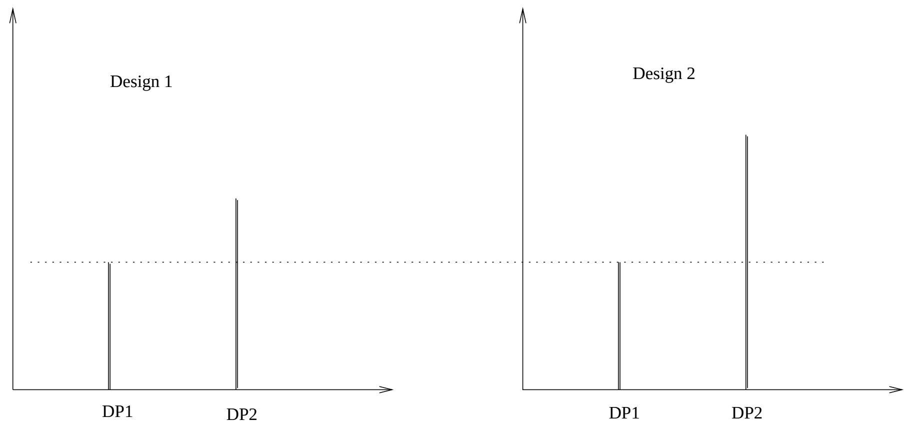

Figure 2.1 Two designs which have the same low preference for a component DP_1, and different (higher) preference for a different component DP_2.

must be continuous. It states only that as any individual goal's preference is continuously changed, the method of combining all the goals' preferences (that is, $\mathcal{P}$) will induce only continuous changes in the overall preference, if it changes at all. If some parameters have preference discontinuities, the method of combining them will continuously propagate the discontinuities. Therefore a design will not be abruptly preferred by slight changes in values, unless the parameterizing expressions dictate this.

These first three axioms present nothing new in terms of inferencing mechanisms under uncertainty. Probability and Bayesian inferencing [31], Dempster-Shafer theory [27], fuzzy sets and triangular norms in general [10], and finally utility theory [15] all conform to these axioms, with slight variations on boundary conditions. The subsequent discussion, however, indicates where these theories diverge among themselves and with the development presented here.

The last axiom in Table 2.1 is an idempotentcy restriction. It states that if a designer has the same preference for all individual concerns in a design, then the overall preference must have this degree of preference as well. This condition must be considered closely, for it is a statement related to rationality. A definition of irrational behavior is to act in a manner which is against one's objectives [31]. In the context of preference, this linguistic definition translates into meaning that an irrational method is to reduce or increase the overall preference beyond what the parameters specify. Formalization of this definition, however, has many possibilities (among which, for example, is probability, or even $\mathcal{P}$ as so far specified) since this definition is linguistic and non-formal.

The last axiom of idempotentcy eliminates any functions which combine preferences in an inherently pessimistic or optimistic manner. Methods which combine individual preferences and artificially reduce or increase the overall preference rank should not be considered: *e.g.*, some of the various triangular norms [10] and power methods [8]. For example, if a design had two goals, each with preference 0.8, one would not expect an overall rating of 0.2, or 1.0, since these results are irrational: they reduced or increased the overall preference beyond what the parameters specified. Idempotentcy eliminates these possibilities.

One axiom absent from the list is strictness:

$$\forall j, \mathcal{P}(\mu_1, ..., \mu_j, ...) < \mathcal{P}(\mu_1, ..., \mu_j', ...) \text{ iff } \mu_j < \mu_j' \tag{2.3}$$

The strictness requirement is unacceptable as *always* being required for *any* design metric. For some design strategies, strictness may be acceptable; for others not. For example, consider one parameter in two different designs which has a low preference of 0.4. The two designs differ only in that a second parameter (different from the one ranked at 0.4) has a preference of 0.6 and 0.8 in the two designs respectively. See Figure 2.1. It is not clear that the designer should *always* distinguish between these two designs, which the strictness requirement requires. Both designs have equally bad components at 0.4, and so the designer may decide to rank both designs as equally poor overall, with a preference of 0.4. Alternatively, the designer may see the first parameter as irrelevant, and rank the second design as better overall. This decision depends on the strategy employed. However, attempting to *always* include the strictness requirement eliminates valid strategies from consideration by always differentiating between Design 1 and 2 in Figure 2.1. The strictness requirement is discussed in [10].

Table 2.1 is a list of necessary requirements to which any global combination design metric which uses preferences must conform. Using fewer constraints on the design metric permits irrational and non-intuitive preference combination functions to be used, based on the informal comparison of these axioms with design decision making. Additional constraints will now be placed on the metric, where these additional constraints imply a particular design strategy.

2.2 Conservative Design

Suppose the designer wishes to trade off to improve the lower performing goals (in terms of preference) when selecting a design parameter set $\overrightarrow{DP}$. Also, assume for the moment that all of the individual goals are equal causes of concern to the designer. This implies that, to improve a design, there must be an increase in the preference level of the goal whose preference is lowest. We refer to this as a conservative design strategy, and the method to use for combining the multiple preferences is $\mathcal{P} = min$. That is,

$$\mu(\overrightarrow{DP}^{*}) = \max_{DPS}\left[\min[\mu(DP_1), ..., \mu(DP_n), \mu(PP_1), ..., \mu(PP_q)]\right] \quad (2.4)$$

where $\overrightarrow{DP}^{*}$ is the most preferred design parameter solution set. Using the *min* as a design metric always improves a design's worst aspect, meaning that aspect with the lowest preference. Whichever parameter has the lowest preference dictates the overall preference. If the designer can improve the design, this parameter will change. Of course, the goal which is the "weakest link" changes with changes in $\overrightarrow{DP}$ (changes of position in the design space). Therefore this metric trades off to improve the lower performing goals.

Finally, Equation 2.4 is exactly the fuzzy set formulation of the design problem [8, 20]. Therefore, using a fuzzy set resolution in the design domain reflects trading off goals conservatively, and without considering importance weightings.

2.3 Aggressive Design

The *min* resolution of Equation 2.4 is not always appropriate, however. If the resulting design is drastically hindered by one parameter and relaxing it a bit greatly increases the others' preference, then the modified design may be considered to produce a higher "overall" performance, even though the lower performing goal was slightly reduced even further. In this case, the hindering parameter should be relaxed and thereby allow other parameters to substantially increase their preference.

This can be accomplished with the use of a product:

$$\mu(\overrightarrow{DP}^{*}) = \max_{DPS}\left[\prod_{j=1}^{q+n} \mu_j\right]^{\frac{1}{q+n}} \quad (2.5)$$

where n is the number of design parameters and q is the number of performance parameters.

This resolution reflects a different design strategy than the *min* resolution presented earlier. Specifically, Equation 2.5 allows higher performing goals to compensate for lower performing goals (in terms of preference). This metric trades off the goals to cooperatively improve the design. We refer to this as an aggressive (or cooperative) trade-off strategy.

2.4 Importance Ratings

Both strategy formalizations presented in the previous two sections assumed all goals were equally important. The formalization of the conservative design strategy as reflected by Equation 2.4 traded off the overall performance to gain in the lower performing goals, as if each were equally important to the designer. The formalization of the aggressive design strategy as reflected by Equation 2.5 did the reverse (traded off the lower performing goals to gain in overall performance), as if each goal were equally important. Yet, in the general case, each goal will not hold equal importance. In this more general case, factors must be included to allow the designer to specify how much concern (or weight) should be allocated to each goal.

The reader is referred to [23, 28] for methods on how to specify weights; this paper will not discuss this aspect of the problem. However, it is noted that there are several reasons why weighting functions present difficulty [28, 30]. We concur, and adopt the standard solution to the problem of specifying weights: iteration. That is, it is not assumed the designer can, *a priori*, specify the final goal weights, only preliminary estimates. The designer then gains insight on how to specify weights through iteration. In any case, techniques for specifying weights from pairwise comparisons of goals are the Analytical Hierarchy Process [23], or the marginal rate of substitution [28]. It is noted that, though theoretical issues remain with weighting functions, they are commonly used in practice [1, 2, 13, 19].

Assigning importance factors to goals is a relative measure: a goal's importance is ranked relative to the rest of the goals in a design. The importance of goal j (either a design parameter or a performance parameter) shall be denoted ω_j. Since importance is a relative measure, the importance factors should always be normalized by their sum; *i.e.*, the ω_j must be such that

$$\sum_{j=1}^{q+n} \omega_j(\vec{p}) = 1 \tag{2.6}$$

where $\vec{p}$ is the vector composed of the design and performance parameters. This allows for non-normalized weights; for example, ω_j might be fuzzy. At each point, the non-normal weights must be normalized.

There is another observation on the importance factors: since it is assumed that no goals are trivial or absolutely dominant, the normalized ω_j must be such that

$$0 < \omega_j(\vec{p}) < 1 \qquad \text{for all } j \tag{2.7}$$

The 0 lower boundary condition is actually not strict: the particular goal j then simply drops out of the consideration ($\mu_j^{\omega_j}$ becomes 1). Further, the 1 upper boundary condition is always ensured by the previous normalization requirement.

A final observation is that importance factors are functions: they can change with changes in the design. If a goal's preference is low, perhaps a designer may wish to change the goal's importance. It is assumed that slight changes in a goal's value do not induce drastic changes in the goal's importance. This is a continuity requirement; *i.e.*, the normalized ω_j must be such that

$$\lim_{\vec{p'} \to \vec{p}} \omega_j(\vec{p'}) = \omega_j(\vec{p}) \tag{2.8}$$

Having made these observations about importance factors, they can now be used in any design strategy. For the conservative design strategy, the design metric becomes:

$$\mu(\overrightarrow{DP^*}) = \max_{DPS} \left[\left(\min_{i \in [1,q+n]} [\mu_i^{\omega_i}] \right)^{\frac{1}{\max_{i \in [1,q+n]} [\omega_i]}} \right] \tag{2.9}$$

This expression reflects trading off the overall performance to gain in the lowest performing goal, with each goal raised to its importance level. In the previous unweighted case (Equation 2.4), each goal had an equal importance of $\frac{1}{q+n}$. Equation 2.9 reduces to Equation 2.4 when all goals have equal importance ($\omega_j = \frac{1}{q+n}$ for all j).

An almost identical function has been proposed by Yager [37] for including weighting functions into fuzzy sets. However, our metric is normalized to maintain consistency with Table 2.1. Therefore it is a normalized metric, enabling direct preferential comparisons with other alternatives for which the designer may not have used the conservative design strategy. A different technique was proposed by Bellman and Zadeh [4] involving the fuzzy linear weighting of goals. Their formalization is not adopted because of its failure to maintain consistency with Table 2.1. They fail to maintain consistency with the boundary conditions. Hence one could select a design parameter set which has no preference for a subset of the goals. As stated, Vincent [32] and Biegel and Pecht [5] also argue this is unacceptable for engineering design. Dubois and Prade extend Bellman and Zadeh's technique into possibility theory [11], where the weights become degrees of possibility. The combination of possibility with preference is an area for future research.

The conservative design strategy is affected by the use of importance factors (ω_j) as will be demonstrated graphically for a simple case. Consider a design with just one parameter which has preferences from two sources, as shown in Figure 2.2. For example, the parameter might be material ultimate strength, μ_1 might be preference for cost (cheaper materials are more preferred), and μ_2 might be preference for strength (stronger materials are preferred more). As the relative importance of the preferences change (ω_1 goes from an importance of 1.0 to an importance of 0.0 as ω_2 goes from 0.0 to 1.0), the resulting peak preference point changes as shown in Figure 2.3. The design strategy will choose the value with maximum preference from the resulting combination. For example, with $\omega_1 = 0.75$ and $\omega_2 = 0.25$, the final parameter value chosen will equal 0.44, with a preference of 0.75 (the boxed point in Figure 2.3).

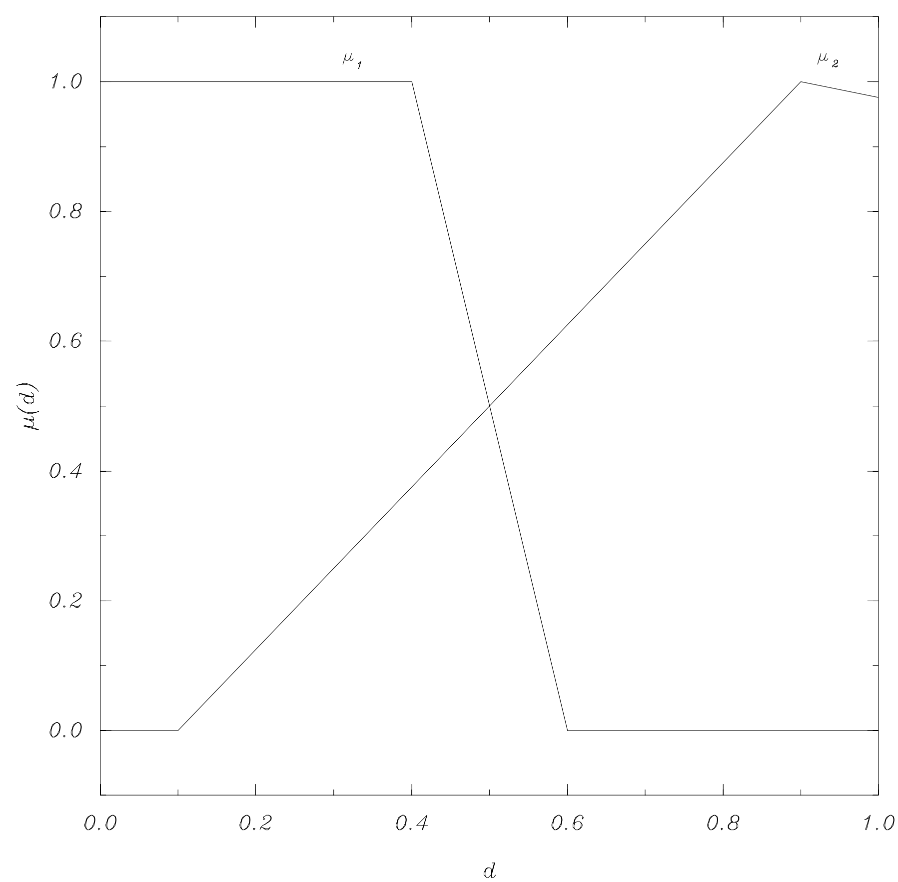

Figure 2.2 Single parameter design with two preference sources.

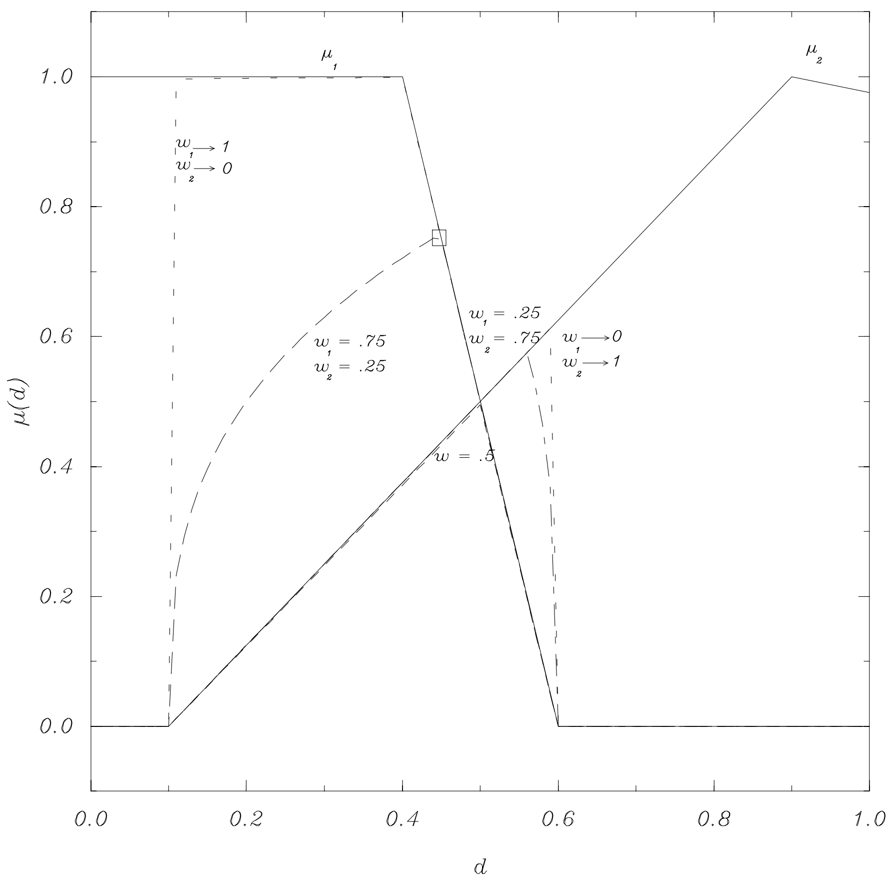

Figure 2.3 Weighted conservative design strategy results.

For the aggressive (cooperating) design strategy case, the design metric will use a variation from the previous unweighted case (Equation 2.5):

$$\mu(\overrightarrow{DP}^*) = \max_{DPS} \left[\prod_{i=1}^{q+n} \mu_i^{\omega_i} \right] \tag{2.10}$$

This expression reflects trading off the goals cooperatively to gain in the overall performance, with each goal raised to its importance level. In the previous unweighted case (Equation 2.5), each goal had an equal importance of $\frac{1}{q+n}$. Equation 2.10 reduces to Equation 2.5 when all goals have equal importance ($\omega_j = \frac{1}{q+n}$ for all j).

Yager presents this resolution in [37] as a method to select a proper course of action based on a set of objectives. We, however, present a justification for its use as reflecting a design trade-off strategy, and apply the method to problems beyond selection from a finite set of alternatives, the thrust of Yager's work.

The aggressive design strategy is also affected by the use of importance factors (ω_j) as will be demonstrated graphically for the same simple example (Figure 2.2). As the relative importance of the preferences change (ω_1 goes from an importance of 1.0 to an importance of 0.0 as ω_2 goes from 0.0 to 1.0), the resulting peak preference point changes as shown in Figure 2.4. The aggressive design strategy will choose the value with maximum preference from the resulting combination. For example, with $\omega_1 = 0.75$ and $\omega_2 = 0.25$, the final parameter value chosen will equal 0.4, with a preference of 0.78 (the boxed point in Figure 2.4).

Note that, for the same problem with the same preferences and importance factors, the two design strategies selected different peak points. The two strategies traded off the goals in different fashions: conservatively or aggressively. In either case, a goal's importance can be handled within the design strategies.

2.5 Hybrid Design Strategies

Generally, a designer may not wish to exclusively trade-off every design component aggressively or conservatively. A subsystem may need to have its weakest goals maximized, but a different subsystem may need to be cooperatively maximized. For these more general cases, a combination of the two methods (the *min* and the *product*) can be performed, and this is consistent with Table 2.1's axioms. The sub-design would use the *min* combination of its preference rankings, and this sub-result would use the *product* to be combined with rest of the design.

In the general case, an entire hierarchy of the parameters' preferences would be constructed into the overall metric. This construction could be aided by using the γ-*level* measure [34] to determine which parameters are critical to the design. The γ-*level* measure provides an indication of how sensitive each

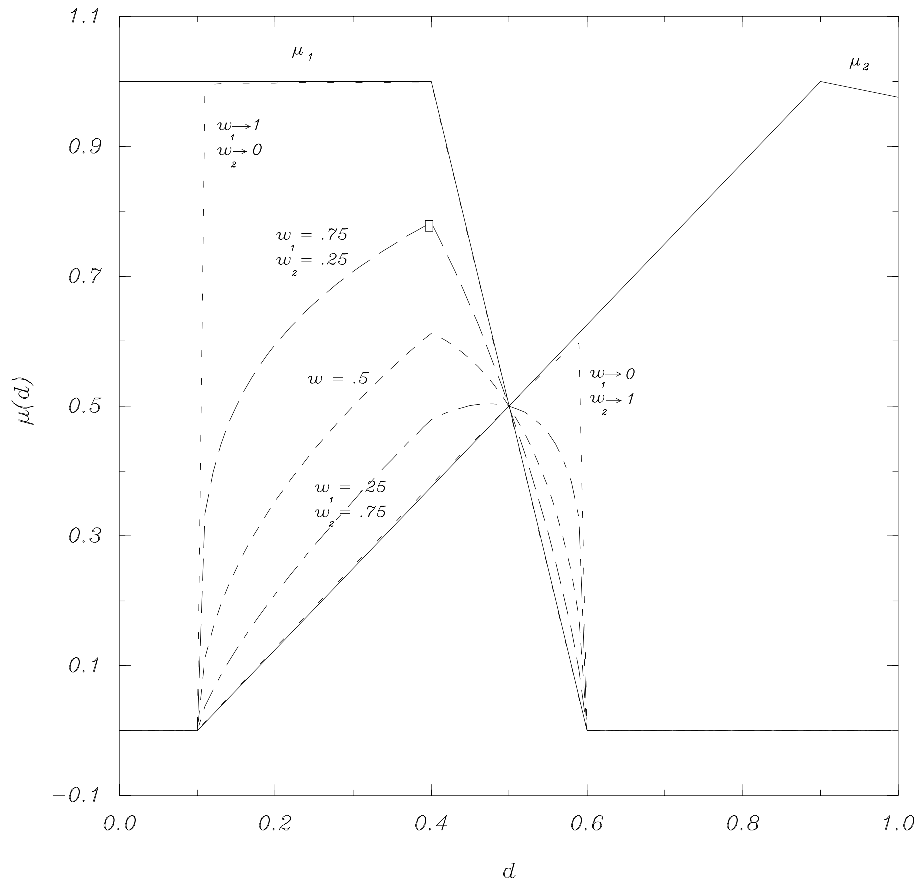

Figure 2.4 Weighted aggressive design strategy results.

design parameter is to each performance parameter, based on their relation and the specified preferences. If the γ-*level* measure indicates a particular design parameter is critical to different performance parameters, the designer could then take extra care when specifying that design parameter's importance. Also note that the importance weightings might also change as the design process progresses to reflect the addition of more information.

2.6 Discussion: What is a Design Strategy?

The term "strategy" has many meanings, both in the research literature, and in engineering practice. This paper has introduced a formalization of one aspect of design strategies: how to make trade off decisions among different goals. Design strategies might also include considerations of performance, safety, importance, or noise variations. They also usually include considerations of the design problem solving methodology or approach. This paper's use of the term "strategy" therefore includes only one aspect: how to make trade-off decisions among multiple, incommensurate goals in a design.

A related question is how to determine what goals should be included in a problem's formalization. This question cannot be answered *a priori*, but will evolve with the design. A preliminary indication of whether a parameter needs consideration can be determined in the same manner as developed for utility theory: using Ellis' "test of importance" [15]. In this method, before determining how the overall metric is to be formulated, the designer asks whether a parameter's inclusion could change the choice of the others. If so, this additional parameter should be included in the formalization. Hence every possibly important parameter in a design is included. This would likely lead to overly complicated forms. Therefore, the γ-*level* measure [34] could be used to eliminate those which are shown to have little consequence.

Another concern involves practically evaluating (or searching for) the most preferred design parameter set, once a strategy has been used to formally specify the multi-criteria objective function. This aspect of the problem will not be elaborated; it is a problem and processor dependent consideration. For example, when the design space is a list of alternative configurations and the goals are features of the design, the problem can be formulated in a matrix format (even for designs involving hundreds of variables) with preferences entered in the matrix. Search is then simply selecting the alternative with maximum $\mathcal{P}$ of the feature preferences, as will be shown in Example 1. In such a case, for a human "processor", the *min* conservative strategy is easier to evaluate than the *product* aggressive strategy.

A different problem may involve goals with explicit performance parameter expressions, as will be shown in Example 2. Here optimization methods [17] can be invoked to search across the design parameter space for the maximum

Table 2.2 Raw designer rankings.

Criteria	Importance	Mechanism	Robot
Ease to get design to satisfy quantity rate	4	5	-1
Ease to ensure operator safety	4	-1	0
Development cost	5	-1	3
Ease to ensure production reliability	5	3	0
Ease to ensure size constraints	2	4	2
Ease to do design by production time	3	0	4
Ease to ensure production quality	4	5	4

preference point, possibly involving penalty methods [17] to ease the search. Here the *product* aggressive strategy may be easier to evaluate due to differentiability. The *min* conservative strategy becomes a traditional *maximin* multiple goal optimization formulation [16]. In any case, iteration will almost certainly be required to ensure the final preferences and weights. Formal iterative methods (see Steuer [28] for a review) could be used.

3. Examples

The first example presented will be in the preliminary design domain. The task is to determine which of two candidate models to pursue into the latter design stages. The second example will be in the parametric design domain. The task is to determine which of two air tanks to manufacture, and which parametric design parameter values to use.

3.1 Example 1: Preliminary Design

Consider the design task involving a selection between two candidate concepts. The candidates are to be used for assembling items in a manufacturing production line. The first candidate design is a special purpose mechanism, the other is a general purpose robotic arm.

The decision criteria for determining which candidate to pursue into the subsequent design stages are listed in Table 2.2. As well, each criterion's importance (on a scale of 0 to 5), and each candidate's ability to satisfy the criterion (on a scale from -5 to 5) is tabulated. Background and details of matrix methods are discussed in [2, 18].

Using the standard weighted sum matrix analysis [2], the mechanism candidate produces an overall rank of 54, and the robot candidate produces an overall rank of 43. This technique guides the designer to pursue the mechanism.

Table 2.3 Imprecise designer rankings.

Criteria	Importance	Mechanism	Robot
Ease to get design to satisfy quantity rate	$\frac{4}{27}$	1.0	0.4
Ease to ensure operator safety	$\frac{4}{27}$	0.4	0.5
Development cost	$\frac{5}{27}$	0.4	0.8
Ease to ensure production reliability	$\frac{2}{27}$	0.8	0.5
Ease to ensure size constraints	$\frac{3}{27}$	0.9	0.7
Ease to do design by production time	$\frac{3}{27}$	0.5	0.9
Ease to ensure production quality	$\frac{4}{27}$	1.0	0.9

Using the method of imprecision, the ranks are normalized by the range of the ranking. As well, the importance ratings are normalized by their sum. The results of this calculation are shown in Table 2.3.

Strategies for resolving these multiple attributes of the candidates can be invoked. Let us assume that the designer wishes to trade-off the criterion in an aggressive, cooperative fashion, meaning that the designer is willing to measure the overall preference of each alternative based on a composite of its attributes. This implies that some goals with high preference can compensate for others with low preference. Then Equation 2.10 can be used to combine the preferences. Doing so results in a rating of 0.65 for the mechanism, and 0.63 for the robot. Again, the mechanism is determined to be the most promising candidate to pursue.

Now instead, let us assume that the designer wishes to trade-off the goals in a conservative, non-compensatory fashion, meaning that the designer will measure the overall performance for each alternative based on the worst (lowest preference) attribute. This implies that the attributes that perform well cannot compensate for those that perform poorly. Then Equation 2.9 can be used to combine the preferences. Doing so results in a rating of 0.40 for the mechanism, and 0.48 for the robotic arm. Here, the robot is determined to be the most promising candidate to pursue. This result is different from the cooperative trade-off strategy result. The new choice was caused by the mechanism being rated poorly at development cost, which was not compensated for by the other superior ratings of the mechanism.

Note the standard matrix method resolved the most promising candidate by selecting the one with the highest weighted performance average across the goals. It did not do so by rating each candidate by the worst aspect. Therefore the standard weighted sum matrix technique invokes a compensating goal trade-off strategy, informally similar to our aggressive trade-off strategy. The developments described here allow for a variety of design strategies. Combinations of conservative and aggressive strategies could be used for different sub-arrangements of the goals, and then these sub-arrangements combined with either a conservative or aggressive strategy, depending on the designer's judgments.

3.2 Example 2: Parametric Design

The example presented below considers a pressurized air tank design, and is the same problem as presented in Papalambros and Wilde [17], page 217. The reader is referred to the reference [17] to see the restrictions applied to the problem to permit it to be solved using crisp constraints and various optimization techniques (monotonicity analysis, non-linear programming). The example is simple and was chosen for that reason, and also the ability of its preferences to be represented on a plane for a visual interpretation.

The design problem is to determine length and radius values in an air tank with two different choices of head configuration: flat or hemispherical. See Figure 2.5.

There are four performance parameters in the design. The first is the metal volume m:

$$m = 2\pi K_s r^2 l + 2\pi C_h K_h r^3 + \pi K_s^2 r^2 l \tag{2.11}$$

This parameter is proportional to the cost, and the preference ranks are set because of this concern. Another performance parameter is the tank capacity v:

$$v = \pi r^2 l + \pi K_v r^3 \tag{2.12}$$

This parameter is an indicator of the design's principle objective: to hold air. This parameter's aspiration level ranks the preference for values. Another parameter is an overall height restriction L_0, which is imprecise:

$$l + 2(K_l + K_h) r \leq L_0 \tag{2.13}$$

Finally, there is an overall radius restriction R_0, which is also imprecise:

$$(K_s + 1) r \leq R_0 \tag{2.14}$$

The last two performance parameters have their preference ranks set by spatial constraints.

The coefficients K are from the ASME code for unfired pressure vessels. S is the maximal allowed stress, P is the atmospheric pressure, E is the joint efficiency, and C_h is the head volume coefficient.

$$K_h = \begin{cases} 2\sqrt{CP/S} & \text{flat} \\ \frac{P}{2S-.2P} & \text{hemi} \end{cases} \tag{2.15}$$

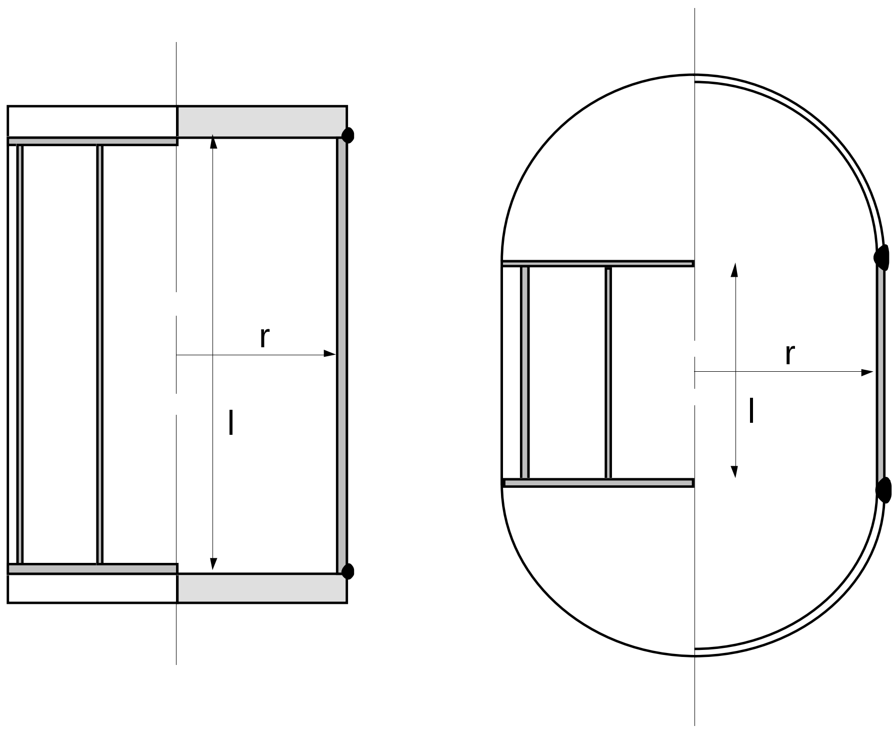

Figure 2.5 Hemispherical and flat head air tank designs.

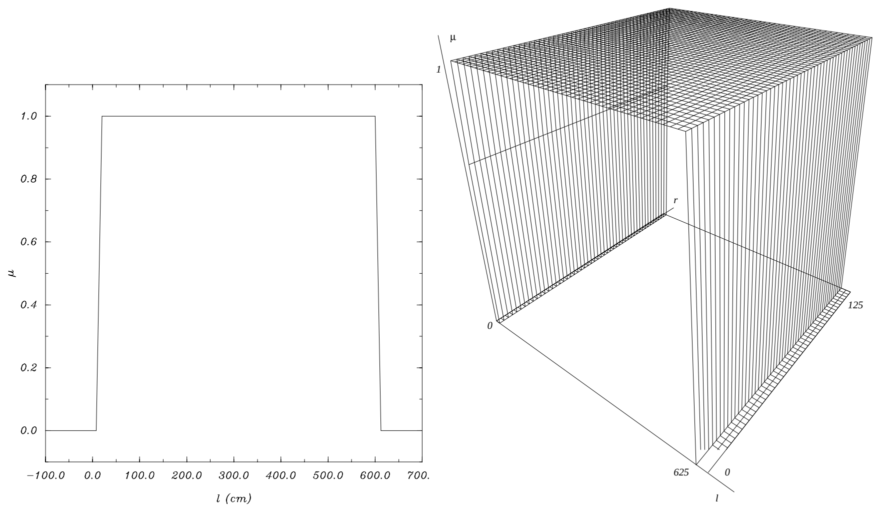

Figure 2.6 Length l preference.

$$K_l = \begin{cases} 0 & \text{flat} \\ 4/3 & \text{hemi} \end{cases} \tag{2.16}$$

$$K_s = \frac{P}{2SE - .6P} \tag{2.17}$$

$$K_v = \begin{cases} 0 & \text{flat} \\ 1 & \text{hemi} \end{cases} \tag{2.18}$$

This example's design space is spanned by 2 design parameters l and r. The preferences for values of these design parameters and the four performance parameters are shown in Figures 2.6 through 2.11 for the hemispherical design; the flat head design space is similar.

The problem is to find the values for l and r which maximize overall preference. For comparison, both a conservative and an aggressive strategy will be presented and contrasted below. Both consider all goals to be equally important.

For the conservative design strategy, l^* and r^* are to be found, where

$$\mu(l^*, r^*) = \max_{l,r} \left[\min[\mu_l, \mu_r, \mu_{v(l,r)}, \mu_{m(l,r)}, \mu_{L_0(l,r)}, \mu_{R_0(l,r)}] \right] \tag{2.19}$$

This will find the l^* and r^* by trading off the goals to improve the lowest performing goal (in terms of preference), even though the design parameters and performance parameters are incommensurate with each other.

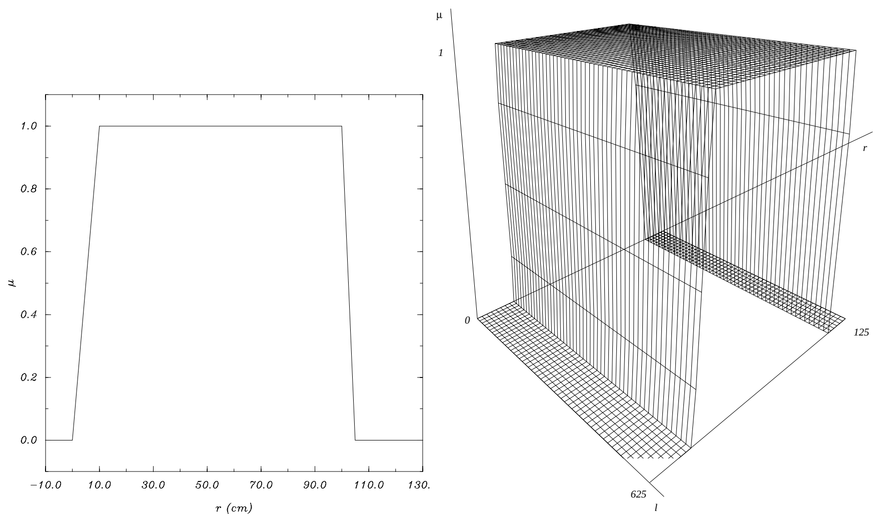

Figure 2.7 Radius r preference.

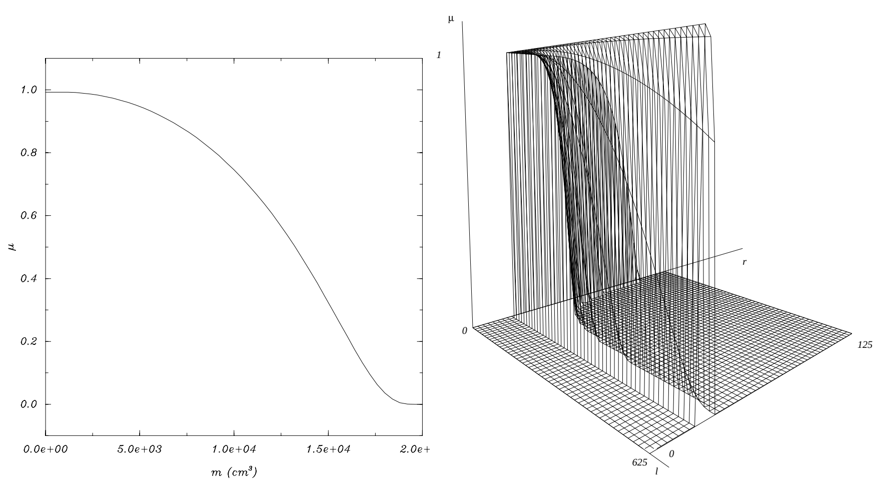

Figure 2.8 Metal volume m preference.

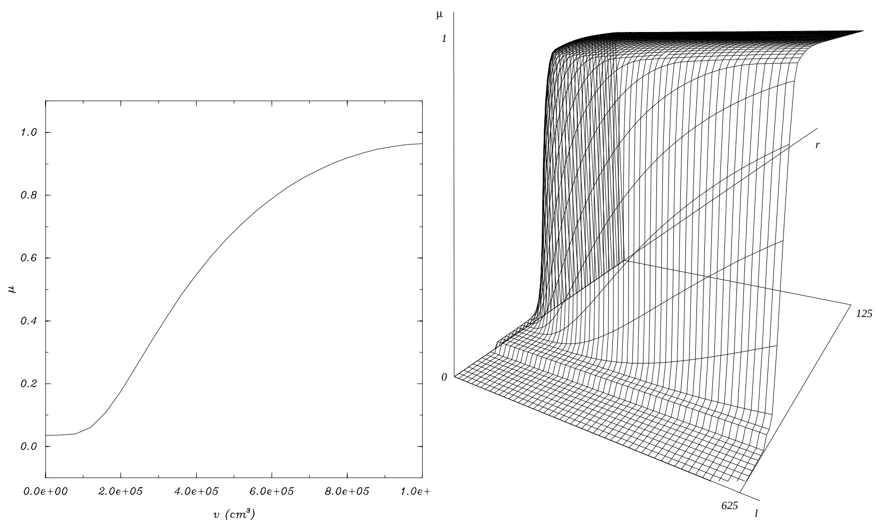

Figure 2.9 Capacity v preference.

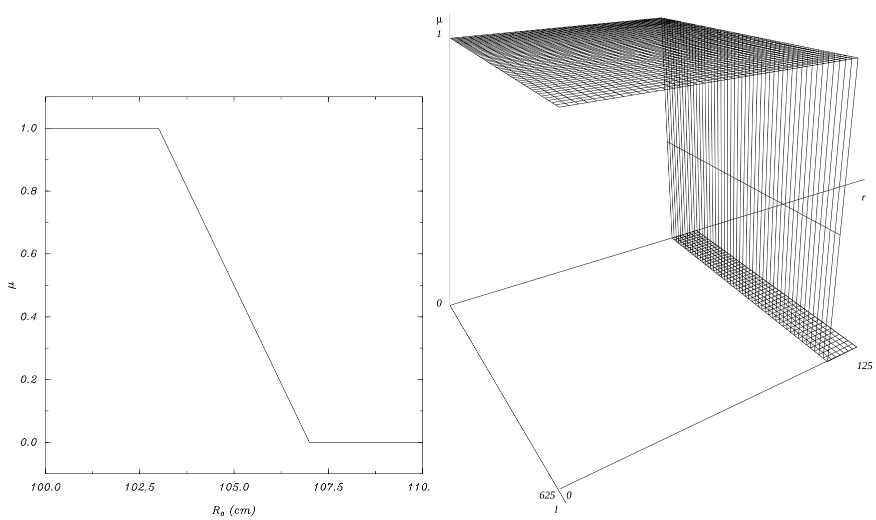

Figure 2.10 Outer radius R_0 preference.

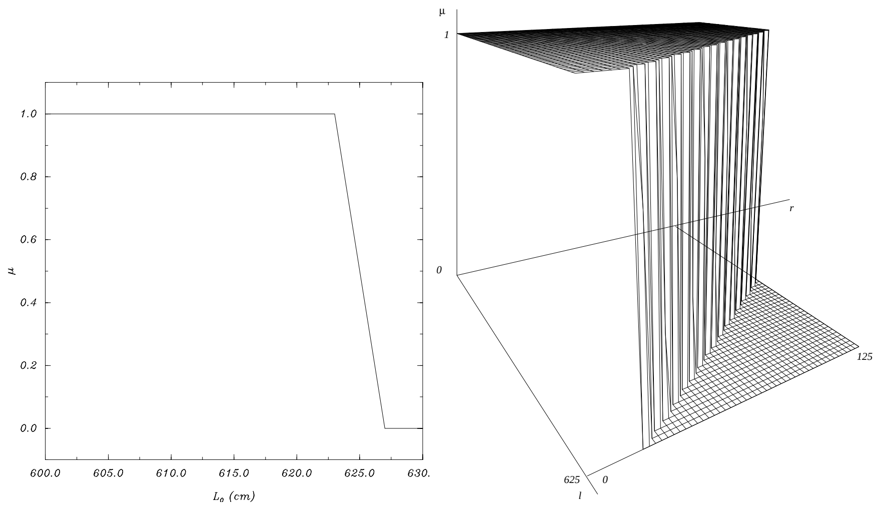

Figure 2.11 Outer length L_0 preference.

For the aggressive design strategy, the problem to be solved is to find l^* and r^* where

$$\mu(l^*, r^*) = \max_{l,r} \left[\mu_l \times \mu_r \times \mu_{v(l,r)} \times \mu_{m(l,r)} \times \mu_{L0(l,r)} \times \mu_{R0(l,r)} \right]^{1/6} \tag{2.20}$$

This will find the l^* and r^* by trading off the goals cooperatively among each other, allowing the higher performing goals to compensate for the lower performing goals (in terms of preference), even though the design parameters and performance parameters are incommensurate with each other.

The preference combination results can be seen graphically in Figures 2.12 through 2.15. For the conservative design strategy, the *min* of each individual preference across the design space is the resulting surface shown. This is shown in Figures 2.12 and 2.13. The surface's maximum value in μ is the solution point to use (the most preferred l and r). For the aggressive design strategy, the individual preference surfaces are multiplied together as a product of powers for all points on the l, r plane. This is shown in Figures 2.14 and 2.15. These overall preference surfaces should be compared with the individual goals' preferences shown in Figures 2.6 through 2.11 to observe the relations between individual goals' preferences over the design space, and the end resulting preference surface.

As can been seen, the aggressive strategy will produce higher overall preference than a conservative strategy, and the two strategies will result in different solution design parameter values for the design: different l^*,r^* have the highest μ on the overall preference surfaces of Figures 2.13 and 2.15 (hemispherical

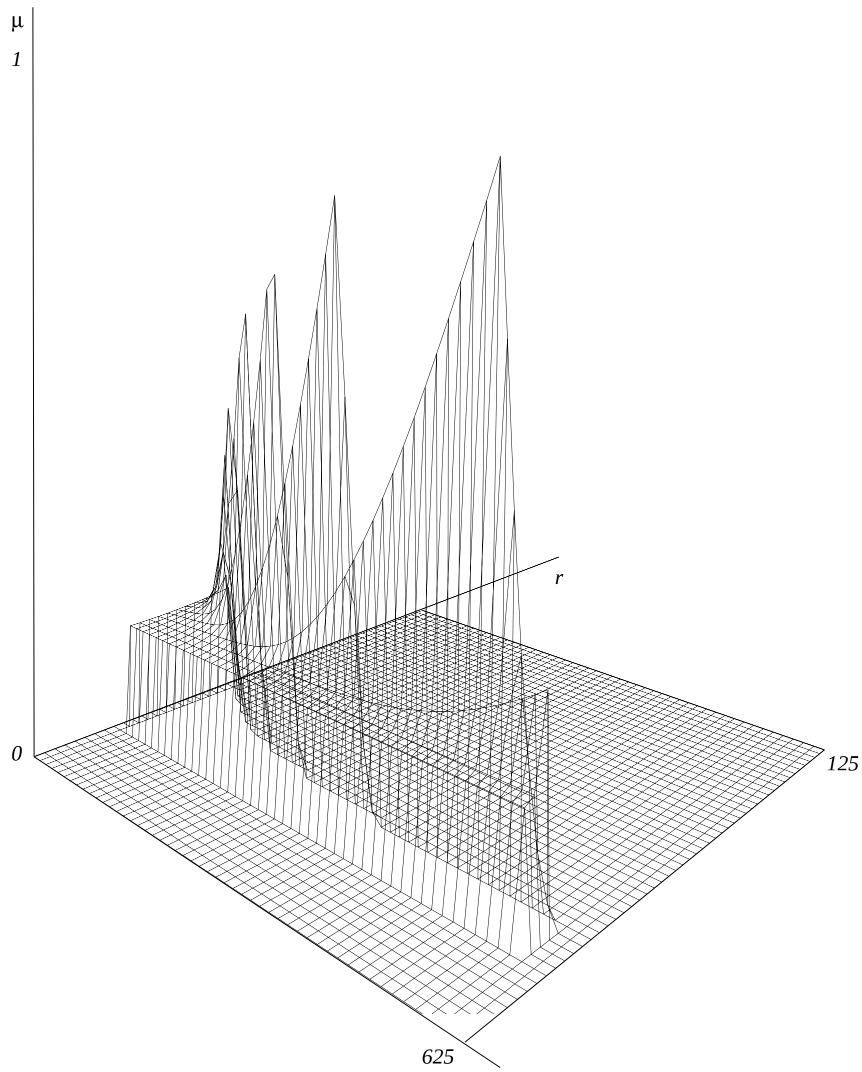

Figure 2.12 Flat head tank design: conservative design strategy results.

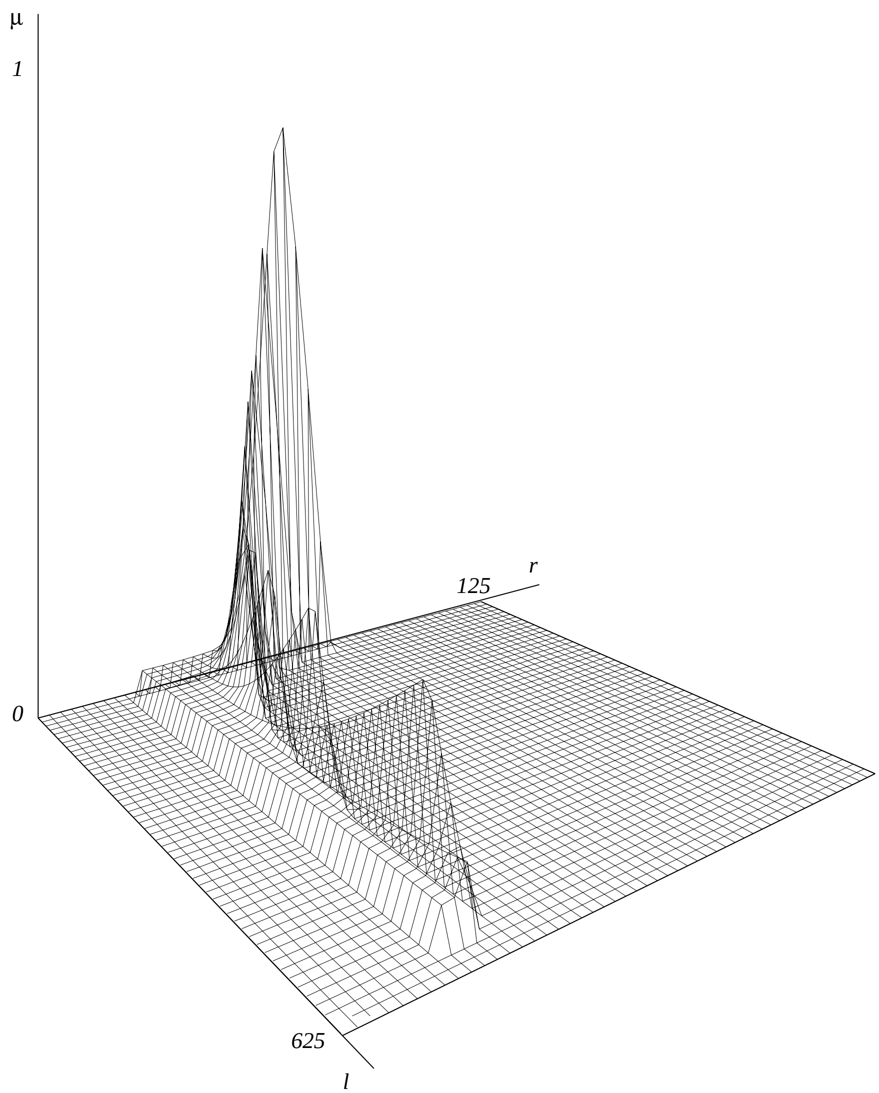

Figure 2.13 Hemi head tank design: conservative design strategy results.

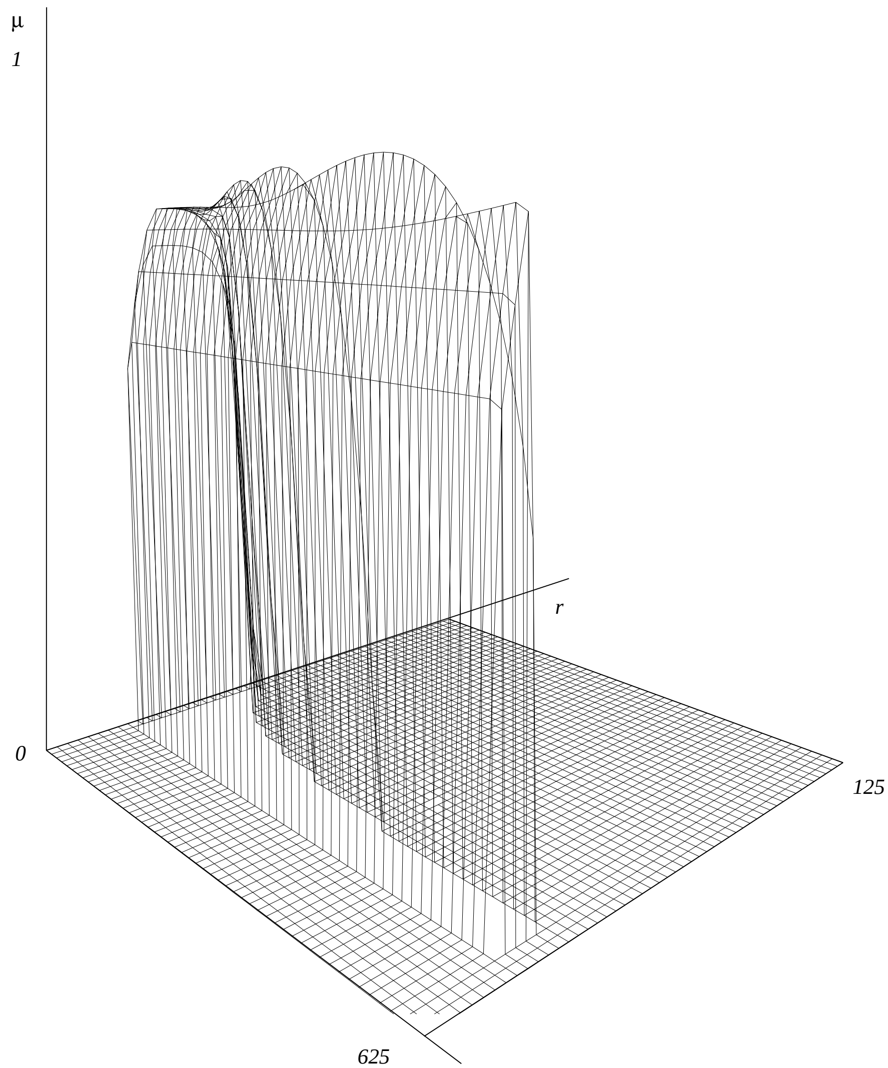

Figure 2.14 Flat head tank design: aggressive design strategy results.

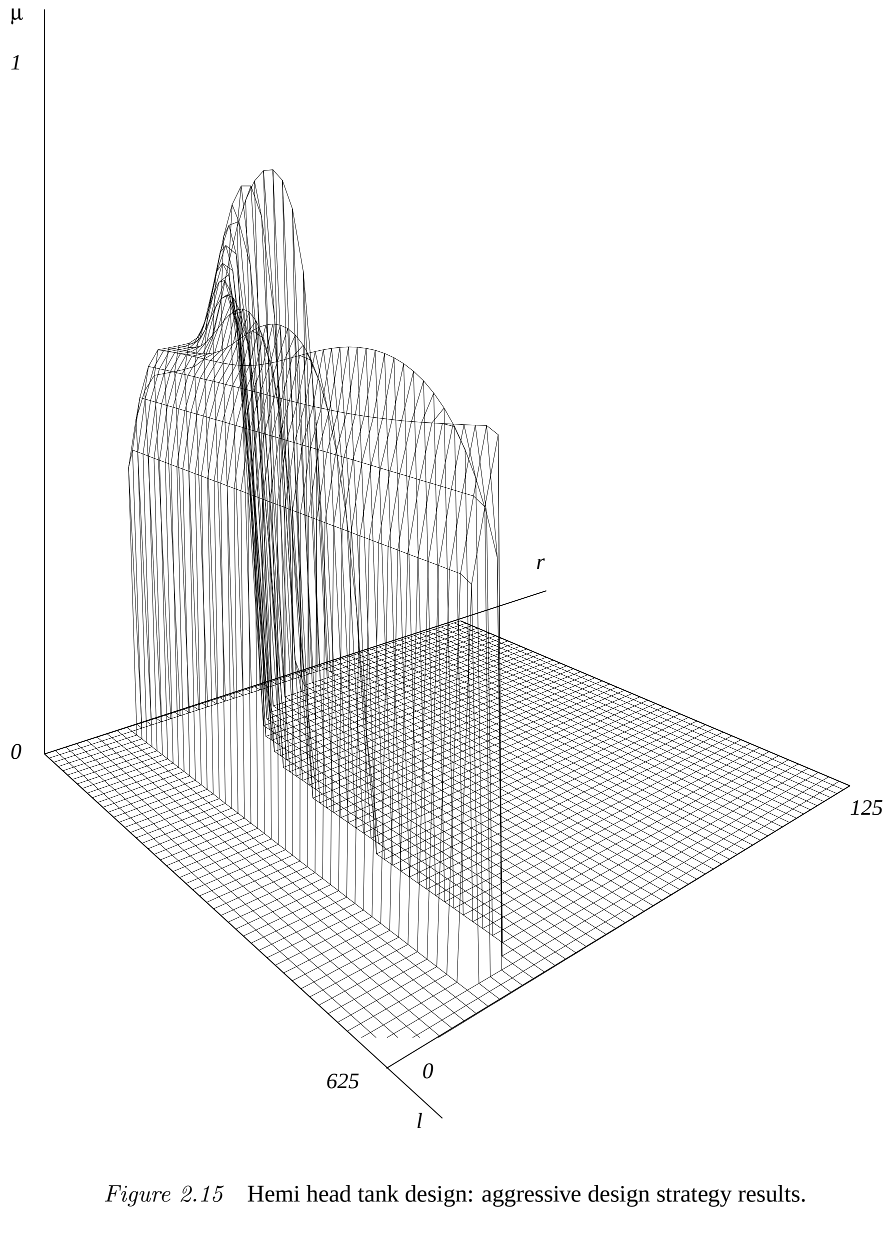

Figure 2.15 Hemi head tank design: aggressive design strategy results.

head design), and likewise for Figures 2.12 and 2.14 (flat head design). The conservative design strategy sacrificed the cost (m) to ensure the capacity (v). Designing aggressively did the reverse: reduced the cost (m) at expense of the capacity (v). Observe that, from an optimization viewpoint, both solutions are in the Pareto-optimal solution set, since both require a reduction in preference of a goal to increase another.

This differs from the results of the various problem formulations presented in Papalambros and Wilde [17]. For example, the non-linear programming formulation solves the problem by minimizing the metal volume with the rest of the goals as crisp constraints. Our formulation allows the constraints to be elastic, as shown in Figures 2.6 through 2.11, so the final design parameter values determined are different than if crisp constraints had been used. If the example had selected step functions for preference curves on the constraint performance parameters, the imprecision results would reduce to the non-linear programming solution for any strategy. This is because, with step functions for the constraint parameters, only one parameter (m) dictates the preference, and so the issue of trade-off between goals is not applicable: there is only one goal. The point of this example is to visually demonstrate the differing overall preference (shown here as surfaces) over the design space, and to demonstrate that different design trade-off strategies can entirely change the solution.

4. Conclusion

This paper presents a method for trading off multiple, incommensurate goals. The definition of "best" in light of the incommensurate goals is determined by specifying a formal, explicit design trade-off strategy.

This work permits direct comparisons of different design configuration alternatives in a formal sense: by comparing their respective overall preference ratings. This is true even if the different alternatives have vastly different physical forms, or even if the different configurations have different parameters. Further, the formalization of these strategies, introduced here, shows that strategies can guide design decisions. Two simple examples were presented to demonstrate the methodology in familiar domains. It was shown that current matrix method formalizations for preliminary design (such as Pugh's method [18], and QFD [1, 13]) use a compensating design strategy to trade-off the different features of alternative configurations. This is because they select a configuration based on the net sum of designer rankings, rather than on worst case. This research will allow designers to apply the same techniques, but with different design strategies, as appropriate.

This new methodology permits the designer to formally implement a design trade-off strategy, and incorporate subjective knowledge and experience (via preference and imprecision). This makes the designer's prejudices and strategy explicit, which can be used to help make, observe, justify, and record design decisions.

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References

[1] AKAO, Y. *Quality Function Deployment: Integrating Customer Requirements into Product Design*. Productivity Press, Cambridge, 1990.

[2] ALGER, J., AND HAYS, C. *Creative Synthesis in Design*. Prentice Hall, Englewood Cliffs, 1964.

[3] ASHLEY, S. The battle to build better products. *Mechanical Engineering 109* (Nov. 1990), 34–38.

[4] BELLMAN, R. E., AND ZADEH, L. A. Decision-making in a fuzzy environment. *Management Science 17*, 4 (Dec. 1970), B–141–163.

[5] BIEGEL, P., AND PECHT, M. Design trade-offs made easy. *Concurrent Engineering 1*, 3 (May/June 1991), 29–40.

[6] COLTON, J., AND DESCANIO, J. An integrated, intelligent design environment. *Engineering with Computers 7* (1991), 11–22.

[7] DIAZ, A. R. Fuzzy set based models in design optimization. In *Advances in Design Automation - 1988* (New York, NY, Sept. 1988), S. S. Rao, Ed., vol. DE-14, ASME, pp. 477–485.

[8] DIAZ, A. R. Modeling of computer-based decisions and optimization in the design process. In *Design Theory '88* (RPI, Troy, NY, June 1988), S. L. Newsome and W. R. Spillers, Eds., NSF, pp. 286–295. 1988 NSF Grantee Workshop on Design Theory and Methodology.

[9] DIAZ, A. R. A strategy for optimal design of hierarchical systems using fuzzy sets. In *The 1989 NSF Engineering Design Research Conference* (College of Engineering, University of Massachusetts, Amherst, June 1989), J. R. Dixon, Ed., NSF, pp. 537–547.

[10] DUBOIS, D., AND PRADE, H. A class of fuzzy measures based on triangular norms. *International Journal of General Systems 8* (1982), 43–61.

[11] DUBOIS, D., AND PRADE, H. A review of fuzzy set aggregation connectives. *Information Sciences 36* (1985), 85–121.

[12] FREIHEIT, T., AND RAO, S. S. A modified game theory approach to multi-objective optimization. In *Advances in Design Automation - 1988* (New York, NY, Sept. 1988), S. S. Rao, Ed., vol. DE-14, ASME, pp. 107–114.

[13] HAUSER, J., AND CLAUSING, D. The House of Quality. *Harvard Business Review 66*, 3 (May/June 1988), 63–73.

[14] HOLMES, M. F. Machine dynamics, The need for greater productivity. In *Research Needs in Mechanical Systems* (New York, NY, 1984), K. N. Reid, Ed., ASME, pp. 140–159.

[15] KEENEY, R., AND RAIFFA, H. *Decisions with Multiple Objectives: Preferences and Value Tradeoffs*. Wiley, New York, NY, 1976.

[16] OSYCSKA, A. *Multi-Criterion Optimization in Engineering with Fortran Examples*. Halstad Press, New York, NY, 1984.

[17] PAPALAMBROS, P., AND WILDE, D. *Principles of Optimal Design*. Cambridge University Press, New York, NY, 1988.

[18] PUGH, S. *Total Design*. Addison-Wesley, New York, NY, 1990.

[19] PUTRUS, R. Non-traditional approach in justifying computer integrated manufacturing systems. In *Autofact '89 Conference Proceedings* (Detroit, MI, Oct. 1989), SME, pp. 8.1–8.26.

[20] RAO, S. S. Description and optimum design of fuzzy mechanical systems. *ASME Journal of Mechanisms, Transmissions, and Automation in Design 109* (Mar. 1987), 126–132.

[21] RAO, S. S., AND DHINGRA, A. K. Integrated optimal design of planar mechanisms using fuzzy theories. In *Advances in Design Automation - 1989* (New York, NY, Sept. 1989), vol. DE-15-2, ASME, pp. 161–168.

[22] RAO, S. S., AND DHINGRA, A. K. Applications of fuzzy theories to multi-objective system optimization. NASA Contractor Report 177573, NASA Ames Research Center, 1991.

[23] SAATY, T. L. Exploring the interface between hierarchies, multiple objectives, and fuzzy sets. *Fuzzy Sets and Systems 1* (1978), 57–68.

[24] SAKAWA, M., AND YANO, H. Interactive decision making for multiobjective nonlinear programming problems with fuzzy parameters. *Fuzzy Sets and Systems 29* (1989), 315–326.

[25] SAKAWA, M., AND YANO, H. Interactive fuzzy decision making for multiobjective nonlinear programming problems with fuzzy parameters. *Fuzzy Sets and Systems 32* (1989), 245–261.

[26] SAKAWA, M., AND YANO, H. An interactive fuzzy satisficing method for multiobjective nonlinear programming problems with fuzzy parameters. *Fuzzy Sets and Systems 30* (1989), 221–238.

[27] SMETS, P. Belief functions. In *Non-Standard Logics for Automated Reasoning* (1988), P. Smets, E. Mamdani, D. Dubois, and H. Prade, Eds., Academic Press, pp. 254–286.

[28] STEUER, R. *Multiple Criteria Optimization: Theory, Computation, and Application*. J. Wiley, New York, NY, 1986.

[29] THURSTON, D. L., AND TIAN, Y. A method for integrating utility analysis into an expert system for design evaluation under uncertainty. In *Proceedings of the Seventh Uncertainty in Artificial Intelligence Conference* (San Mateo, CA, 1991), B. D'Ambrosio, Ed., Morgan Kaufmann Publishers, pp. 398–405.

[30] TRIANTAPHYLLOU, E., AND MANN, S. An evaluation of the eigenvalue approach for determining the membership values in fuzzy sets. *Fuzzy Sets and Systems 35* (1990), 295–301.

[31] TRIBUS, M. *Rational Descriptions, Decisions, and Designs*. Pergamon Press, New York, NY, 1969.

[32] VINCENT, T. Game theory as a design tool. *ASME Journal of Mechanisms, Transmissions, and Automation in Design 105* (June 1983), 165–170.

[33] WOOD, K. L. *A Method for Representing and Manipulating Uncertainties in Preliminary Engineering Design*. PhD thesis, California Institute of Technology, Pasadena, CA, 1989.

[34] WOOD, K. L., AND ANTONSSON, E. K. Computations with Imprecise Parameters in Engineering Design: Background and Theory. *ASME Journal of Mechanisms, Transmissions, and Automation in Design 111*, 4 (Dec. 1989), 616–625.

[35] WOOD, K. L., AND ANTONSSON, E. K. Modeling Imprecision and Uncertainty in Preliminary Engineering Design. *Mechanism and Machine Theory 25*, 3 (Feb. 1990), 305–324. Invited paper.

[36] WOOD, K. L., OTTO, K. N., AND ANTONSSON, E. K. A Formal Method for Representing Uncertainties in Engineering Design. In *First International Workshop on Formal Methods in Engineering Design* (Fort Collins, Colorado, Jan. 1990), P. Fitzhorn, Ed., Colorado State University, pp. 202–246.

[37] YAGER, R. Fuzzy decision making including unequal objectives. *Fuzzy Sets and Systems 1* (1978), 87–95.

Chapter 3

DESIGN PARAMETER SELECTION IN THE PRESENCE OF NOISE

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Abstract

The *method of imprecision* is a design method whereby a multi-objective design problem is resolved by maximizing the overall degree of *designer preference*: values are iteratively selected based on combining the degree of preference placed on them. Consider, however, design problems that exhibit multiple uncertainty forms (noise). In addition to degrees of preference (*imprecision*) there are *probabilistic* uncertainties caused by, for example, measuring and fabrication limitations. There are also parameters that can take on any value *possible* within a specified range, such as a manufacturing or tuning adjustment. Finally, there may be parameters which must *necessarily* satisfy all values within the range over which they vary, such as a horsepower requirement over a motor's different speeds. This paper defines a "best" set of design parameters for design problems with such multiple uncertainty forms and requirements.

1. Introduction

Much of the process of formalizing design decision making has yet to be understood. Rigorous methods for representing and manipulating the concerns and unknowns of a design need to be advanced for a robust automation. The ability to make such decisions, however, is hampered by uncertainties (noise) in the variables used, both in their values and in the manner in which they should be manipulated. This paper will introduce methods that appropriately model the various forms of uncertainty to define an overall "best" set of parameters for use.

Parameter types are broken down by characterizing some model variables as *design parameters*, or those parameters in the engineering model for which the designer must select values, such as geometric sizes. *Performance parameters* are those the designer uses to indicate a design's ability to satisfy *functional requirements*, such as stress or deflection. Finally, there are usually *noise parameters* that introduce uncertainty in the designer's ability to measure, such as manufacturing errors. Note this use of the term *parameter* is therefore different from its usage in statistics, where it is used to describe a entire set of noise values in some respect, usually with a single number (*e.g.*, an expected value).

Initially, design parameter values are uncertain: the designer does not know what values to use. Consequently, the performance parameter values are also uncertain. As a design process proceeds, values are determined more and more precisely in an iterative test and refine fashion. Noise parameter uncertainty, however, always exists, and requires changes in measuring and manufacturing processes to improve.

In addition to these basic uncertainties, there are several different types of design parameters [21]. Some parameters may have absolute, rigid functional requirements. Others may be flexible; targets only express what is desired, not required. Similarly, tolerances may have strict limits placed on them; failure to be within the tolerances is a failed design. Other tolerances may be flexible. This paper introduces a modeling scheme for both parameter types, and defines methods for selecting an overall best design parameter set with such influences. Those parameters that have strict requirements are termed *necessary parameters*, and may be of the design, performance, or noise variety. The mathematics of necessity was introduced to engineering design by Ward and Seering [20] for interval mathematics. The concept is extended here to different uncertainty forms.

Modeling of Uncertainty

Every uncertainty form discussed above shall be directly modeled. That is, the initial design parameter uncertainty shall be modeled using *the method of imprecision* [8, 22] where each design parameter value is given a rank from zero to one to indicate degree of preference. This forms a preference function μ over each design parameter and performance parameter indicating degree of preference for values. Co-dependencies are possible, see [8].

Similarly, probabilistic noise parameters [10] shall have their values ranked with degrees of probability. Finally, possibilistic noise parameters [10] shall have their values ranked with degrees of possibility. All these uncertainties reflect different phenomena, and consequently will have different derived mathematics. Of course, these uncertainties can interact. A design parameter may have both a preference function and a probability density function.

This paper focuses on making the determination of the "best" overall design parameter set given such influences, and presents a method which can be used to solve for the maximum overall preference for a design parameter set, even with confounding probabilistic and possibilistic uncertainties, as well as necessary requirements.

Section 2.1 briefly reviews a metric to define the highest preference, see [8] for a complete discussion of its iterative specification. The effects of uncertainty on the highest overall preference concept are discussed in Section 2.2 for individual forms of confounding influences. Section 3. discusses incorporation of necessary parameters. Finally, Section 4. discusses design problems with combinations of these effects.

2. Parameter Selection in the Design Process

2.1 Global Preference Functions and Design Strategies

This work addresses the stage of the engineering design process where the designer is selecting a configuration. Therefore, the designer has developed a formal design parameter space (DPS) consisting of alternative configurations to choose among. The DPS will be characterized by *design parameters*, $d_1, \ldots, d_n$. For a design process of selection among alternatives, each d_i represents an alternative, so the DPS $\simeq \mathbb{Z}_n$ (the finite set of integers up to n). For determination of values in a design model, there are usually multiple parameters each of which could be thought of as a continuum, and so each d_i might thought of as a value of a vector within a DPS $\simeq \mathbb{R}^n$, where the DPS is represented in some basis with coordinates d^{i}, for example.

Given that an overall best design parameter set is to be found, the "best" concept must be defined. Unfortunately, the various performance parameters usually involve incommensurate concepts. A traditional approach to combining incommensurate parameters is to use a normalization and a weighting. Instead, incommensurate parameters can be combined more usefully using a common trait: designer preference. This means that preference information (μ) on the design parameters (d^i) and requirement preferences on the the performance parameters ($f_j(\vec{d})$) must be combined into an overall preference rating for that design parameter set ($\vec{d}$). The vector notation is meant to suggest the typical design scenario of a multi-component design, but this development is applicable to singleton designs.

To reflect the designer's overall preference, a global *design metric* (which ranks each combination of possible design parameter arrangements) must exist across the design parameter space, expressed as a function of the known preferences of the design:

$$\mu(\vec{d}) = \mathcal{P}\left(\mu(d^1), \ldots, \mu(d^n), \mu(f_1(\vec{d})), \ldots, \mu(f_q(\vec{d}))\right) \tag{3.1}$$

where $\vec{d}$ is a design parameter arrangement, and f_1 through f_q are the performance parameters. This definition is the least possibly restrictive method of combining incommensurate concepts. It is a formalization of the notion of combining incommensurate parameters based on the degree that each parameter satisfies the designer. Hence the best design parameter set to use is defined by the maximum of this preference function:

$$\mu(\vec{d^*}) = \sup\left\{\mathcal{P}(\mu(d^1), \ldots, \mu(d^n), \mu(f_1(\vec{d})), \ldots, \mu(f_q(\vec{d}))) \;\middle|\; \vec{d} \in \text{DPS}\right\} \tag{3.2}$$

The multi-objective design problem with multiple constraints then becomes to find $\vec{d^*}$ as reflected in Equation 3.2, by maximizing the design metric $\mathcal{P}$ over the design space (DPS).

This problem statement, however, is incomplete: $\mathcal{P}$ is unspecified. The choice of a *trade-off strategy* desired to be used by the designer will specify $\mathcal{P}$. Formal trade-off strategies for engineering design are introduced in [8]. For example, use of the minimum function ($\mathcal{P} \equiv$ *min*) reflects a *conservative* or *non-compensating* strategy of the designer to improve the design by always improving the weakest design aspect. Use of a normalized multiplication reflects an *aggressive* or *compensating* strategy of the designer to develop a maximally performing design. Of course, non-compensating and compensating strategy hybridization for different aspects of the design are possible [8].

2.2 Uncertainty Effects

Having formulated the overall preference function, there may still be uncertainties to confound the search for the design parameter set which provide the highest global preference. This will be resolved, however, by assuming that the designer wants the design parameter set that provides the best "quality" in light of the possible variations, using the following interpretation of quality: overall preference despite variations. This is similar to the view of quality that Taguchi uses [17], where he instead applies the view on a single performance parameter, rather than on preference over many parameters [9].

Confounding Influences

In an engineering design problem, noise is typically characterized by *noise parameters,* $n_1, \ldots, n_k, \ldots, n_q$. A noise parameter n_k might be the possible positioning of an operator switch, and so the alternatives may be finite. Alternatively, n_k might be a value of a manufacturing error on a design parameter, and so the NPS may have a continuum of possibilities. In this case, n_k might be thought of as a value of a vector within an NPS $\simeq \mathbb{R}^q$ represented with coordinates n^κ in some basis, for example.

Though these specific, simple examples illustrate the concept of noise, the structure needed to model noise can be developed more generally. Thus, the NPS is defined to be an *uncertainty measure space (NPS, $\mathcal{B}$, g)*, which is a set NPS of elements n, a σ-algebra $\mathcal{B}$ of sets over the NPS (also called a *Borel field*), and an uncertainty measure $g : \mathcal{B} \to [0,1]$. Notice that $\mathcal{B}$ is a set of sets. $N \in B$ is an event: a set of possible values n. This is in keeping with the historical development of statistical noise, where an event is a possible outcome or outcomes.

An uncertainty measure is different from a Lebesgue measure [6] or a fuzzy measure [4, 5, 14, 15]. An uncertainty measure is a function $g : \mathcal{B} \to [0,1]$ intended to measure the effects of noise. Three specific measures will be developed: the probability measure Pr, the possibility measure Π, and a necessity measure N_α.

In keeping with the terminology of probability, an element of $\mathcal{B}$ is called an *event*. The measure g is to be interpreted as a formalization of the ability of an event to occur.

The σ-algebra $\mathcal{B}$ is determined by the designer. $\mathcal{B}$ characterizes the ability of the designer to make statements about the NPS. The number of subsets within $\mathcal{B}$ is determined by the designer's ability to characterize the NPS.

That events in the NPS have the structure of a σ-algebra must be justified. Formally, this means the NPS has an associated collection of subsets $\mathcal{B}$ that satisfy, $\forall\, N_i,\, N_j \in \mathcal{B}$:

$$\begin{array}{rl} i) & \overline{N_j} = \text{NPS}\backslash N_j \in \mathcal{B} \\ ii) & \bigcup_{j \in J} N_j \in \mathcal{B} \end{array} \tag{3.3}$$

where J is an index set. Thus, $i)$ states that for all events N_j in the set $\mathcal{B}$, not-an-event ($\overline{N_j}$) is equal to the Noise Parameter Space with the event (N_j) removed. It also states that if event N_j can occur, then $\overline{N_j}$ can also occur (or, stated differently: N_j can not occur). Also, $ii)$ states that the union of all events N_j (where j is in the index set J) are in the set $\mathcal{B}$. If index set J contains 1 and 2, and N_1 and N_2 are contained in $\mathcal{B}$ (which means that N_1 and N_2 can occur separately), then the union of N_1 and N_2 can occur. Stated differently: either N_1 or N_2, or both can occur. These are true for any sequence of events. DeMorgan's laws also hold on subsets:

$$\begin{array}{rl} iii) & \overline{(N_i \cup N_j)} = \overline{(N_i)} \cap \overline{(N_j)} \\ iv) & \overline{(N_i \cap N_j)} = \overline{(N_i)} \cup \overline{(N_j)}. \end{array} \tag{3.4}$$

Thus $iii)$ states that the complement (negation) of the union of N_i and N_j is the same as the intersection of not-N_i and not-N_j. Stated differently: if either N_i or N_j do not occur, then both N_i and N_j do not occur. Finally, $iv)$ states that the complement (negation) of the intersection of N_i and N_j is the same as the union of not-N_i and not-N_j. Stated differently: if neither N_i or N_j can occur together, then either N_i or N_j can not occur.

These assumptions are sufficient to show the collection $\mathcal{B}$ forms an algebra over the subsets [6]. Thus, events in $\mathcal{B}$ also satisfy:

$$\begin{aligned} ii) &\quad \textstyle\bigcup_{j \in J} N_j \in \mathcal{B} \\ v) &\quad \textstyle\bigcap_{j \in J} N_j \in \mathcal{B} \end{aligned} \tag{3.5}$$

where J is an index set. The algebraic structure of a noise space is not new to this work, it is historically well developed [6] (for probability).

Given an uncertain space (NPS,$\mathcal{B}$,g), the NPS is characterized. What is desired is to rate a design configuration, given these noise effects. To do so, a disjoint collection of subsets whose union is the entire NPS, or a *partition*, is needed. On each subset in the partition, the effects of the noise will be determined, and then each rating on each subset in the partition will be incorporated into an overall rating across the partition (across the NPS). Not just any partition is used, but the limit in refinement of any sequence of partitions within $\mathcal{B}$. Thus, the most accurate rating of the noise is used, given what the designer can state about the noise.

Probabilistic Uncertainty A particular uncertainty form that can be used to model the NPS occurs when the events are random. For example, inaccuracies in measurements and manufacturing are usually modeled as random. Such inaccuracies form what is now termed a *probability space*. The probability space will be denoted NPS, meaning all uncertainties in the NPS are now considered probabilistic, for this section.

Given the probability space, an uncertainty measure g is constructed, and denoted Pr. Pr measures the probability of an event occurring. A restriction of the Pr measure is that the probability of an event occurring and not occurring must equal the probability of the certainty event (assumed to be 1) under real addition. Further, the probability of either of two disjoint events occurring must equal the real additive probability of the two events. These restrictions are sufficient to derive an uncertainty measure Pr [2, 3, 4, 7, 18].

Thus, for events N_j, $N_k \in \mathcal{B}$ such that $N_j \cap N_k = \emptyset$, if g is restricted to obey:

$$\begin{aligned} g(N_j) + g(\text{NPS} \backslash N_j) &= 1 \\ g(N_j) + g(N_k) &= g(N_j \cup N_k) \end{aligned} \tag{3.6}$$

then g is a probability measure of classical probability theory [2, 7, 18].

If uncertainty information is included in design decision making, it is because the variational effects are to be minimized by proper selection of the nominal value of the imprecise variables. This is determined by weighting the preference of a design parameter set by its probability of occurring through the probabilistic uncertainty. This implies that given a probability space $(\text{NPS}, \mathcal{B}, Pr)$, the preferential performance of a point $d \in \text{DPS}$ is defined by

$$\mu(d) = \int_{NPS} \mu(d, n)\; dPr \tag{3.7}$$

where $\mu(d, n) = \mathcal{P}(\mu_1, \ldots, \mu_N)$. This integral is the standard Lebesgue integral from measure theory [6, 12]. Thus, $\mu(d)$ is the probabilistic expectation of $\mu(d, n)$ across the probability space with respect to the probability measure Pr. The design configuration d^* to use is the one which maximizes the expected performance of Equation 3.7 across the DPS.

If the NPS is a discrete NPS $\simeq \mathbb{Z}_q$, then each of the individual events $\{n_j\}$ forms a suitable partition, where each has an associated discrete $Pr(\{n_j\})$. The integral of Equation 3.7 then becomes a simple finite sum. If the NPS $\simeq$ $\mathbb{R}$, then a partition of the form $[n, n + dn)$ might be used. This can usually be characterized by a probability density function $pdf(n)$. The integral of Equation 3.7 then becomes

$$\mu(d) = \int_{NPS|d} \mu(d, n)\; pdf(n|d)\; d(n|d) \tag{3.8}$$

a Riemann integral over the NPS. Since the distributions over n can, as before, vary with d, we here use the notation $n|d$. Of course the PPS, preferences, and density functions must all be Riemann integrable for this to hold.

This definition will produce a change in the "best" overall solution from the case without probabilistic uncertainty. Consider a one parameter design, with a preference μ and a pdf probability density as shown in Figure 3.1. The pdf is the same for all design parameter values. The maximal *expected* preference, *i.e.*, the maximum of the $E[\mu]$ curve (which is the result of applying Equation 3.7), not the μ maximum, is found. This set (d^*) is shown on the design parameter axis.

The resulting $\mu(d)$ determined from the integral will be the expected value of preference given the probabilistic uncertainty. One could also evaluate the higher moments of this relation to determine the standard deviation, skew, and kurtosis. Indeed, one could perform a Monte-Carlo simulation for each design parameter set $d \in \text{DPS}$ or, if possible, analytically determine the probabilistic uncertainty distribution of the $\mu(d)$. This additional information, however, is usually less important and is computationally expensive.

For designs with no probabilistic uncertainty, the parameters in the design model that are not design parameters are all crisp numbers. In such circumstances, Equation 3.7 reduces to Equation 3.1, since the probability density

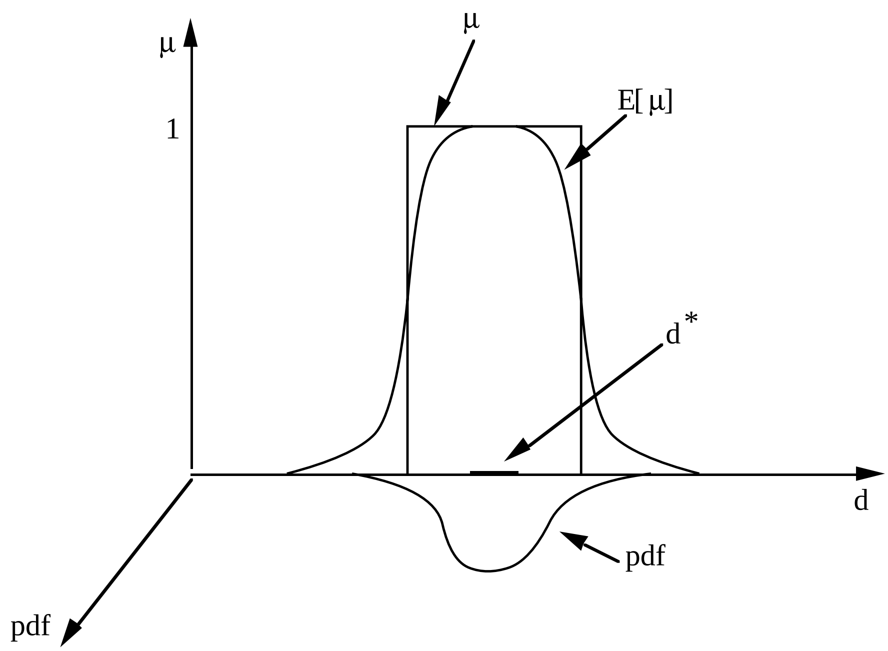

Figure 3.1 Parameter resolution with probabilistic uncertainty.

functions reduce to delta functions at the crisp values, which the integral of Equation 3.7 isolates to produce Equation 3.1.

The relation between this evaluation and Taguchi's method and experimental design in general can also be demonstrated. The solution to Taguchi's method has been shown to be an approximation to the solution which gives the highest quality (with a suitable definition of quality) [9]. The definition in Equation 3.7 and Taguchi's method are similar in this quality concept: Taguchi's method incorporates an experimental approximation to the integral in Equation 3.7 across probabilistic noise. The difference is that Taguchi's method is finding the mean of a single f (the S/N ratio), not preference over many design parameters and performance parameters.

The relationship this method and experimental design have is demonstrated by observing that the integral of Equation 3.7 can be approximated by experimental points in the noise space; *i.e.*, experimental design techniques can be used. These points can be chosen using a factorial method. Fractional factorial methods are possible and suggested. See [1, 9] for a discussion of factorial methods and methods for determining experimental points.

Possibilistic Uncertainty Possibility is the uncertainty in the limits of capacity within a formal model. Possibility can be used to represent parameters within a formal model that the designer does not have choice over, and that are not characterized by probability. Subjective choices of others (not the designer), for example, can be modeled with possibility. Thus, a possibilistic variable can have a range of values, but the range is limited by another person's choices.

Similarly to the previous form of probability, possibility will be formalized into what is now termed a *possibility space*. The possibility space will be denoted NPS, meaning all uncertainties in the NPS are now considered possibilistic, for this section.

Given the possibility space, an uncertainty measure g is constructed, and denoted Π which measures the possibility of an event occurring. For all events, either the event or not-the-event is possible. Also, if two events are possible, a choice must be made between them (but not by the designer). If two events can occur, it is assumed that some other person or agent will choose to maximize the performance (such as an adjustment or tuning during manufacture, testing, or operation). This means that when using either of two events, the performance will be the greater of the two, since this option would be chosen. Therefore, for possibilistic events $N_j, N_k \in \mathcal{B}$:

$$\begin{aligned} \max\{g(N_j), g(\text{NPS}\backslash N_j)\} &= 1 \\ g(N_j \cup N_k) &= \max\{g(N_j), g(N_k)\}. \end{aligned} \tag{3.9}$$

Equations 3.9 defines a possibility measure [5].

This formalism allows the measurement of the effects of noise on the performance, given any fixed design parameter arrangement d. Across the NPS, a disjoint collection of subsets $N \in \mathcal{B}$ whose union is the whole NPS is used (a partition of the NPS). The effect of each possible event N can be accounted for by determining the performance μ at a point $n \in N$, and then ensuring $\Pi(N)$ is within capacity at this evaluation point. Since the design is limited by the capacity Π, the design can be rated no better than the capacity Π. After this attenuation, the best possibility in the NPS can be used.

Thus, the integral of performance across the possibility space becomes the maximum possible performance, with performance attenuated to be possible to the degree specified by the possibility measure. Given a possibility space $(\text{NPS}, \mathcal{B}, \Pi)$, the preferential performance of a point $d \in \text{DPS}$ is

$$\mu(d) = \sup\left\{\min\{\mu(d,n), \Pi(N)\} \mid N \in \{N_j\} \text{ disjoint} \subset \mathcal{B}\right\} \tag{3.10}$$

where $\mu(d,n) = \mathcal{P}(\mu_1, \ldots, \mu_N)$, $n \in N$. This integral is a form of the Sugeno integral of possibility theory [14]. Note the *sup* is across the subsets of the partition $\{N_j\}$, and the limit as the partition becomes finer in $\mathcal{B}$ is used. Thus, $\mu(d)$ is the possibilistic expectation of $\mu(d,n)$ across the possibilistic noise space with respect to the possibility measure Π.

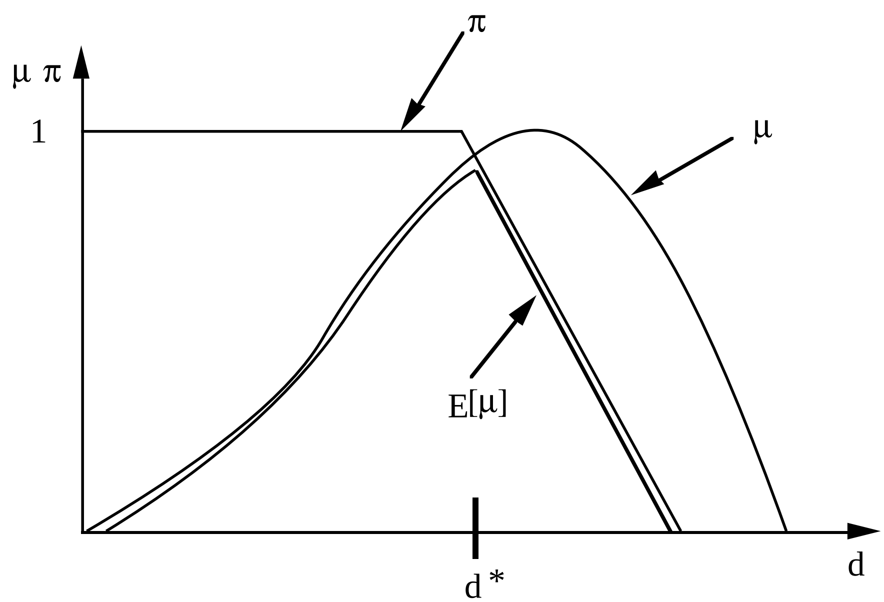

Figure 3.2 Parameter resolution with possibilistic uncertainty.

This definition resolves a different best solution. Consider a one parameter design, with a μ preference and a π possibility density as shown in Figure 3.2. The maximum expected preference, *i.e.*, the maximum of the $E[\mu]$ curve (using the possibilistic expectation of Equation 3.10), not the μ maximum, is found. This set is a point (d^*) as shown on the design parameter axis.

If all the points in the noise space are equally possible, ($\Pi(N) = 1 \; \forall \; N \in \mathcal{B}$), then the development reduces to a simple max of $\mu(d, n)$ across the noise and design space, *i.e.*, this reduces to finding the max of $\mu(d)$ across the design space, as shown in Equation 3.1.

3. Necessary Parameters

So far this presentation has assumed that all the parameters can take on any value within their distribution ranges. Alternatively, the design may need to satisfy *every* value of a noise parameter to the degree specified by the distribution. Such parameters are termed *necessary parameters* [19].

A design's noise parameters can be grouped into those that must be satisfied across their entire range of variation, and those which any single value in the allowable range can be used. This distinction corresponds to whether a parameter should be modeled as a necessary parameter.

Two forms of necessary parameters are recognized: probabilistic necessity and possibilistic necessity. Probabilistic necessity arises when a parameter varies probabilistically and the designer wishes to ensure the design will function for the entire range of variation. Possibilistic necessity arises in a similar situation, but when the parameter varies possibilistically.

When using necessity, the degree of satisfaction desired for a necessary parameter must be specified, and shall be denoted α. α is therefore a number in $[0, 1]$, it is not a distribution, and reflects the domain percentage, as measured by the underlying uncertainty measure (either Pr or Π), that the designer feels must be ensured. α will be termed the *confidence factor*. If $\alpha = 0$, then only the most likely value is considered. If $\alpha = 1$, then all values in the NPS must be satisfied. For some design problems, α might need to be expressed absolutely, *i.e.,* $\alpha = 0.999999$ to satisfy designs to six standard deviations. For others, α might be made a function of the preference μ, so when the achievable preference is high, the necessity range is increased (α is increased); when the achievable preference is low, the necessity range is decreased (α is decreased), easing the degree of difficulty in satisfying the design.

Thus, there is a requirement of determining the domain (NPS) percentage required to be satisfied. A subset $\mathcal{N}_\alpha \in \mathcal{B}$, called the *necessary set*, is defined as the subset of the NPS that is desired to be satisfied.

Given the necessary subset $\mathcal{N}_\alpha$, any other subset N in $\mathcal{B}$ can be identified as necessary. If N lies within $\mathcal{N}_\alpha$, then the subset N is necessary, otherwise it is not. Thus, a necessity measure of each subset N can be constructed given the necessary subset $\mathcal{N}_\alpha$. This measure can be defined by

$$\mathrm{N}_\alpha(N) = \begin{cases} 0 & N \subseteq \mathcal{N}_\alpha \\ 1 & N \nsubseteq \mathcal{N}_\alpha. \end{cases} \tag{3.11}$$

Any measure g satisfying Equation 3.11 will be called a *necessity measure*, and is denoted by N_α. Thus, only if all points within a set N are necessary does the set N become necessary.

The necessity measure is a $\{0, 1\}$ placed measure. The designer uncertainty will be incorporated into $\alpha \in [0, 1]$, to determine the extent of $\mathcal{N}_\alpha$. Thus the uncertainty is incorporated into specification of $\mathcal{N}_\alpha$ (the necessary set), not in N_α (the measure of a set in $\mathcal{B}$).

An uncertainty integral can be constructed with the necessity measure. The integral of performance becomes the worst case performance across the necessity space, as measured by the necessity measure. Given a necessity space $(\text{NPS}, \mathcal{B}, \mathrm{N}_\alpha)$, the preferential performance of a point $d \in \text{DPS}$ is defined by

$$\mu_\alpha(d) = \inf \{\max\{\mu(d, n), \mathrm{N}_\alpha(N)\} \mid N \in \{N_j\} \text{ disjoint} \in \mathcal{B}\} \tag{3.12}$$

where $\mu(d,n) = \mathcal{P}(\mu_1, \ldots, \mu_N)$, $n \in N$. Again, the limit as the partition becomes finer in $\mathcal{B}$ is used. Thus, $\mu_\alpha(d)$ is the necessary expectation of $\mu(d,n)$ across the necessary space with respect to the necessary measure N_α.

The problem is thus well formed, provided the set $\mathcal{N}_\alpha$ can be identified. How much of the NPS should be ensured? For an NPS $\simeq \mathbb{R}^q$ independent, this has typically been done with *confidence intervals* [16]. This formalism, however, assumes that the underlying uncertainty measure (Pr or Π) is constructed from a density function (pdf or π), which requires an ordering on the NPS. In any case, the portion of the uncertainty space to be satisfied is identified as the *confidence factor*: α.

3.1 Probabilistic Necessary Parameters

When the underlying uncertainty is probabilistic, the density function that can be used to partition an ordered NPS is the probability density function, pdf : NPS $\rightarrow \mathbb{R}^+ \cup \{0\}$. The pdf can be used to define the necessary subset of the NPS, for any confidence factor α:

$$\mathcal{N}_\alpha = \{n \in \text{NPS} \mid pdf(n) \geq \Theta\} \tag{3.13}$$

where

$$\Theta = \inf \left\{ \theta \;\middle|\; pdf(n) \geq \theta \text{ and } \int_{NPS} \chi_{\{pdf(n)\geq\theta\}}(n)\; dPr \leq \alpha \right\}. \tag{3.14}$$

Thus, Θ is the lowest level pdf value with α equal to the Pr of all n whose $pdf \geq \Theta$. This forms the set of elements n whose total $Pr = \alpha$, and whose n all have $pdf \geq \Theta$.

For example, in the case of NPS $\simeq \mathbb{R}$, and pdf as the normal distribution, $\mathcal{N}_\alpha$ becomes a class interval:

$$\mathcal{N}_\alpha = [E - r, E + r]$$

where E is the expected value, and r is a radial distance from the expected value such that $Pr([E-r, E+r]) = \alpha$.

The most preferred design parameter set and the maximal preference in light of probabilistic necessary parameters n then becomes:

$$d^* : \mu_\alpha(d^*) = \sup \{\inf \{\mu(d,n) \mid n \in \mathcal{N}_\alpha\} \mid d \in DPS\}. \tag{3.15}$$

To see this definition's meaning, consider a one variable design with a probabilistic necessary distribution, as shown in Figure 3.3. Each value along the design parameter axis is uncertain because of the normal probabilistic variation as shown. Therefore the preference curve μ must be reduced at each point

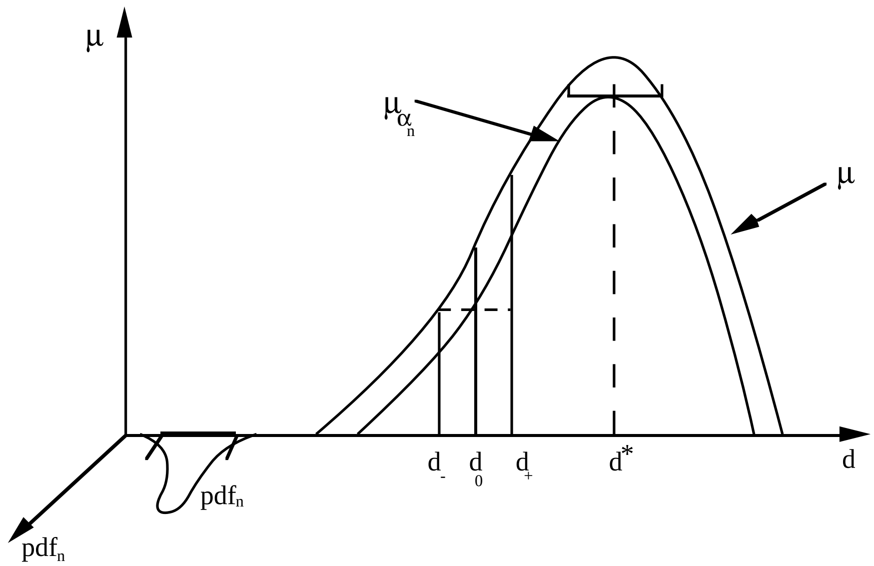

Figure 3.3 Probabilistic necessary parameter preference resolution.

to the lowest preference it can take on, given a probabilistic variation, and a degree. For example, as shown in Figure 3.3, at a design parameter value d_0 a 95% confidence interval ($\alpha = 0.95$) can produce variations in the range from d_- to d_+. The lowest preference μ in that range $[d_-, d_+]$ becomes the μ_α for d_0. This is repeated for all design parameter points to obtain the μ_α curve. The maximum of this μ_α curve is the "best" solution, that is, the most preferred design parameters subject to the necessary probabilistic distribution.

3.2 Possibilistic Necessary Parameter

When the underlying uncertainty is possibilistic, the density function which can be used to partition the NPS is the possibility density function π : NPS $\rightarrow$ $[0, 1]$. π can be used to define the necessary subset of the NPS, for any confidence factor α:

$$\mathcal{N}_\alpha = \{n \in \text{NPS} \mid \pi(n) \geq 1 - \alpha\}. \tag{3.16}$$

The most preferred design parameter set and the maximal preference in light of possibilistic necessary parameters n then becomes:

$$d^* : \mu_\alpha(d^*) = \sup\left\{\inf\left\{\mu(d, n) \mid \pi(n|d) \geq 1 - \alpha, n \in \text{NPS}\right\} \mid d \in \text{DPS}\right\}. \tag{3.17}$$

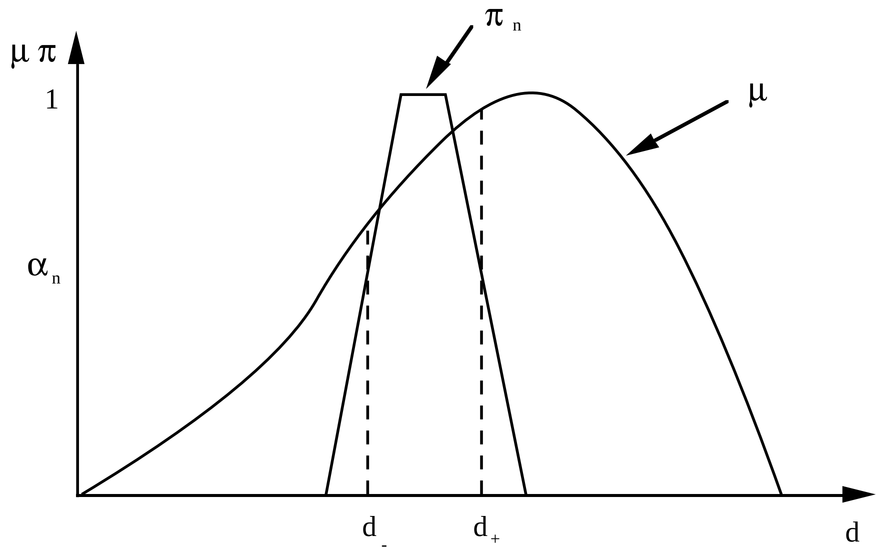

Figure 3.4 Possibilistic necessary parameter preference resolution.

This definition is illustrated in Figure 3.4. Here the π denotes the necessary region, and n is considered as the design parameter ($n = d$). Therefore the solution is the entire support of the π distribution, and the object is to rank the degree of preference for the range. Therefore the preference μ_α must be the lowest preference within the support of π. For example, as graphed in Figure 3.4, at a degree of necessity α, the domain of necessity is from d_- to d_+. The lowest preference μ in that range $[d_-, d_+]$ becomes the μ_α for the necessary range.

4. Hybrid Uncertainty

For problems with multiple uncertainty forms, Equations 3.7 through 3.17 must be combined. Such a combination is possible, but requires making explicit the *precedence relation* among the parameters. This is discussed in [9, 10] for Taguchi's method and optimization.

As an example, consider the design of a uni-directional accelerometer, which indicates accelerations above a threshold with a switch closure. It can be modeled as a simple mass spring system, as shown in Figure 3.5. Under specified accelerations, the accelerometer mass must contact a switch within specified time durations. Suppose, however, that the spring is a thin metal sheet manufactured by a stamping procedure. The inaccuracies introduced by the stamping manifest themselves as a variation in the value for k, the spring constant. This uncertainty occurs randomly. Hence due to the manufacturing process, it

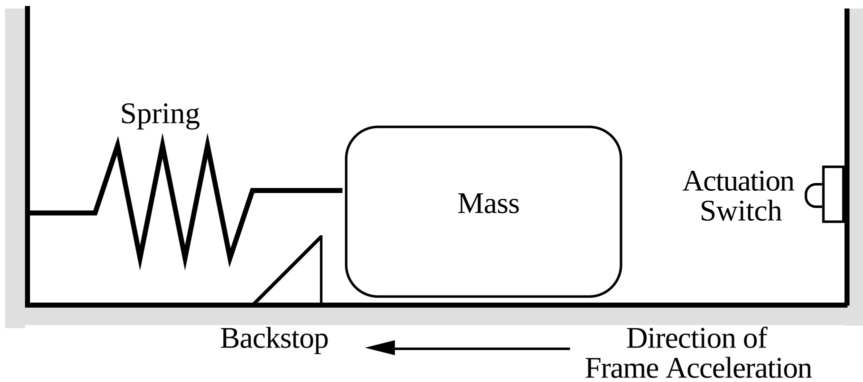

Figure 3.5 Example: accelerometer.

is difficult to set precise actuation times (time for the mass to move into contact with the actuation switch). The design has, however, a method to overcome these manufacturing errors in the spring. Specifically, during manufacturing, the backstop of the mass can be adjusted to compensate for variations in k. This backstop positioning distance is a tuning parameter of the design. During manufacture, the spring constant of every accelerometer is measured, and the backstop of each accelerometer is positioned accordingly to meet the specified actuation times.

Such parameters are denoted *tuning parameters*, and are introduced in [10]. They are not an artifact of the imprecision formulation, they exist in any formulation. They must be accounted for, however, when selecting the design parameters.

Such relations between design, confounding, and tuning parameters are readily modeled using imprecision. A tuning parameter's range of possibility forms a possibilistic uncertainty. Therefore one can combine Equations 3.7 through 3.17, but care must be taken that the equations are combined in their proper order: design parameters on the outside, and tuning parameters on the inside (relative to the confounding noise parameters).

Thus, with multiple forms of uncertainty, the d^* are chosen as

$$\mu(d^*) = \sup\left\{ \int_{Pr(\delta d)} \sup\left\{\mu(d, \delta d, t) \mid t \in \text{TPS}\right\} \times dPr(\delta d) \,\middle|\, d \in \text{DPS} \right\} \tag{3.18}$$

where δd are the manufacturing errors and t are the tuning parameters. The maximization of the tuning parameter selection occurs inside the integral across the manufacturing errors, and the maximization of the design parameter selection occurs outside the integral across the manufacturing errors.

It is not always true that possibilistic uncertainty has its evaluation inside the probabilistic integral. If there had been possibilistic uncertainty associated with the design parameters d to limit the designer's choice, then this combination occurs outside the manufacturing error integral. Similarly, if the designer had particular preferences for tuning parameter values, then this combination occurs inside the integral. The precedence relation among the variables is determined by the variable type (design, noise, and tuning parameters) not on the uncertainty forms associated with each parameter (imprecision, probability, and possibility).

For a general design problem, the evaluation order of the maximizations, minimizations, and integrals will depend on the precedence relation among the variables. This is not an artifact of the imprecision formulation. The same problem occurs with any other formulation (such as probabilistic optimization or an extended form of Taguchi's method); the reader is referred to [10] for demonstration of the precedence relation ordering in these other formulations. Imprecision simply sets the metric across the space to be preference (μ) rather than, for example, a single performance parameter expression.

Finally, the tuning parameter's value is a possibilistic uncertainty from the design engineer's perspective. It has a range of possible values and the value that should be used cannot be set by the design engineer, since it will depend on the manufacturing errors. But the expected value of the probabilistic manufacturing error can be determined. Since the possibilistic tuning parameter's values depend on the probabilistic manufacturing error, then from the design engineer's viewpoint (pre-manufacturing) the tuning parameter expected value can also be found. That is, the possibilistic tuning parameter will adjust its value based on the probabilistic manufacturing error. Hence, there will be, from the design engineer's viewpoint, a probabilistic distribution for the tuning parameter as well, even though it has no probability aspect associated with it at all. Taking a view of the tuning parameter before the noise occurs, it can be said to have a distribution. But, inherent in the parameter itself (*i.e.*, from the manufacturing engineer's perspective who must actually set the variable's value after the noise has occurred), the parameter has absolutely nothing to do with probability.

5. Example

The example presented herein considers the design of a pressurized air tank, and is the same problem as presented in Papalambros and Wilde [11], page 217. The reader is referred to reference [11] to see the restrictions applied to the problem to permit it to be solved using crisp constraints and an optimization methodology. The same problem is considered in a previous paper with no uncertainty, only imprecision and preferences [8]. The example is simple

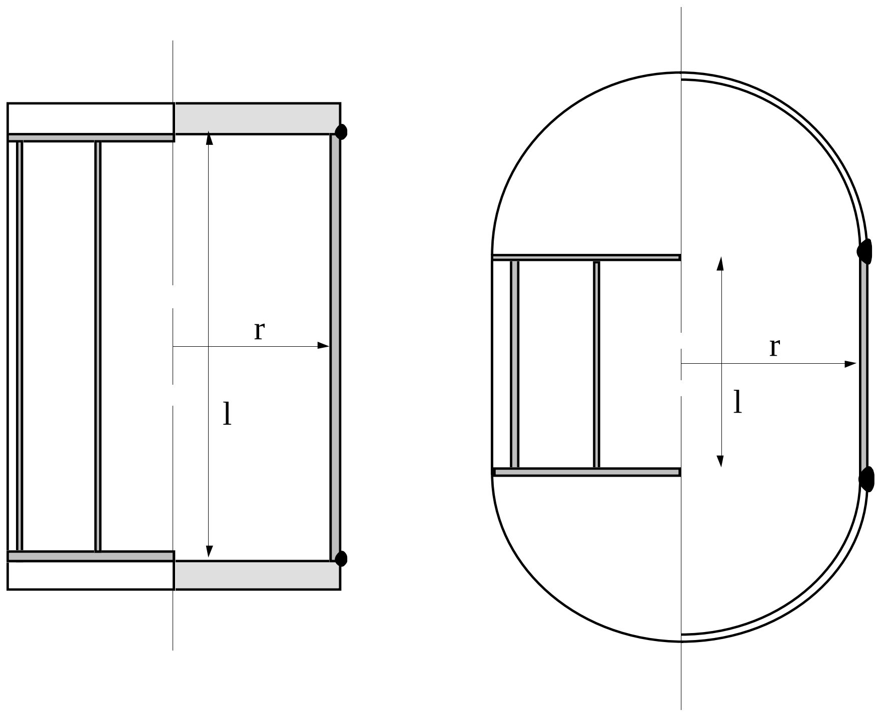

Figure 3.6 Hemispherical and flat head air tank designs.

but was chosen for that reason, and also the ability of its preferences to be represented on a two-dimensional plane for a visual interpretation.

The design problem is to determine length and radius values in an air tank with two different choices of head configuration: flat or hemispherical. See Figure 3.6.

There are four performance parameters in the design. The first is the metal volume m:

$$m = 2\pi K_s r^2 l + 2\pi C_h K_h r^3 + \pi K_s^2 r^2 l \tag{3.19}$$

This parameter is proportional to the cost, and the preference ranks of m are set because of this concern. Another is the tank capacity v:

$$v = \pi r^2 l + \pi K_v r^3 \tag{3.20}$$

This parameter is a measure of the attainable performance objective of the tank: to hold air, and the desired level of this performance ranks the preference for values. Another performance parameter is an overall height restriction L_0, which is imprecise:

$$l + 2(K_l + K_h) r \leq L_0 \tag{3.21}$$

Finally, there is an overall radius restriction R_0, which is also imprecise:

$$(K_s + 1)r \leq R_0 \tag{3.22}$$

The last two performance parameters have their preference ranks set as a result of spatial constraints.

The coefficients K are from the ASME code for unfired pressure vessels. S is the maximal allowed stress, P is the atmospheric pressure, E is the joint efficiency, and C_h is the head volume coefficient.

$$K_h = \begin{cases} 2\sqrt{CP/S} & \text{flat} \\ \frac{P}{2SE-.6P} & \text{hemi} \end{cases} \tag{3.23}$$

$$K_l = \begin{cases} 0 & \text{flat} \\ 4/3 & \text{hemi} \end{cases} \tag{3.24}$$

$$K_s = \frac{P}{2SE - .6P} \tag{3.25}$$

$$K_v = \begin{cases} 0 & \text{flat} \\ 1 & \text{hemi} \end{cases} \tag{3.26}$$

This example's design space is spanned by 2 design parameters l and r. The preferences for values of these design parameters and the four performance parameters are shown in Figures 3.7 through 3.12.

The problem, however, is confounded by noises. There are manufacturing errors on l and r that limit how well one can specify their values. We assume, for this example, that this error is Gaussian, however, noise with any distribution can be incorporated in the same manner. Error is also introduced by the supplied material variability. This error is manifested in the allowable stress S, which varies (in this example) with a beta distribution. The effects of these errors are desired to be minimized.

Finally, there is error introduced in the variability of the welds made. This error is manifested in the joint efficiency E, which varies (in this example) with a beta distribution. The effects of these errors must be reduced such that even the least efficient weld, within the chosen tolerance, will not fail, since failure in a weld represents a safety concern. Therefore, this error is modeled as a necessary probabilistic uncertainty.

The other unknown in the problem is the applied pressure P, which can vary with use. This is represented as a range of possibilistic necessity from -15 and 120 psi, since this design must satisfy all these pressures.

These distributions are shown in Figures 3.13 through 3.16. The necessary parameters used a value of $\alpha = 0.9$, or the variables were satisfied 90 percent of the time. These delimiters are also shown in the figure.

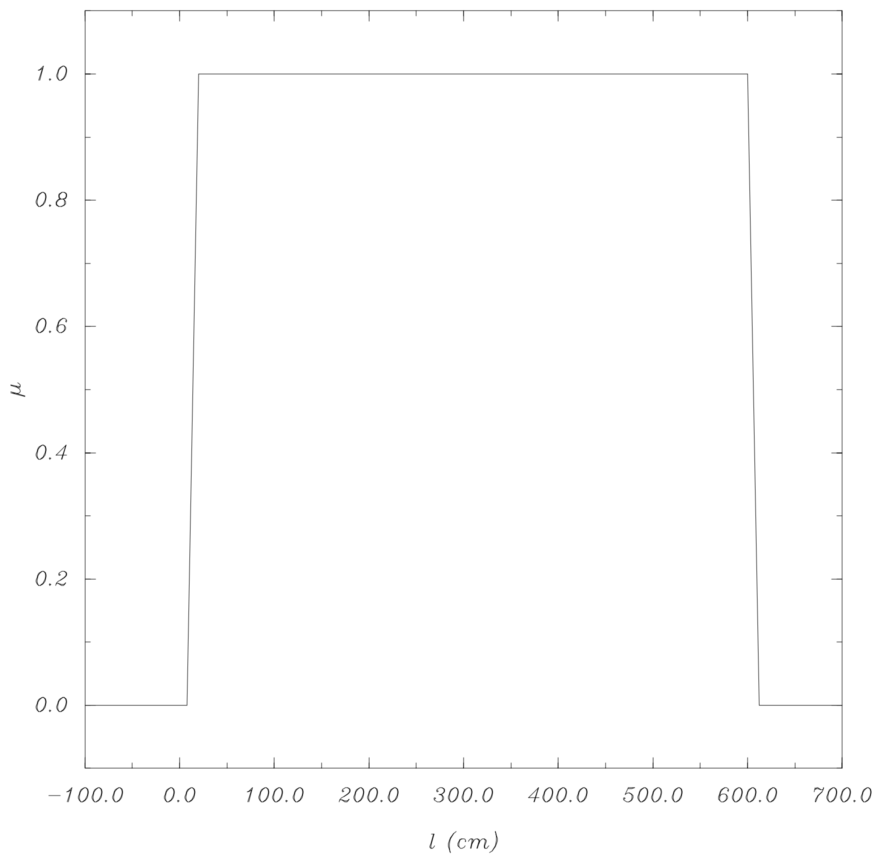

Figure 3.7 Length l preference.

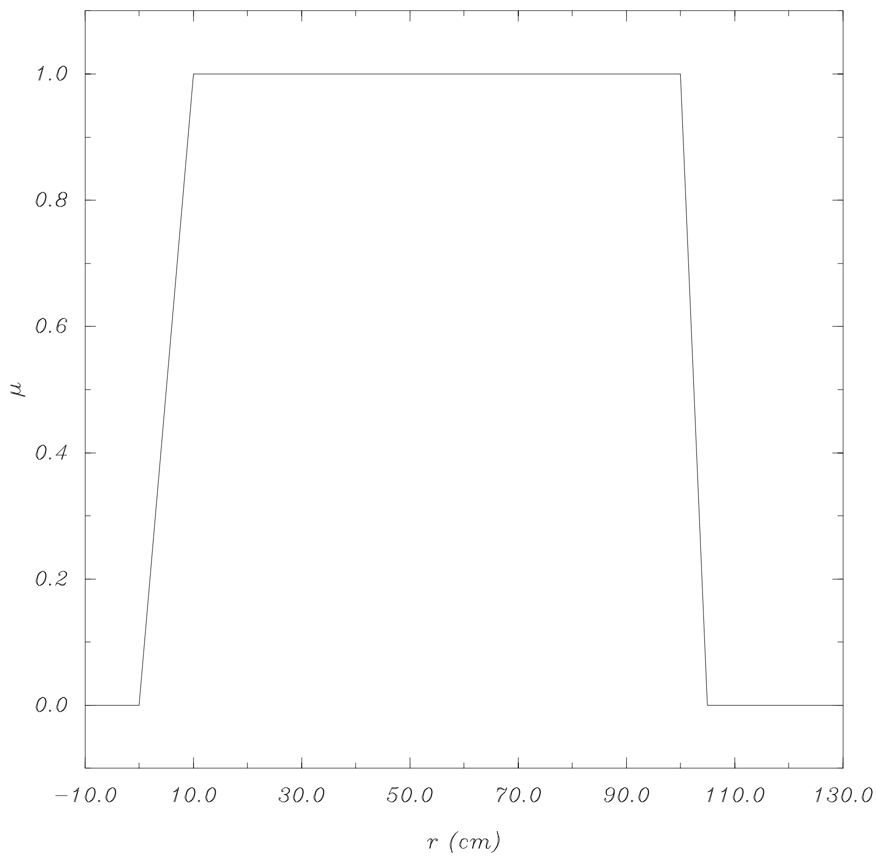

Figure 3.8 Radius r preference.

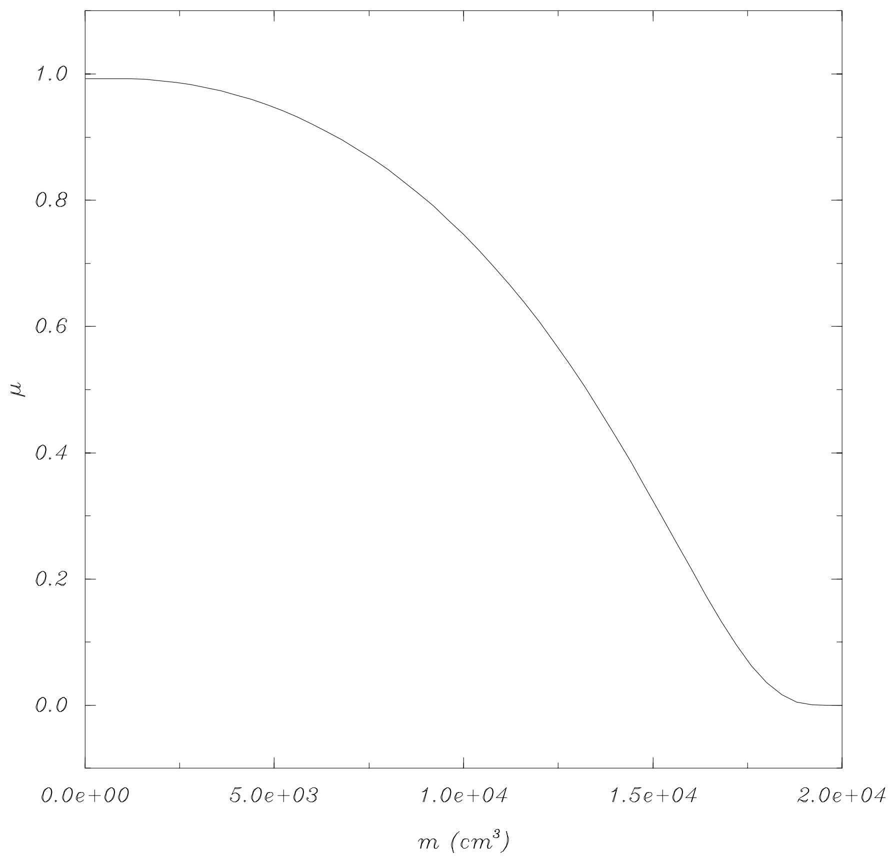

Figure 3.9 Metal volume m preference.

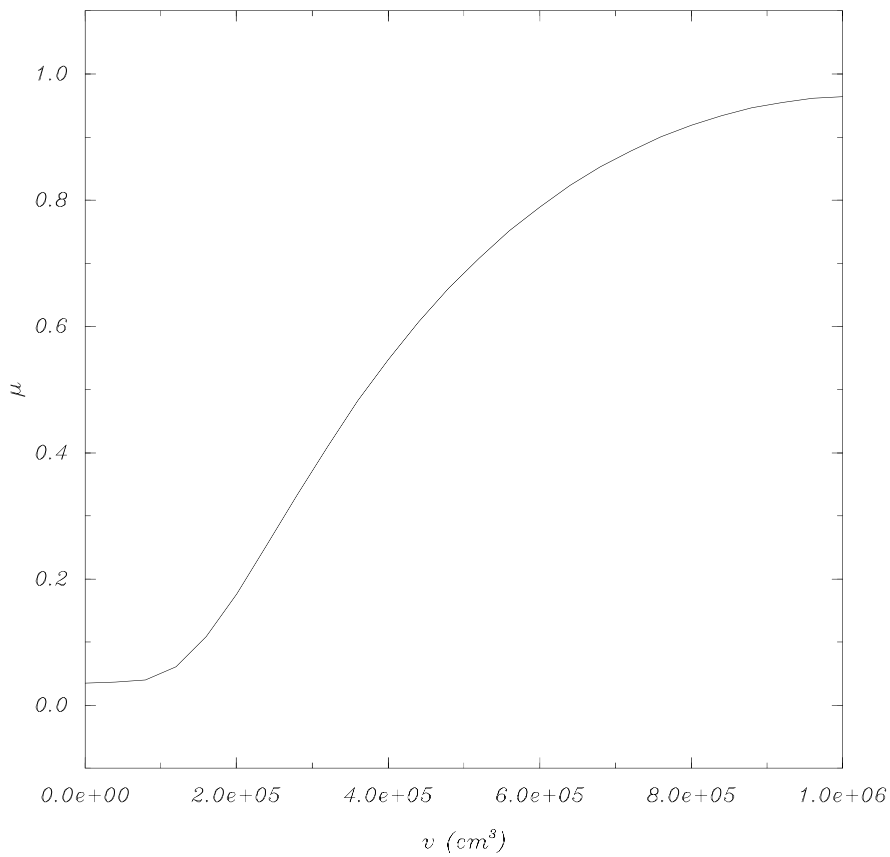

Figure 3.10 Capacity v preference.

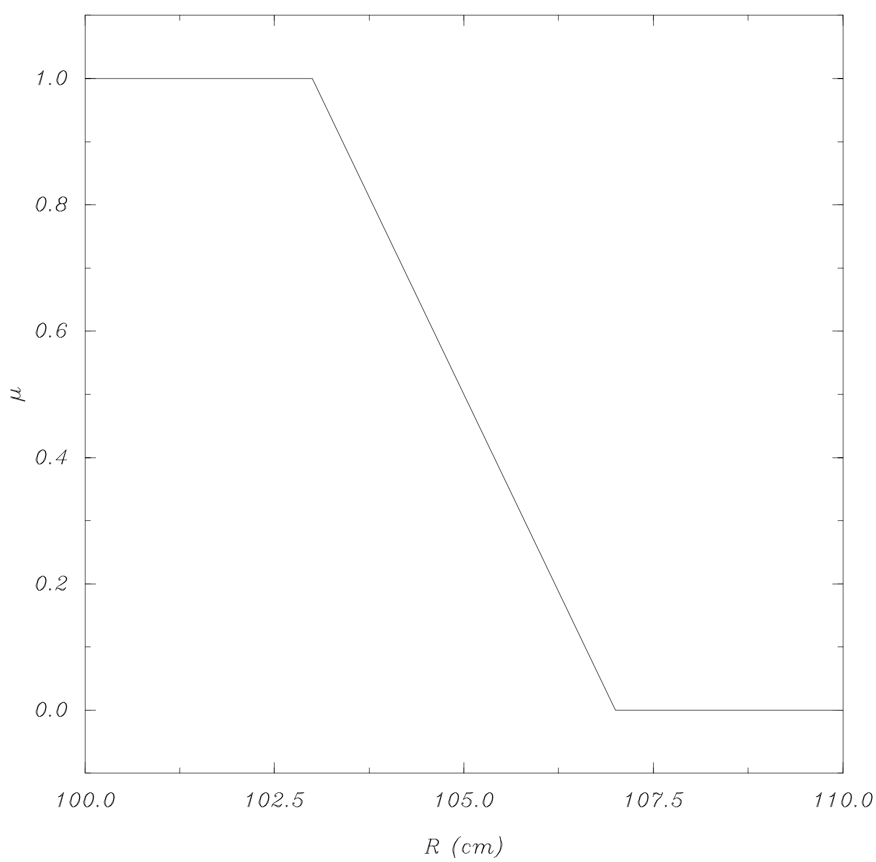

Figure 3.11 Outer radius R_0 preference.

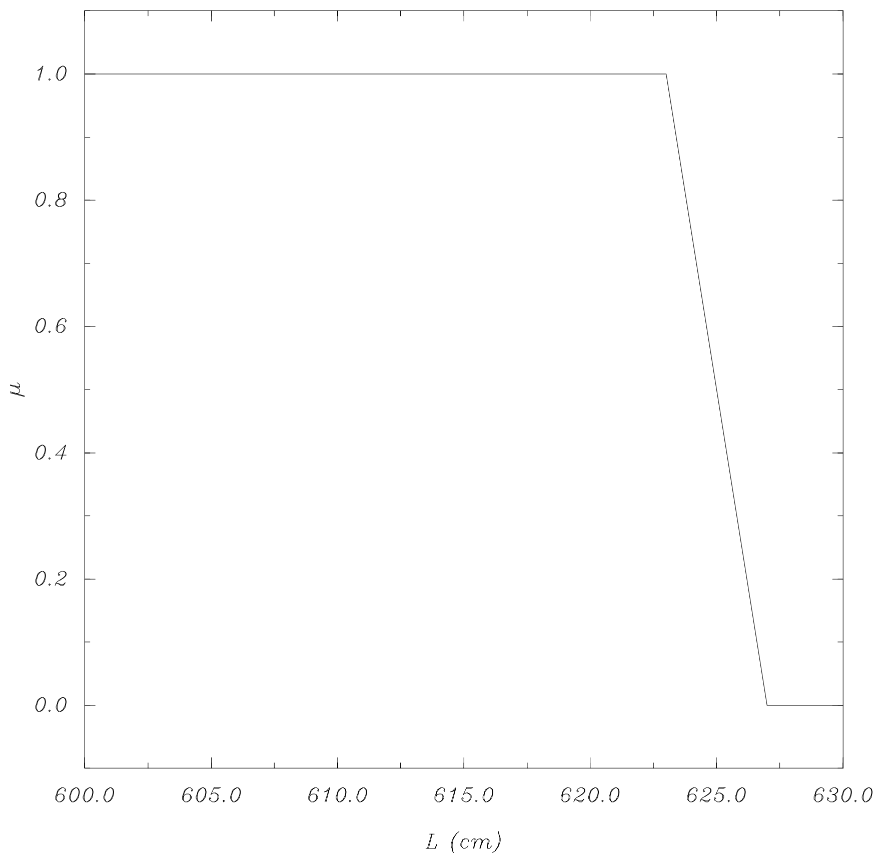

Figure 3.12 Outer length L_0 preference.

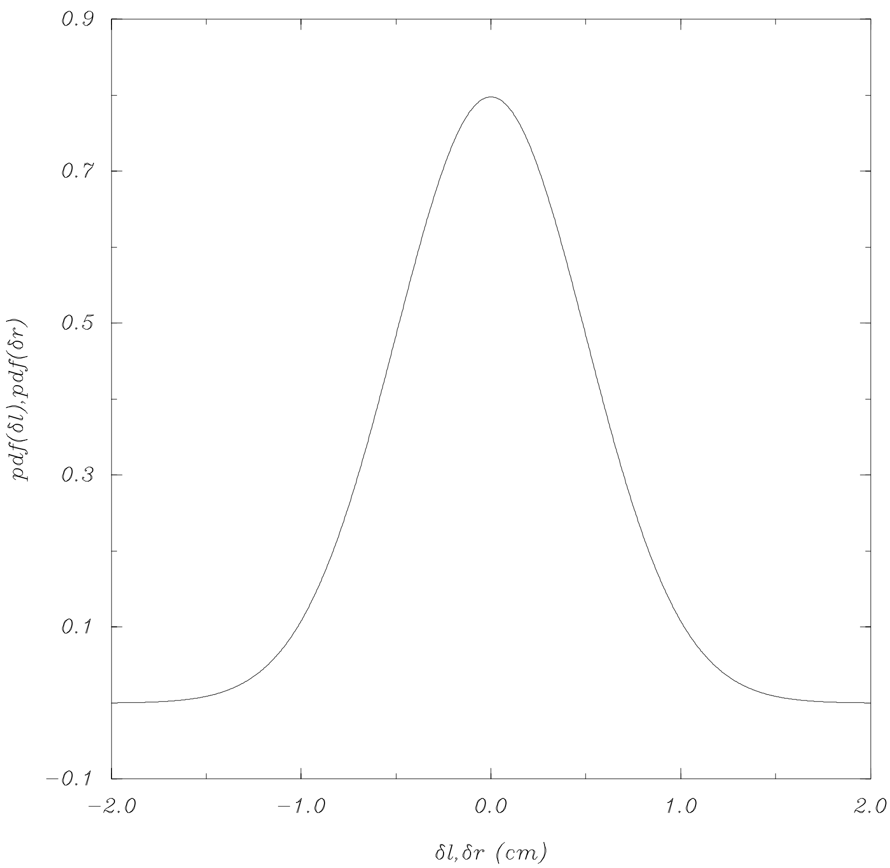

Figure 3.13 Length l and radius r uncertainty distribution

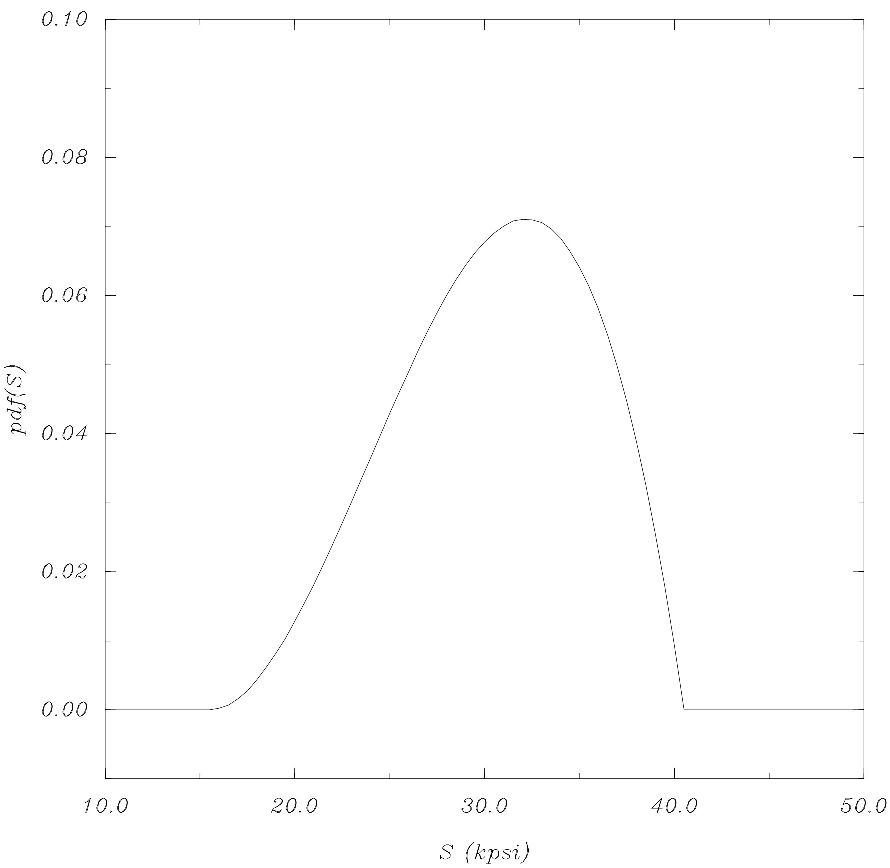

Figure 3.14 Allowable stress S uncertainty distribution

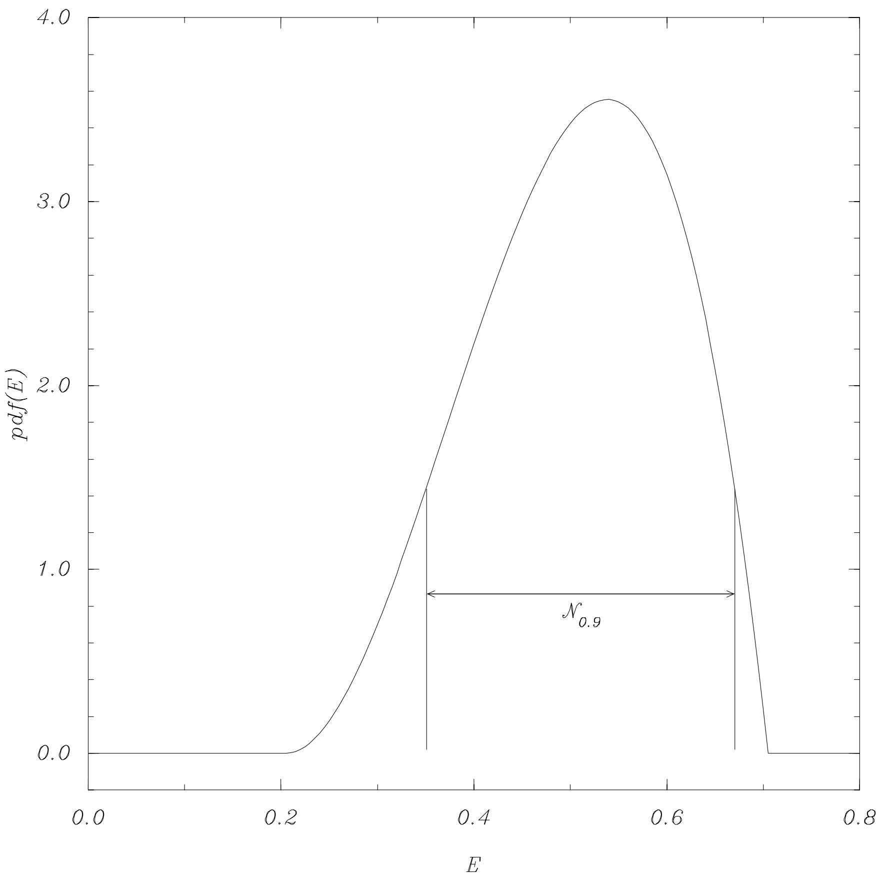

Figure 3.15 Joint efficiency E uncertainty distribution

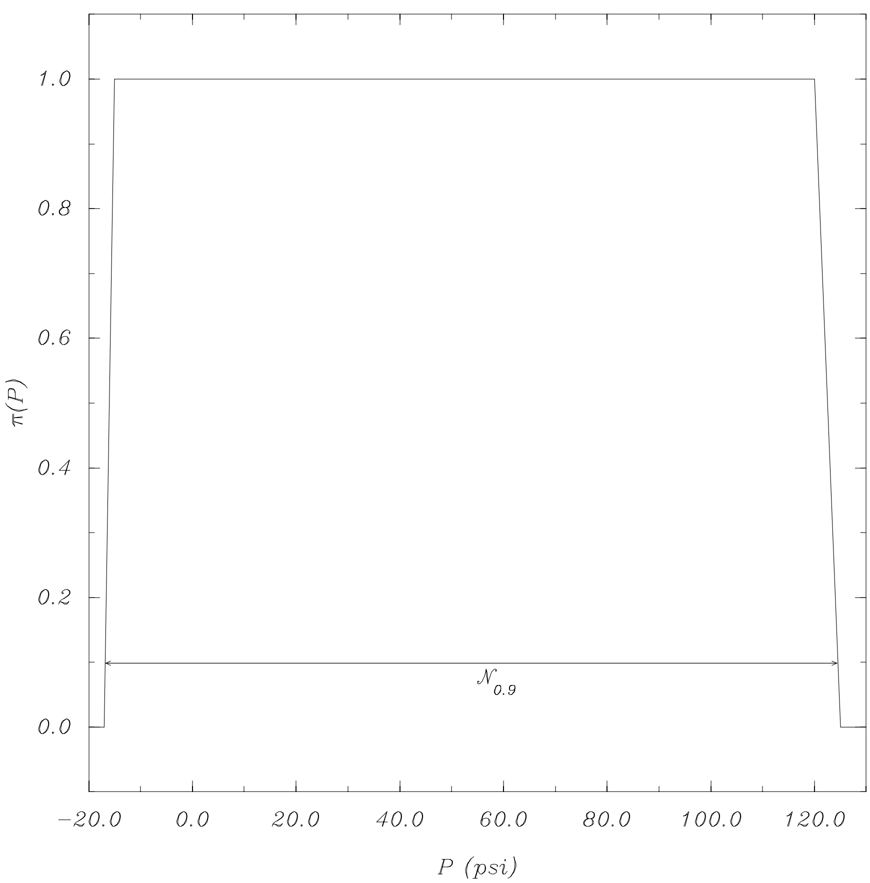

Figure 3.16 Applied Pressure P possibility distribution

The problem, then, is to find the values for l and r that maximize overall preference. It is yet to be determined how to evaluate this objective – *i.e.*, a strategy must be set [8]. In this particular example the non-compensating design strategy will be used. This means that among the multiple goals of the design, the worst performing goal will be improved, if any improvement can be made at all, by changing values of the design parameters l and r.

For a non-compensating design strategy, the problem to be solved is to find l^*, r^*, where:

$$\mu(l^*, r^*) = \sup\left\{\inf\left\{\int_S \int_{\delta r} \int_{\delta l} \min\{\mu_l, \mu_r, \mu_v, \mu_m, \mu_{L_0}, \mu_{R_0}\} \; pdf(S)\, pdf(\delta r)\, pdf(\delta l)\;\, dS\; d(\delta l)\; d(\delta r) \;\middle|\; (P, E) \in \mathcal{N}_{0.9}\right\} \;\middle|\; (l, r) \in \mathbb{R}^2\right\} \tag{3.27}$$

This will determine the l^* and r^* that maximizes the poorest design aspect's preference, yet considers the confounding probabilistic noise effects, and satisfies the necessary parameters 90 percent of the time.

The design space is shown in Figure 3.17 for the flat head design and in Figure 3.18 for the hemispherical head design. The peak preference point represents the design parameter values to use: those with maximum expected preference, given the designer specified preference curves, necessary distributions, and the noise distributions. The results are different from the case when no noise or necessity was considered. When only preference information is considered (the expected values are considered for the noise parameters), the resulting l and r values are chosen directly on the imprecise constraint boundaries, as shown in a previous paper [8]. The consideration of noise moves the chosen parameter values from the imprecise preference curves to more robust values, as shown in Figure 3.17.

This differs from the results of the various problem formulations presented in Papalambros and Wilde [11]. For example, the non-linear programming formulation solves the problem by minimizing the metal volume with the rest of the goals as crisp constraints. The preference formulation allows the constraints to be elastic, so the final design parameter values determined are different than if crisp constraints had been used. If the example had selected step functions for preference curves on the constraint performance parameters and no noise was considered, the results would reduce to a non-linear programming solution. The addition of noise to the problem with step function constraints would reduce the problem to a probabilistic optimization formulation, as discussed in Siddall [13].

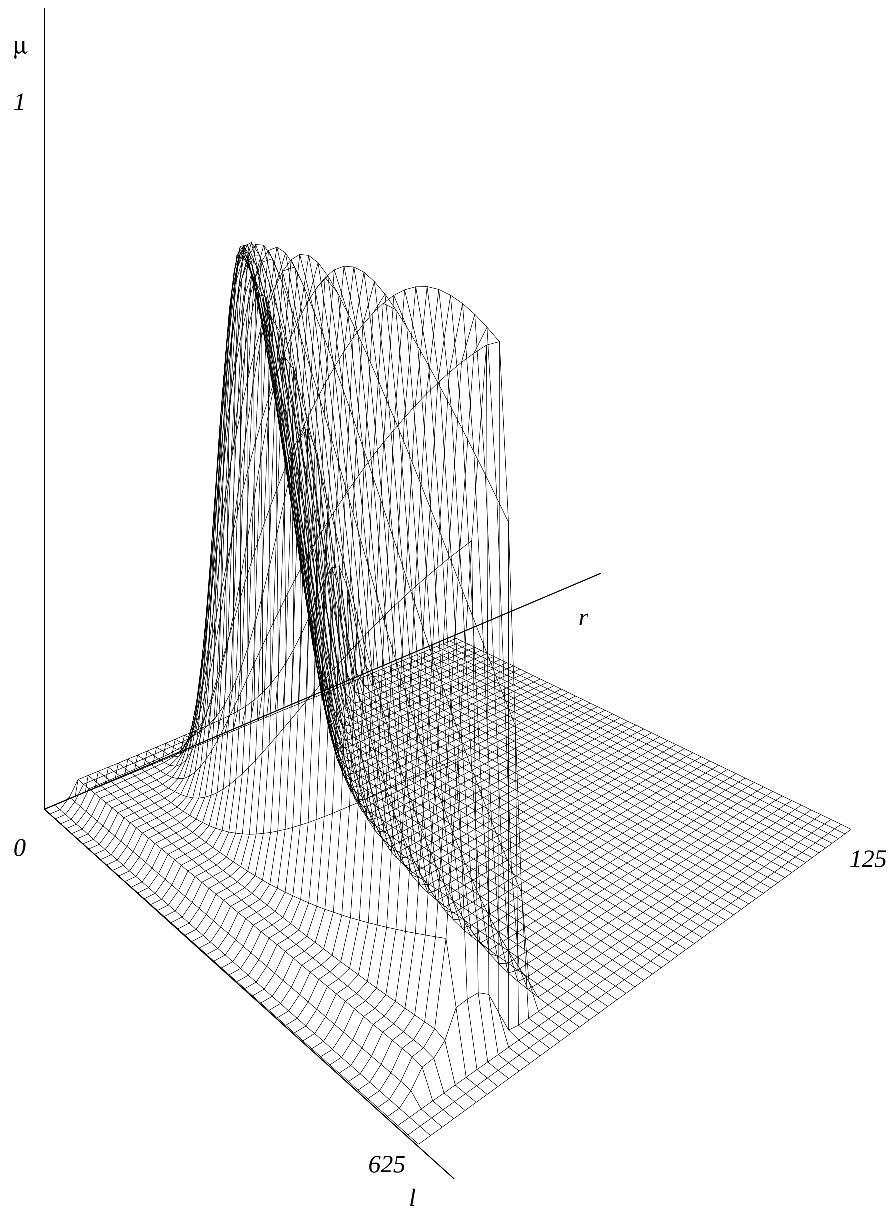

Figure 3.17 The expected preference across the design space (r,l) for the Flat head design.

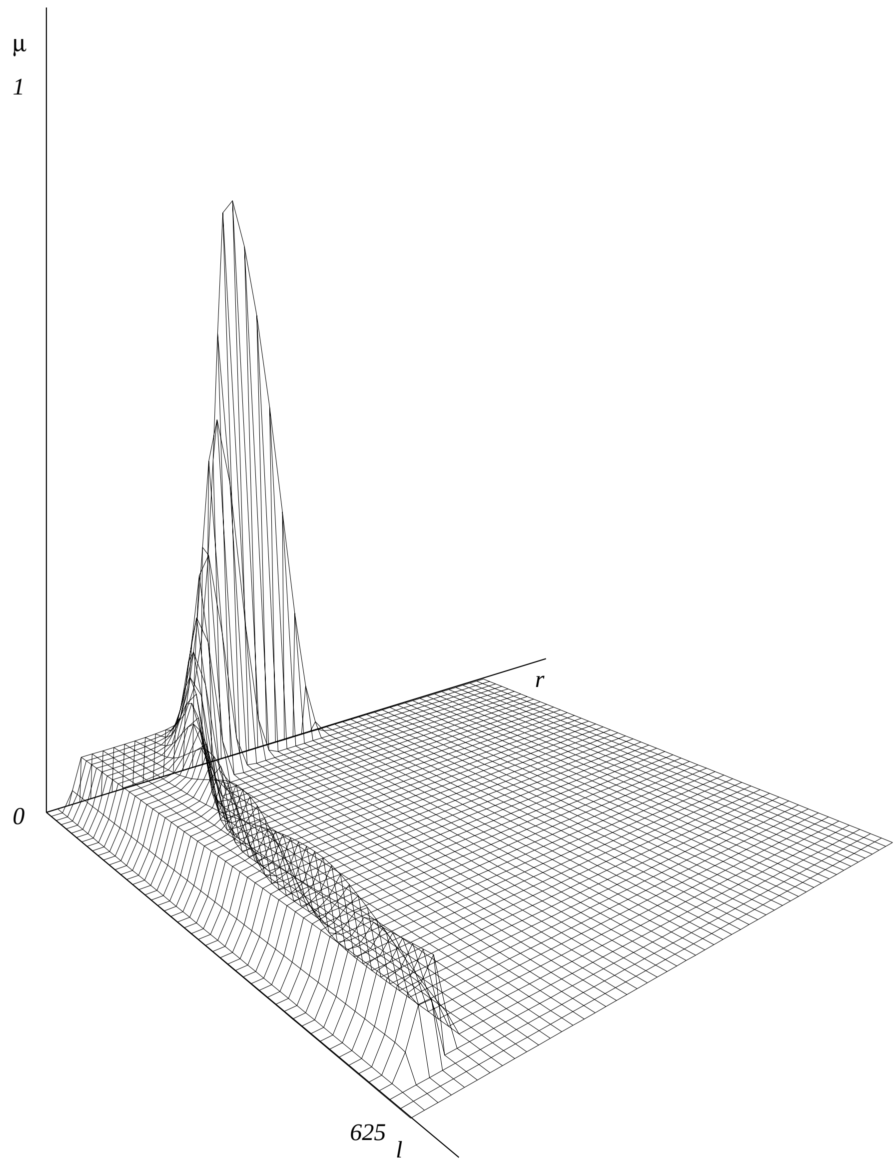

Figure 3.18 The expected preference across the design space (r,l) for the Hemispherical head design.

6. Conclusion

Imprecise preference functions have been used to solve design problems with multiple, incommensurate goals. A global metric is constructed across the design space from the preference rankings of the parameters. The result of applying the design's global metric has a simple interpretation as the overall preference for the design.

The imprecision methodology of resolving design parameter values is extended to include different confounding noise forms. Probabilistic noise requires the use of the probabilistic expectation process. This allows for the determination of the design parameter values that produce the highest overall quality. Possibilistic uncertainties, on the other hand, require the use of the possibilistic mathematics to confine the design to select the best value among only those that are possible.

The formulation is also extended to include necessary requirement forms. Necessary parameters can be either probabilistic or possibilistic in nature, and the parameter's necessity range is determined accordingly. Non-linear formulations are possible that tie α to μ. Such a formulation will permit trade-offs between preference and degree of necessity.

Design problems that have different variable types (design parameters, noise parameters, and tuning parameters) can now be solved. This work allows the designer to determine the "best" design parameter set to use, given uncertain design specifications and uncertainty in manufacturing processes.

Acknowledgments

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References

[1] BOX, G. E. *Statistics for Experimenters*. J. Wiley and Sons, New York, 1978.

[2] COX, R. T. *The Algebra of Probable Inference*. Johns-Hopkins University Press, Baltimore, MD, 1961.

[3] DUBOIS, D., AND PRADE, H. *Fuzzy Sets and Systems: Theory and Applications*. Academic Press, New York, 1980.

[4] DUBOIS, D., AND PRADE, H. A class of fuzzy measures based on triangular norms. *International Journal of General Systems 8* (1982), 43–61.

[5] DUBOIS, D., AND PRADE, H. *Possibility Theory: An Approach to the Computerized Processing of Information*. Plenum Press, New York, 1988.

[6] HALMOS, P. *Measure Theory*. Springer-Verlag, New York, 1974.

[7] KRANTZ, D., LUCE, R., SUPPES, P., AND TVERSKY, A. *Foundations of Measurement*, vol. I. Academic Press, New York, NY, 1971.

[8] OTTO, K. N., AND ANTONSSON, E. K. Trade-Off Strategies in Engineering Design. *Research in Engineering Design 3*, 2 (1991), 87–104.

[9] OTTO, K. N., AND ANTONSSON, E. K. Extensions to the Taguchi Method of Product Design. *ASME Journal of Mechanical Design 115*, 1 (Mar. 1993), 5–13.

[10] OTTO, K. N., AND ANTONSSON, E. K. Tuning Parameters in Engineering Design. *ASME Journal of Mechanical Design 115*, 1 (Mar. 1993), 14–19.

[11] PAPALAMBROS, P., AND WILDE, D. *Principles of Optimal Design*. Cambridge University Press, New York, NY, 1988.

[12] ROYDEN, H. *Real Analysis*. Macmillan, New York, 1988.

[13] SIDDALL, J. N. *Probabilistic Engineering Design; Principles and Applications*. Marcel Dekker, New York, NY, 1983.

[14] SUGENO, M. *Theory of fuzzy integrals and its applications*. PhD thesis, Tokyo Institute of Technology, 1974.

[15] SUGENO, M. Fuzzy measures and fuzzy integrals – a survey. In *Fuzzy Automata and Decision Processes* (New York, 1977), M. Gupta et al., Eds., North-Holland, pp. 89–102.

[16] SVESHNIKOV, A. *Problems in Probability Theory, Mathematical Statistics, and the Theory of Random Functions*. Dover Publications, New York, 1968.

[17] TAGUCHI, G. *Introduction to Quality Engineering*. Asian Productivity Organization, Unipub, White Plains, NY, 1986.

[18] TRIBUS, M. *Rational Descriptions, Decisions, and Designs*. Pergamon Press, New York, NY, 1969.

[19] WARD, A. C. *A Theory of Quantitative Inference for Artifact Sets, Applied to a Mechanical Design Compiler*. PhD thesis, MIT, 1989.

[20] WARD, A. C., LOZANO-PÉREZ, T., AND SEERING, W. P. Extending the constraint propagation of intervals. *Artificial Intelligence in Engineering Design and Manufacturing 4,* 1 (1990), 47–54.

[21] WILSON, I., AND WILSON, M. *From Idea to Working Model*. J. Wiley and Sons, New York, NY, 1970.

[22] WOOD, K. L., AND ANTONSSON, E. K. Computations with Imprecise Parameters in Engineering Design: Background and Theory. *ASME Journal of Mechanisms, Transmissions, and Automation in Design 111*, 4 (Dec. 1989), 616–625.

Chapter 4

TUNING PARAMETERS IN ENGINEERING DESIGN

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Abstract

In the design and manufacture of mechanical devices, there are parameters whose values are determined by the manufacturing process in response to errors introduced in the device's manufacture or operating environment. Such parameters are termed *tuning parameters*, and are distinct from *design parameters* which the designer selects values for as a part of the design process. This paper introduces tuning parameters into the design methods of: optimization, Taguchi's method, and the method of imprecision [10]. The details of the mathematical formulation, along with a design example, are presented and discussed. Including tuning parameters in the *design process* can result in designs that are more tolerant of variational noise.

1. Introduction

In the design, development, and manufacture of mechanical devices, parameter values are determined by different mechanisms. The device's geometry, power, etc. are chosen during the design process. However, there are usually variations on these values which are determined by mechanisms such as random manufacturing errors. There are also variations in the device's operating environment which are operator induced. But there are also parameters whose values are set during the manufacturing process in response to the previous variations. We denote these parameters as *tuning parameters*. Though they are common to the practicing industrial engineering community, we have found no formal models which incorporate their behavior.

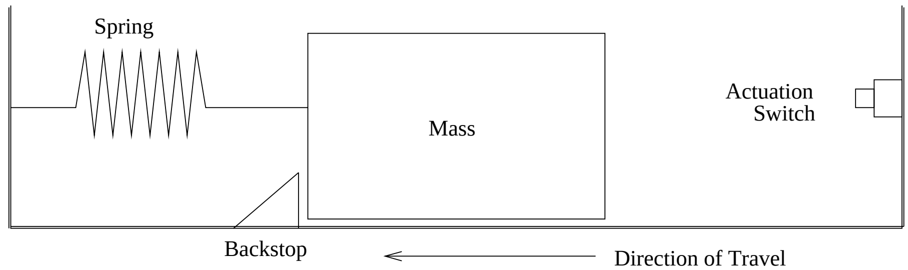

Figure 4.1 Accelerometer design.

As an example of a tuning parameter, consider the design of a uni-directional accelerometer, which indicates accelerations above a threshold with a switch closure. It can be modeled as a simple mass spring system, as shown in Figure 4.1. Under specified accelerations, the accelerometer mass must contact a switch within specified time durations. However, suppose the spring is a plate manufactured by a stamping procedure. The inaccuracies introduced by the stamping manifest themselves as an inaccurate value for k, the spring constant. This uncertainty occurs randomly. Hence due to the manufacturing process, it is difficult to set precise actuation times (time for the mass to touch the actuation switch). However, the design has a method to overcome these manufacturing errors in the spring. Specifically, during manufacturing, the backstop of the mass can be adjusted to compensate for variations in k. This backstop positioning distance is a tuning parameter of the design. During manufacture, the spring constant of every accelerometer is measured, and the backstop of each accelerometer is positioned accordingly to meet the specified actuation times.

This example demonstrates what is meant by a tuning parameter. Tuning parameters are also observed in other engineering problems such as: automotive carburetor idle positioning, radio or television signal tuning circuit adjustments, etc. They are characterized by the tuning parameter's ability to compensate for noise.

Note that the term "noise" is used in the sense of Taguchi [8], meaning that there are three types of noise observed: external, internal, and variational. External noise errors are due to environmental fluctuations, such as operating temperatures, humidities, etc. Internal noise errors are inherent in the design, such as wear, storage deterioration of materials, etc. Variational noise errors are due to variations in the supplied materials and manufacturing processes.

Tuning parameters are those which are introduced to overcome the confounding influences of the noise parameters. Tuning parameters are characterized by being set *after* the confounding effects of the noise have occurred. This distinguishes tuning parameters from design parameters, which are set *before* the noise has occurred. Hence design engineers do not set the values of tuning parameters, the manufacturing engineer (or even the customer) sets their values. But when the design parameter values are chosen, the design engineer should take into consideration that a tuning parameter can later be adjusted.

Given such an inherently common concept of engineering, it is surprising that none of the formal techniques of engineering design incorporate tuning parameters. It is the objective of this paper to introduce tuning parameters, and to demonstrate how they can be modeled in various existing formalized methods of engineering design. We will demonstrate how optimization [4], Taguchi's method [1], and the method of imprecision [10, 11] can incorporate tuning parameters.

2. Modeling Tuning Parameters

In addition to tuning parameters, two other types of parameters are used in the design process: design parameters and noise parameters. Design parameters are those for which the designer selects values as a part of the design process. When the design is finished, exact nominal values are specified for design parameters. Noise parameters, on the other hand, confound the ability of the designer to specify nominal values for the design parameters. As stated earlier, noise parameters model uncertainty in the design, they behave randomly. Typically, parameters in a design will have both a design and a noise parameter aspect. That is, a designer will specify a nominal value, and there will be (manufacturing) noise associated with trying to achieve the specified value.

In the design process, the design parameter values are chosen such that the design goals are maximized despite the noise parameter actions. In the accelerometer example, the design parameters (the nominal spring constant K and the mass M) would be chosen such that the desired performance is achieved as often as possible. This is the extent to which formal techniques such as Taguchi's method [1] and probabilistic optimization [6] currently model parameter types: select the design parameter values which maximize the output despite the noise.

With tuning parameters, however, more freedom is provided to the designer. The above techniques choose the design parameters to maximize the output despite the expected noise. Tuning parameters, however, are set after the noise has occurred, and are set to overcome the noise parameters' influences.

Utilizing this observation, we can introduce tuning parameters into three current formal methods of engineering design: optimization, Taguchi's method, and the method of imprecision. The first requirement is to establish (or determine) the order in which the various parameters are fixed. We refer to this order as the *precedence relation* between tuning and noise parameters. This order is important because not all tuning parameters are set before all of the probabilistic noise has occurred.

Consider again the accelerometer design. There were manufacturing errors in the spring constant k which could be overcome by the backstop position tuning. However, if the spring material is sensitive to the operating temperature, k will also vary with the operating temperature. Temperature is another noise parameter. Yet the tuning parameter (backstop positioning) cannot be adjusted to overcome this noise, the backstop is already positioned. All that can be done is to adjust the tuning parameter to maximize the performance across the *expected* temperatures, paralleling what a designer does when selecting design parameters. Hence relative to this component of the noise (temperature), the tuning adjustment is not a tuning parameter, but rather a "design parameter" of the manufacturing engineer. It's value is set (by the manufacturing engineer) to maximize the expected performance over the temperature noise. So the precedence relation in this design is: the manufacturing noise occurs, then the tuning parameter adjustment occurs, and then the temperature noise occurs.

Having made this observation, that the precedence relation must be known among tuning and noise parameters, different formal engineering design methods will now be extended to model tuning parameters. Section 3. will then formalize the accelerometer example to demonstrate tuning parameters.

2.1 Optimization

Consider a single objective function $f(\vec{x}, \vec{p}, \vec{t})$ to be minimized, where $\vec{x}$ are the design parameters, $\vec{p}$ are the noise parameters, and $\vec{t}$ are the tuning parameters. The problem is to choose values for $\vec{x}$ which minimize f, given that there is random noise $\vec{p}$ which varies according to specified probability distributions, and that there are tuning parameters $\vec{t}$ which can be adjusted after the noise has occurred (in their proper precedence). Also, consider constraint equations $\vec{g}(\vec{x}, \vec{p}, \vec{t})$ which all must be less than or equal to zero. This problem is an extended form of a probabilistic non-linear programming problem [6].

To use tuning parameters within optimization, observe that tuning parameter values are determined based on noise parameter values. That is, given values for the noise parameters, a value for each tuning parameter is selected. This can be directly formalized into the following statement. Find:

$$\vec{x}^* = \min_{\vec{x}} \left[\int_{\vec{p}|\vec{x}} \min_{\vec{t}|\vec{x},\vec{p}} \left[f(\vec{x}, \vec{p}, \vec{t}) \right] dPr(\vec{p}|\vec{x}) \right] \tag{4.1}$$

subject to:

$$Pr\left(\vec{g}(\vec{x}, \vec{p}, \vec{t}) \leq \vec{0} \right) \geq D \tag{4.2}$$

where the $\vec{x}$, $\vec{p}$, and $\vec{t}$ used to evaluate the constraints $\vec{g}$ are also used simultaneously in evaluating f.

In Equation 4.1, the inner *min* minimizes the objective function across the tuning parameters: the best tuning parameter arrangement is used. The integral across the noise parameters finds the expected value of the minimized objective function (over the tuning parameters), thus the average case of performance is considered (as in traditional probabilistic optimization [6]). The outer *min* minimizes the expected performance across the design parameters.

In Equation 4.2, the Pr in the expression of $\vec{g}$ is the probability that any of the constraints are less than or equal to zero (*i.e.*, are satisfied). D is a specified probability requirement to satisfy the constraints. The solution will therefore be the set of design parameters which minimize the expected f and satisfies the constraints D % of the time, given that the tuning parameters can be adjusted. Within the constraint equations $\vec{g}$ will be the bounds on allowable ranges of $\vec{x}$ and $\vec{t}$.

The differences between tuning, noise, and design parameters are as follows. Noise parameters confound the solution, and their modeling requires an expectation process. Design parameters values are selected to minimize the objective function. Tuning parameters, on the other hand, are adjusted to minimize the output after the noise is set. Hence the tuning $\vec{t}$ variation occurs inside the expectation integral across the noise $\vec{p}$. Notice that at the end of the designer's nominal design process, values for $\vec{x}$ have been selected. But values for the tuning parameters $\vec{t}$ have not; this will occur subsequently after the noise has occurred (and will be selected by the manufacturing engineer). But the design engineer has incorporated the fact that the tuning parameters can be adjusted when the selection was made for the design parameters $\vec{x}$. Also notice that there may be environmental or operating noise that occurs after the tuning parameters are set. This is noise for which the tuning parameters cannot compensate, they can only be chosen to minimize the objective function.

Thus, it is not always the case that all of the tuning parameters t_k in the tuning parameter vector $\vec{t}$ will be chosen after the all of the noise parameters p_i in the noise vector $\vec{p}$ have been set. Some p_i will perhaps occur after all of the $\vec{t}$, and hence the expectation across that p_i must occur inside the minimization across $\vec{t}$ as well. In the accelerometer example, the expectation across the temperature effects will occur inside the tuning parameter (backstop positioning) adjustment minimizations. Hence the order among the expectation integrals of $\vec{p}$ and the minimizations of $\vec{t}$ depends on the precedence relation among the variables.

For example, if the precedence relation in a problem was: $\vec{p}_1\ \vec{t}_1\ \vec{p}_2\ \vec{t}_2$ (noise $\vec{p}_1$ is compensated by tuning parameters $\vec{t}_1$, then noise $\vec{p}_2$ occurs and is compensated by $\vec{t}_2$), then first the $\vec{p}_1$ integral would be expressed, within which the $\vec{t}_1$ would be minimized, and likewise for $\vec{p}_2$ and $\vec{t}_2$. The objective function would then be

$$\int_{\vec{p_1}|\vec{x}} \left\{ \min_{\vec{t_1}|\vec{x},\vec{p_1}} \left[\int_{\vec{p_2}|\vec{x},\vec{p_1},\vec{t_1}} \left(\min_{\vec{t_2}|\vec{x},\vec{p_1},\vec{t_1},\vec{p_2}} \left[f(\vec{x},\vec{p},\vec{t}) \right] \right) dPr(\vec{p_2}|\vec{x},\vec{p_1},\vec{t_1}) \right] \right\} dPr(\vec{p_1}|\vec{x}) \tag{4.3}$$

Notice that the formulation is flexible enough to incorporate any order of noise and tuning.

2.2 Taguchi's Method

Having extended the formulation of probabilistic optimization to include tuning parameters, extending Taguchi's method is also possible. Taguchi's method selects among different design parameters arrangements (DPAs) determined by considering nominal values of a design, and perturbations from these nominal values. The method also considers different noise parameter arrangements (NPAs), and selects the DPA which minimizes variance across the NPAs. See [1] or [5].

The basic Taguchi method selects the DPA defined by:

$$DPA^* = \max_{DPA} \left[-10 \log \left[\sum_{NPA} \left(f(DPA, NPA) - \tau \right)^2 \right] \right] \tag{4.4}$$

for a design in which an objective f must be maintained as close to τ as possible.

With tuning parameters, however, the output can be maximized after the noise has occurred, or after the NPA has been set. Therefore, Taguchi's method can also be extended by forming a tuning parameter array, similar to the inner array (design parameter array) and outer array (noise parameter array) [1]. The tuning parameter array would list tuning parameter values versus tuning parameter arrangements (TPAs). Then, the extended Taguchi method would select the DPA defined by:

$$\max_{DPA} \left[-10 \log \left[\sum_{NPA} \min_{TPA} \left[\left(f(DPA, NPA, TPA) - \tau \right)^2 \right] \right] \right] \tag{4.5}$$

The example in Section 3. will be solved using Taguchi's method with tuning parameters, and will demonstrate the method. Note that the order of the summation across the NPAs and the minimization across the TPAs varies according to the precedence relation in the same fashion as with optimization, where the precedence order varied the order of integrations across the p_i and minimizations across the t_k. In Taguchi's method, the precedence relation requirement means that the noise parameters and tuning parameters cannot be simply formed into single arrays. Rather, each must be split into sub-arrays according to the precedence relation.

For example, if the precedence relation in a problem was: $\vec{p}_1, \vec{t}_1, \vec{p}_2, \vec{t}_2$, then first the $\vec{p}_1$ would be formed into an array, with NPAs denoted NPA_1, then the $\vec{t}_1$ would be formed into an array, with TPAs denoted TPA_1, and likewise for $\vec{p}_2$ and $\vec{t}_2$. The solution would then be the DPA with the maximum of

$$-10\log\left[\sum_{NPA_1}\min_{TPA_1}\left[\sum_{NPA_2}\left(\min_{TPA_2}\left[(f(DPA, NPA, TPA)-\tau)^2\right]\right)\right]\right] \tag{4.6}$$

2.3 Method of Imprecision

The method of imprecision determines design parameter values based on maximizing designer preferences (as introduced and developed by Wood and Antonsson [10, 11]). It has a much richer set of modeling capabilities than the two methods previously described, and can model tuning parameters in a more detailed fashion. With the method of imprecision, tuning parameters are modeled as possibilistic uncertainties. Tuning parameters have a range over which they can vary, and any value within that range can be used, yet the designer does not specify their values. This is by definition a possibilistic uncertainty [12]. Further, degrees of possibility can be introduced. That is, not only is a range of possibility given for tuning parameters, but every value within the range is given a normalized rank indicating how possible the value is.

The method of imprecision as a preliminary design methodology is presented in [9, 10, 11]. Design parameters values are ranked, by the designer, as to the degree to which they are preferred. These design parameter preferences are then propagated into preferences on multiple performance parameters. The formalism considers multiple uncertainty forms: imprecision (uncertainty in choice), probability (stochastic uncertainty), and possibility (uncertainty due to freedom). Each uncertainty form is propagated from the design parameters into performance parameter uncertainty via their respective mathematics: imprecision and possibility use the fuzzy mathematics, and probability uses the probabilistic mathematics.

Inclusion of tuning parameters within the formalism requires no modification of the calculation procedures, since tuning parameters are forms of possibilistic uncertainty. Thus the method of imprecision can easily be extended to incorporate tuning parameters. The only change required is a modification of how the results of the calculations are interpreted. The possibilistic uncertainty from tuning parameters is uncertainty which can *improve* the solution, *i.e.*, among the possibilistic variations, the design can use the best among the

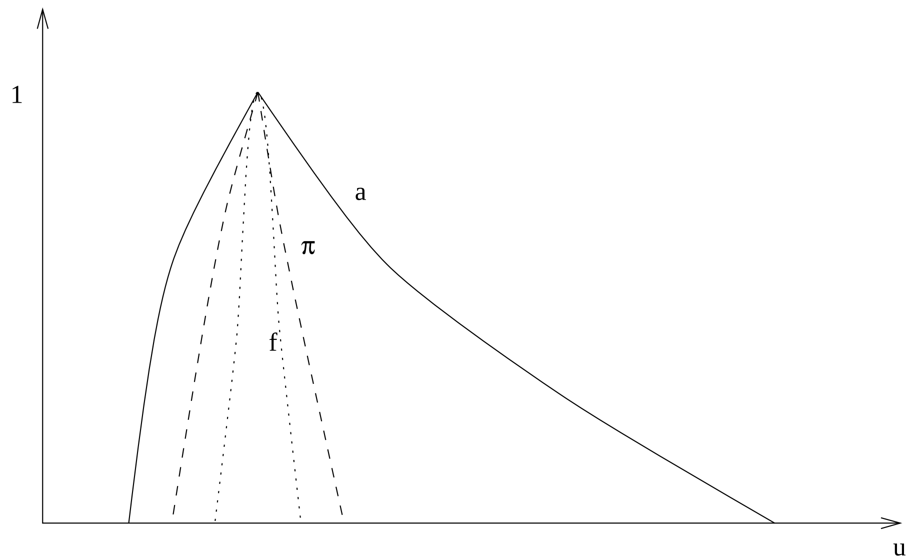

Figure 4.2 Method of Imprecision: Tuning parameters completely overcome the noise.

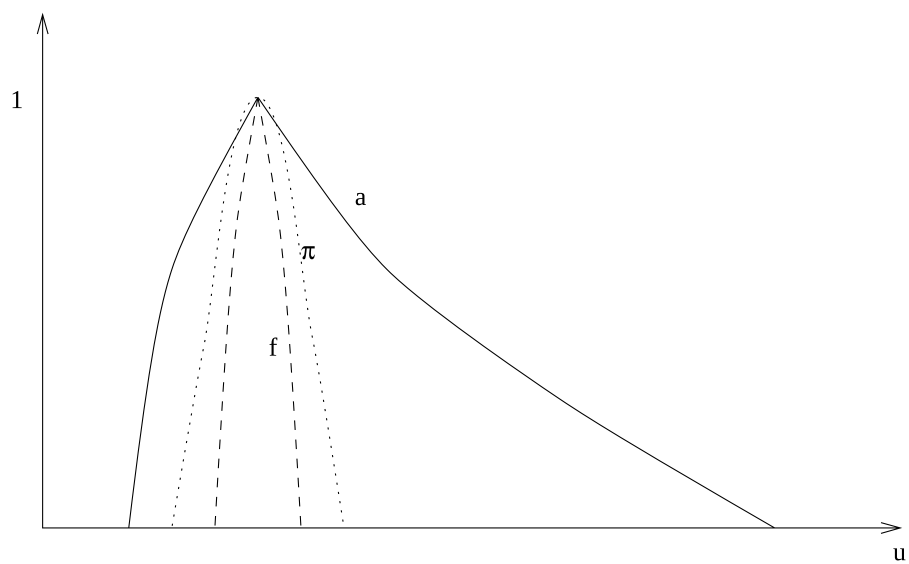

Figure 4.3 Method of Imprecision: Tuning parameters fail to completely overcome the noise.

variation. This is different from basic possibilistic uncertainty (not caused by tuning parameters) which is a possible variation *away* from the nominal.

For example, consider a design which has a possibilistic uncertainty from the tuning parameters which is greater than the probabilistic uncertainty, as

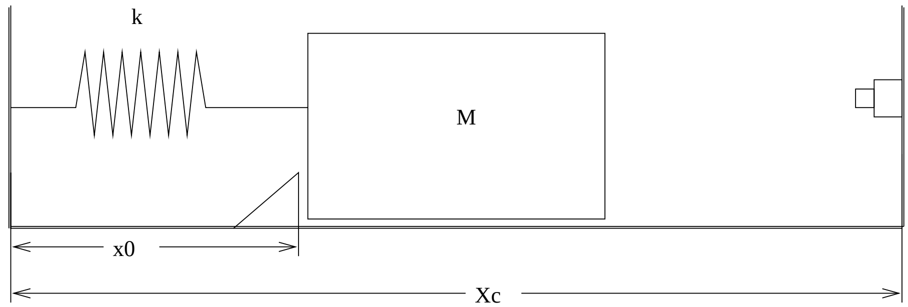

Figure 4.4 Accelerometer model.

shown in Figure 4.2. The spread of the possibilistic uncertainty π is larger than the spread of the probabilistic uncertainty pdf. Hence no matter what the probabilistic uncertainty, the possibilistic uncertainty can overcome the probabilistic uncertainty. Therefore the designer can confidently use the values of μ.

On the other hand, consider a design where the possibilistic uncertainty from the tuning parameters is less than the probabilistic uncertainty, as shown in Figure 4.3. The spread of the probabilistic uncertainty pdf is larger than the spread of the possibilistic uncertainty π. Hence no matter how much tuning occurs, there will always be some residual probabilistic uncertainty error remaining. The tuning parameter's range is not sufficient to overcome all the probabilistic error. In this case, the designer can use the values of μ only to within the difference between the probabilistic uncertainty pdf and the tuning parameter's correcting ability π.

3. Example

Consider again the accelerometer design introduced earlier. There is a mass M attached to a spring k attached to the ground. The ground is accelerated. With sufficient acceleration, the mass must displace a specified distance to make contact with a switch. There is also a backstop placed against the mass, to which the spring k pulls against with pre-load P under no acceleration. Refer to Figure 4.4.

There are two goals in this design: to maintain a specified preload P, and to close the switch in time τ under a specified acceleration. The parameter τ reflects the desired actuation time, and the pre-load P reflects the desired insensitivity to weak accelerations. As a part of these goals, the designer needs to determine whether the design can be made sufficiently tolerant to variational noise to satisfy the customer.

There are two design parameters, mass M and spring constant K. There is, however, uncertainty in the manufacture of the spring: a random variation on K, denoted δk. Finally, to assist in maintaining the targets on the goals, the manufacturing line can position the backstop based on measurements made of the total spring constant ($k = K + \delta k$) of each accelerometer. This backstop distance is denoted x_0, and is a tuning parameter. The switch distance is denoted x_c. The position of the mass at any given time is denoted x. The mass is to make contact with the switch when subjected to acceleration a.

To determine the time to actuate the switch, the differential equation of motion of the mass must be solved. It is:

$$M(\frac{d^2x}{dt^2} + a) \times H(x - x_0) + kx = P \times H(x_0 - x) \tag{4.7}$$

where H is a step function, $x(0) = x_0$, and $\dot{x}(0) = 0$. This can be solved for the time to actuation:

$$\tau = \sqrt{\frac{M}{k}} \times \arccos\left(\frac{Ma - (x_c - x_0)k}{Ma}\right) \tag{4.8}$$

This solution assumes, of course, a is sufficiently large to move the mass (i.e., the *arccos* is defined). The other goal is the pre-load P, whose equation is:

$$P = kx_0 \tag{4.9}$$

Maintaining a specific pre-load helps eliminate spurious switch closures.

3.1 Optimization Solution Formulation

Having formulated the problem, it can now be solved by optimization methods. The problem shall be formulated using an objective function consisting of a weighted sum of the two goals: time to actuation and spring pre-load. Both the target actuation time variation and the target pre-load variation shall be simultaneously minimized, and hence there is a relative coefficient needed between the two goals to ensure the variances are of the same order. The relative weighting coefficient used here was $\sqrt{5}$. Determining weighted sum coefficients is incidental to the tuning parameter formulation, refer to [2, 7] for multi-objective function optimization formulations. Substituting Equations (4.8) and (4.9) into (4.1) produces the problem to be solved as:

$$M^*, K^* = \min_{M,K}\left[\int_{\delta k} \min_{x_0}\left[\frac{|\tau - \tau_s|}{\tau_s} + \sqrt{5} \times \frac{|P - P_s|}{P_s}\right] \times pdf(\delta k) d\delta k\right] \tag{4.10}$$

subject to:

$$0.0135 \leq x_0 \leq 0.0165 \tag{4.11}$$

	M (kg)	K (N/m)
DPA_1	0.013	1.75
DPA_2	0.013	2.0
DPA_3	0.013	2.25
DPA_4	0.015	1.75
DPA_5	0.015	2.0
DPA_6	0.015	2.25
DPA_7	0.017	1.75
DPA_8	0.017	2.0
DPA_9	0.017	2.25

Table 4.1 Inner (design parameter) array.

	δk (N/m)
NPA_1	-0.15
NPA_2	0.0
NPA_3	0.15

Table 4.2 Outer (noise parameter) array.

$$0.0125 \leq M \leq 0.0175 \tag{4.12}$$

$$1.625 \leq K \leq 2.375 \tag{4.13}$$

where $\tau_s = 5$ milliseconds is a specified actuation time under an acceleration of $a = 20$ g's, $P_s = 0.02$ N is the specified preload under no acceleration. The results of the formulation will be shown below.

3.2 Taguchi's Method Solution Formulation

The problem can also be formulated in a Taguchi method formulation. Consider a 3 factorial design. The inner (design parameter) array is shown in Table 4.1. The outer (noise parameter) array is shown in Table 4.2. The tuning parameter array is shown in Table 4.3. Unfortunately, the experimental matrix cannot be drawn, since it would have to be three dimensional, with the new tuning parameter arrangement dimension.

	x_0 (m)
TPA_1	0.014
TPA_2	0.015
TPA_3	0.016

Table 4.3 Tuning parameter array.

Substituting Equations (4.8) and (4.9) into (4.5) produces the problem to be solved as:

$$M^*, K^* = \max_{DPA}\left[-10\log\left[\sum_{NPA}\min_{TPA}\left[\left(\frac{|\tau-\tau_s|}{\tau_s}\right)^2 + 5\times\left(\frac{|P-P_s|}{P_s}\right)^2\right]\right]\right] \tag{4.14}$$

3.3 Modeling without Tuning Parameters

The designer might instead model the backstop position as a design parameter, rather than a tuning parameter. This more traditional formulation will be presented below, and the results will show that the variation in performance (due to the variational noise) will be greater. Substituting Equations (4.8) and (4.9) into a traditional probabilistic optimization formulation [6], the model of this problem would be:

$$M^*, K^*, x_0^* = \min_{M,K,x_0}\left[\int_{\delta k}\left(\frac{|\tau-\tau_s|}{\tau_s} + \sqrt{5}\times\frac{|P-P_s|}{P_s}\right)\times pdf(\delta k)d\delta k\right] \tag{4.15}$$

subject to:

$$0.0135 \le x_0 \le 0.0165 \tag{4.16}$$

$$0.0125 \le M \le 0.0175 \tag{4.17}$$

$$1.625 \le K \le 2.375 \tag{4.18}$$

Similarly, Substituting Equations (4.8) and (4.9) into the traditional Taguchi's method formulation (4.4), the model of this problem would be:

$$M^*, K^*, x_0^* = \max_{DPA}\left[-10\log\left[\sum_{NPA}\left(\frac{|\tau-\tau_s|}{\tau_s}\right)^2 + 5\times\left(\frac{|P-P_s|}{P_s}\right)^2\right]\right] \tag{4.19}$$

These solutions will be compared with the tuning parameter formulation.

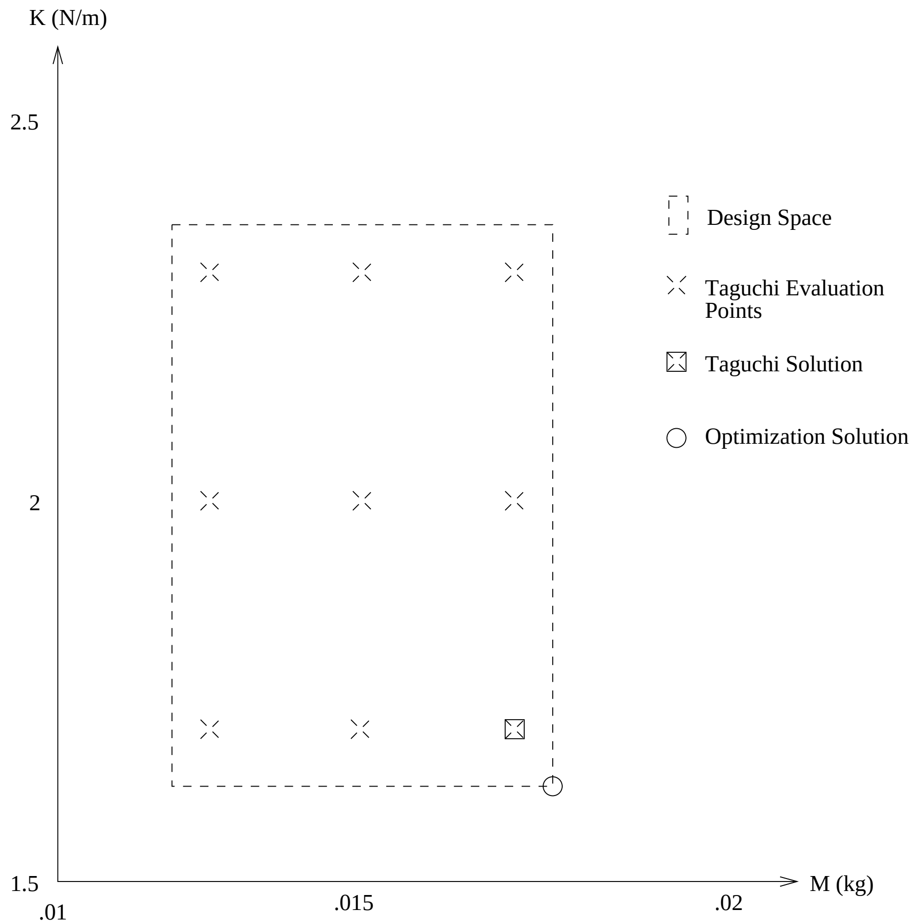

Figure 4.5 Design space K,M with results from the different solution procedures.

3.4 Results

The solutions for the Taguchi and optimization formulations are shown in Figure 4.5. Both methods pick the same solution region. This is due to the fact that both use the same multi-objective function (which Taguchi's method simply transformed through a -10*log*). Taguchi's method picked the closest experimental point to the optimal solution. Different results would have occurred if the optimization formulation used either P or τ as a constraint equation (using the aspiration level), and a single objective function optimization performed. Taguchi's method, of course, cannot incorporate constraint equations [3].

But one additional important question is the tolerance of the device to variational noise. Modeling with tuning parameters allows for more variability in δk, since x_0 can be tuned to keep P and τ on target. Modeling x_0 as a tuning parameter shows that with ± 0.2 N/m variation in δk, the pre-load P was within 0.006 N, and that the time to actuation τ was within 5 milliseconds. A traditional model in which x_0 is treated as a design parameter showed that the pre-load P was within 0.008 N, and that the time to actuation τ was within 6 milliseconds. Therefore a traditional model without tuning parameters results in a needlessly tight tolerance on δk.

4. Conclusion

This paper has introduced a formalization of tuning parameters common in engineering design. Tuning parameters are characterized by being set after the effects of noise have occurred. Tuning parameters are set to overcome these noise effects. The formal models of optimization and Taguchi's method have been extended to include tuning parameters. The method of imprecision inherently incorporates a modeling scheme for tuning parameters (using possibility).

One important observation regarding tuning parameters is that they are not under the direct control of the designer. Their values are, in the case of manufacturing noise, set by the manufacturing engineer. Therefore care must be used by the designer when incorporating tuning parameters into a design model.

In the case of variational noise (noise due to variations in supplied material, manufacturing, etc.), the use of tuning parameters is justifiable – the designer can ensure that tuning will occur, and the assumption of finding the best performance across the tuning parameters' ranges is correct. In the case of external noise (noise due to environmental or user fluctuations) and internal noise (noise due to wear or storage deterioration), the modeling of tuning parameters which can overcome these noises may not be justifiable (in general). The tuning *must* occur for the tuning parameter model to be correct. In the case of a closed loop controller, for example, the control response is a valid tuning parameter. Operator adjustment variables, on the other hand, perhaps should not be modeled as tuning parameters, even though they can be adjusted to increase performance. The designer cannot ensure that tuning will occur. Hence adjustment variables should only be modeled as tuning parameters when the designer is certain the adjustment will occur.

A final point should also be made with respect to modeling tuning parameters in Taguchi's method. Doing so is counter to the Taguchi philosophy, which states that one should eliminate tuning parameters altogether (by proper selection of the design parameters), since these are aspects of tolerancing design [8] and increase cost. This is indeed true; tuning parameters do increase cost. It is almost always the case that the manufacturing process should be kept as simple as possible, *i.e.*, that one should not use tuning parameters. However, if it is known that the design goals cannot be achieved by proper selection of the design parameter values and that tuning parameters will be required, then this fact should be incorporated into the design process. Doing so can allow for the selection of less expensive design parameters and tolerances, given that the design will be tuned during manufacture. Hence this is a more complete formulation of Taguchi's method, incorporating into the method the effects of tolerancing design on the design parameter selection.

This paper has introduced tuning parameters into formal methods of engineering design. Including tuning parameters in the design process can result in designs that are more tolerant of variational noise.

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References

[1] BYRNE, D. M., AND TAGUCHI, S. The Taguchi approach to parameter design. In *Quality Congress Transaction – Anaheim* (May 1986), ASQC, pp. 168–177.

[2] OSYCSKA, A. *Multi-Criterion Optimization in Engineering with Fortran Examples*. Halstad Press, New York, NY, 1984.

[3] OTTO, K. N., AND ANTONSSON, E. K. Extensions to the Taguchi Method of Product Design. *ASME Journal of Mechanical Design 115*, 1 (Mar. 1993), 5–13.

[4] PAPALAMBROS, P., AND WILDE, D. *Principles of Optimal Design*. Cambridge University Press, New York, NY, 1988.

[5] PHADKE, M. *Quality Engineering Using Robust Design*. Prentice Hall, Englewood Cliffs, NJ, 1989.

[6] SIDDALL, J. N. *Probabilistic Engineering Design; Principles and Applications*. Marcel Dekker, New York, NY, 1983.

[7] STEUER, R. *Multiple Criteria Optimization: Theory, Computation, and Application*. J. Wiley, New York, NY, 1986.

[8] TAGUCHI, G. *Introduction to Quality Engineering*. Asian Productivity Organization, Unipub, White Plains, NY, 1986.

[9] WOOD, K. L. *A Method for Representing and Manipulating Uncertainties in Preliminary Engineering Design*. PhD thesis, California Institute of Technology, Pasadena, CA, 1989.

[10] WOOD, K. L., AND ANTONSSON, E. K. Computations with Imprecise Parameters in Engineering Design: Background and Theory. *ASME Journal of Mechanisms, Transmissions, and Automation in Design 111*, 4 (Dec. 1989), 616–625.

[11] WOOD, K. L., AND ANTONSSON, E. K. Modeling Imprecision and Uncertainty in Preliminary Engineering Design. *Mechanism and Machine Theory 25*, 3 (Feb. 1990), 305–324. Invited paper.

[12] ZADEH, L. A. Fuzzy sets as a basis for a theory of possibility. *Fuzzy Sets and Systems 1* (1978), 3–28.

Chapter 5

IMPRECISION IN ENGINEERING DESIGN

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ASME *Journal of Mechanical Design*[1]
Volume 117(B) (1995), pages 25-32. Invited paper.

Abstract

Methods for incorporating imprecision in engineering design decision-making are briefly reviewed and compared. A tutorial is presented on the Method of Imprecision (MoI), a formal method, based on the mathematics of fuzzy sets, for representing and manipulating imprecision in engineering design. The results of a design cost estimation example, utilizing a new informal cost specification, are presented. The MoI can provide formal information upon which to base decisions during preliminary engineering design and can facilitate set-based concurrent design.

Introduction

One of the most critical problems in engineering design is making early decisions on a sound basis. However, the early stages of design are also the most uncertain, and obtaining precise information upon which to base decisions is usually impossible. The primary reason for this difficulty is that imprecision is an integral part of the engineering design process. Not imprecision in thought or logic, but rather the intrinsic vagueness of a preliminary, incomplete description. At the concept stage, the design description is nearly completely vague or imprecise (fuzzy). The design process reduces this imprecision until ultimately the final description is precise (crisp), except for tolerances, which represent the allowable limits on stochastic manufacturing variations.

[1]Special Combined Issue of the Transactions of the ASME commemorating the 50^{th} anniversary of the Design Engineering Division of the ASME.

Despite this evolution of imprecision, engineering design methods and computer aids have nearly all utilized precise information (though some can include stochastic effects). Solid modeling CAD systems, for example, require precise geometry; there is no option to indicate that a dimension is imprecise or only vaguely known.[1]

The need for a methodology to represent and manipulate imprecision is greatest in the early, preliminary phases of engineering design, where the designer is most unsure of the final dimensions and shape, materials and properties, and performance of the completed design. Additionally, the most important decisions, those with the greatest effect on overall cost, are made in these early stages [35, 89, 103, 96].

> "If a major project is truly innovative, you cannot possibly know its exact cost and its exact schedule at the beginning. And if in fact you do know the exact cost and the exact schedule, chances are that the technology is obsolete." [27][2]

This paper will review imprecision and uncertainty methods in engineering design, then present a brief tutorial of the *Method of Imprecision* (MoI), followed by a few recent advances and some thoughts on future research.

Review of Methods

Methods to represent uncertain variables as real numbers, and then perform an aggregation (as a sum, product, integral, *min*, *etc*.), for decision-making purposes are not new. Uncertainty may be: uncontrolled stochastic variations in variable values, design imprecision as described above, variable values to be chosen by optimization, *etc*. Probability and Bayesian inferencing [37, 95, 104], Dempster-Shafer theory [83, 87], fuzzy sets and triangular norms in general [20, 42, 44, 54, 56, 113], and finally utility theory [24, 26, 43] are among the existing formal[3] methods for representing uncertainty.

These methods all represent uncertainty with a range for each variable and a function defined on that range. An illustration is shown in Figure 5.1, where d is an uncertain variable and μ_d is the uncertainty on variable d. They are also similar in that they all conform to the first three restrictions of Table 5.1. The axioms shown in Table 5.1 have been proposed as the minimum set of restrictions for an engineering design combination (aggregation) function $\mathcal{P}$ [58], where μ_i is the uncertainty associated with the i^{th} aspect of the design. The discussion below indicates where these theories diverge among themselves and with optimization theory, matrix methods, and the MoI.

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[1] Variational and parametric modeling systems begin to approach this, by letting the designer specify a dimension precisely, but with the idea that it will be modified later.

[2] Joseph G. Gavin, Jr., discussing the design of the lunar module that landed NASA astronauts on the moon on July 20, 1969.

[3] Here the term "formal" is used to mean computable, in the sense that a design method could be automated.

$\mathcal{P}(0,\ldots,0)=0 \quad \mathcal{P}(1,\ldots,1)=1$	(boundary conditions)
$\forall\, k, \mathcal{P}(\mu_1,\ldots,\mu_k,\ldots,\mu_N) \leq \mathcal{P}(\mu_1,\ldots,\mu'_k,\ldots,\mu_N)$ iff $\mu_k \leq \mu'_k$	(monotonicity)
$\forall\, k, \mathcal{P}(\mu_1,\ldots,\mu_k,\ldots,\mu_N) = \lim_{\mu'_k \to \mu_k} \mathcal{P}(\mu_1,\ldots,\mu'_k,\ldots,\mu_N)$	(continuity)
$\mathcal{P}(\mu_1,\ldots,0,\ldots,\mu_N)=0$	(annihilation)
$\mathcal{P}(\mu,...,\mu)=\mu$	(idempotency)

Table 5.1 Overall preference resolution axioms.

Imprecision vs. Uncertainty. Uncertainty, which usually represents uncontrolled stochastic variations with the mathematics of probability, is distinct from imprecision. Uncertainty occurs throughout engineering design, in the form of manufacturing variations, material property variations, *etc*. Including uncertainty in engineering design decision-making can help produce robust designs by assessing the expected size of variations and determining the risk of failure. Many design methods have been developed specifically to address these calculations, including Taguchi's method, probabilistic optimization, and utility theory.

In the context of engineering design, the term imprecision is used to mean uncertainty in choosing among alternatives. An imprecise variable in preliminary design is a variable that may potentially assume any value within a possible range because the designer does not know, *a priori*, the final value that will emerge from the design process. The nominal value of a length dimension is an example of an imprecise variable. Even though the designer is uncertain about what length to specify, she usually has a preference for certain values over others. This preference, which may arise objectively (*e.g.*, cost or availability of components or materials) or subjectively (*e.g.*, from experience), is used to quantify the imprecision with which design variables are known. Once one or more alternative design concepts are available, each can be described by a collection of (imprecise) variables.

To represent an imprecise variable, d, a range of real numbers could be used, in the style of interval analysis [101, 102, 100]. Alternatively, imprecision could be represented by a range as well as a function, μ_d, defined on this range to describe the desirability of or preference for particular values, as illustrated in Figure 5.1. In this way variables whose values are not known precisely can be formally represented.

Combination Functions. Nearly all formal design methods for representing uncertainty or imprecision utilize one or more functions to aggregate information from multiple attributes. The combination calculation performs a trade-off, such that some aspects of a design may contribute more heavily to the combined result than others. Combination functions are also referred to as *metrics*.

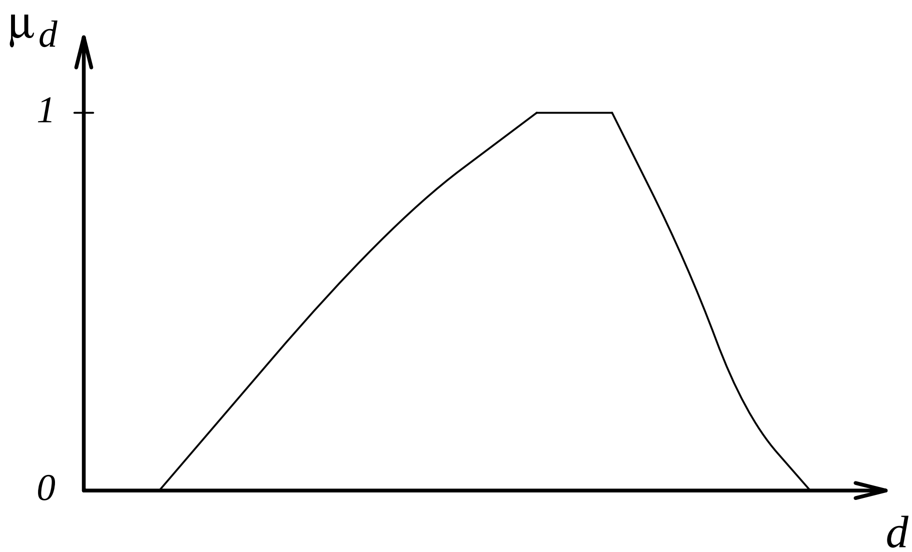

Figure 5.1 Example imprecise design variable

A combination function $\mathcal{P}$ is a formalization of the process of trading-off competing design attributes, and should satisfy the restrictions for engineering design proposed in [58] and shown in Table 5.1. Combination functions can be divided into two classes: compensating and non-compensating. A compensating combination function (*e.g.*, sum) will produce an overall measure of a design alternative where aspects that perform well can compensate for aspects that perform poorly. For example, a potential customer of a new car may prefer plenty of legroom and good fuel economy, and be willing to let a bit more legroom partially compensate for poor fuel economy when creating an aggregate evaluation of a particular new car. A non-compensating combination function (*e.g.*, *min*) will produce an overall measure of a design alternative that is limited by the most poorly performing aspect.

Formalizing the combination of attributes permits trade-off strategies that are determined informally or implicitly to be decided rationally and explicitly. A formal trade-off method also permits design decisions to be clearly understood and recorded for later retrieval and examination. When a question regarding a particular design trade-off arises at a later stage in the design process, a formal method can provide a clear and complete picture of how the decision was reached. Moreover, the trade-off can be repeated with revised information, thus confirming or refuting the original decision.

Methods for representing and manipulating uncertainty and imprecision, and for combining multiple attributes of an engineering design, are reviewed below.

Utility Theory. Utility theory [24, 26, 43] was developed to assist in selecting among a choice of distinct actions, given uncertainty (noise) in the outcome of each action. In utility theory, each aspect of a decision (*e.g.*, each design variable) is assigned a function representing utility, as illustrated in Figure 5.1. The utilities for the individual aspects are aggregated to determine the overall utility, and the combination of variable values that maximizes the overall utility is used.

Utility theory is restricted to decision problems in which the individual preferences can be modeled as additive, either with a weighted sum, or with a "multiplicative" form which is also additive but includes nonlinear terms [26]. Since the overall metric is additive, utility theory always reflects a compensating strategy, allowing the higher preference of some goals to offset the lower preference of others.

Because utility theory is additive, it fails the annihilation restriction shown in Table 5.1. This means that the utility of one aspect of an alternative can be zero, but the overall utility of the alternative will be non-zero. To surmount this difficulty, Thurston has applied utility theory to design by dividing the complete set of goals into two classes: objective constraints and subjective goals [94, 92, 93]. Objective constraints have crisp achievement levels that must be satisfied, and thus become standard constraints as in non-linear programming formulations. Subjective goals are those which can be traded-off, and are modeled using standard utility theory. The subjective goals are traded-off among themselves in a compensating manner (using either of the formulations of utility theory), and the objective constraints are traded-off informally (by iteratively refining the constraint values). The overall strategy, therefore, remains informal.

Utility theory was originally developed for management decisions, not for engineering design, and requires that all attributes be aggregated into a single goal (utility). Economists generally believe every aspect of a decision can always be translated into a monetary cost. Monetary costs are additive. Aspects which are not additive, or which cannot be "bought off", are not deemed possible. The axiom of utility theory which creates the demand that a gain in any aspect must be able to compensate for any loss in any other aspect is the Archimedian property. This restriction requires that any decrease in overall preference caused by changes in the performance of one variable must *always* be able to be balanced by an increase in performance in any of the other variables. Clearly this is not the case in engineering design, as others have argued [11, 97]. For example, given a fixed material, the tensile strength limits cannot be exceeded no matter the reduction in the design's cost. Material stress simply cannot always be traded-off in a compensating fashion. The Archimedian property and annihilation cannot be simultaneously satisfied. This implies that utility theory will not permit a worst case analysis, which is required in many instances in engineering design [33, 49].

Fuzzy Sets. Fuzzy sets have been used to represent imprecision in (non-design) decision-making [20, 42, 44, 54, 56, 113]. Fuzzy sets are intended to model subjective uncertainty for use in logic, constructing subjectively uncertain versions of "and" and "or" of classical logic. In the first paper describing the use of fuzzy sets for decision-making [9], a decision was defined as a convolution of the constraints and goals, using *min* as a non-compensating metric. They also suggested that at other times simple multiplication of the individual preferences might be appropriate.

The overall metrics of *min* and multiplication have been expanded to the more general class of *t-norms*, first proposed by Menger [51], and reviewed in Dubois and Prade [21]. *T-norms* are bounded above by *min*, and are the uncertain version of conjunction. *T-norms* are not appropriate as an overall design metric because they do not, in general, satisfy the restrictions of Table 5.1. Probabilistic reasoning, Dempster-Shafer theory, and fuzzy sets all employ *t-norms* [21]. Related to each *t-norm* is an associated *t-conorm* (or *s-norm*), which is the uncertain version of disjunction. *T-conorms* are bounded below by *max*. The set of functions between *t-norms* and *t-conorms* are the *mixed connectives*, bounded between *max* and *min*, and are the class of combination functions used by the MoI and utility theory.

Using fuzzy sets for decision-making has received further attention. [22, 38, 109, 110, 111, 112], for example, all discuss converting linguistic expressions into fuzzy sets, and then using the fuzzy mathematics to make decisions. An excellent review of fuzzy multiple attribute decision-making is presented in [14]. Bellman and Zadeh suggested using a weighted sum of the preferences. [39] and [7] also develop additive metrics using fuzzy sets beyond Bellman and Zadeh's original work. [110] observed the "softness" of multiplication as a connective, and proposed it (and *min*) in conjunction with weights.

Completely independent of fuzzy set formulations, [32] (the same year as Zadeh's initial paper on fuzzy sets) proposed a product of powers as a "desirability function" in chemical process problems. He observed that the annihilation condition is required for engineering design.

[15, 16] and [69, 70, 71, 72, 73] have applied fuzzy set formulations to design optimization problems in mechanical engineering. [55] has applied fuzzy sets to nuclear radiation cover design. All have used only a non-compensating trade-off strategy for making decisions. Other applications of fuzzy optimization are reviewed by [50].

Optimization. Other design methodologies exist that do not explicitly represent uncertainty or preferences on variables. For example, optimization formulations (linear, non-linear, integer, and mixed integer programming) [1, 4, 65, 66, 74] assume a relationship between preference and the objective function: the lower the function, the higher the preference. Also, a relation is assumed between preference and the constraint functions: if a constraint is satisfied, the preference is high. If any constraint is slightly violated, that constraint alone dictates the preference for the design is zero. Single objective optimization utilizes a non-compensating strategy: at any point in the space of design variables, one goal determines the preference (either the objective function, or a constraint).

Instead of a single objective formulation with constraints, others have proposed multi-objective optimization. Here, strategies are formally explicit only when a norm across the goals is used [79]. For example, weighted sum techniques [17, 23, 25, 57, 84, 88, 108] are compensating formulations: the higher performing objectives are averaged with the lower performing objectives, with the incorporation of importance weighting coefficients. As a specific example, the Archimedian "goal programming" [57, 80] formulation is a weighted sum technique, with target values and nonlinear weights. Additive metrics have been discussed above in relation to utility theory. Other formulations can be found in [82, 84].

[11] and [97], argue that the formulations that fail the annihilation condition (*e.g.*, addition) are not well suited for engineering design.

Typically, multi-objective formulations are used iteratively, without specifying a formal strategy. Such methods have been used in design [5, 68]. There are also algorithms for such iteration. STEM [10], GDF [28], and the VI algorithm [45], for example, interactively question a decision maker about relative trade-off preferences. Such algorithms are based on an informal overall metric consistent with utility theory [88], and thus exhibit an informal compensating design strategy.

Matrix Methods. Concept selection charts [2, 3, 6, 11, 67] are commonly used in engineering design decision-making. When using a formal chart, alternatives are listed versus evaluation criteria. Each alternative is ranked on each criterion, and the alternative with the best aggregated (weighted sum) score is selected. [67] presents an alternative technique of summing the negative and positive aspects of each alternative, and then making an informal decision based on these ranks. In an analysis of these methods, [52] considers a choice among four alternatives, and demonstrates that four different metrics selected each of the four different alternatives. The choice of decision-making method can entirely change the outcome, which confirms the importance of utilizing a method appropriate for engineering design.

The Analytic Hierarchy Process, or AHP, originally developed by Saaty [29, 76, 77], is a formal method for determining relationships between discrete alternatives, each of which can be rated by one or more attributes. A tutorial example is presented in [31]. AHP has an axiomatic foundation [78]. Similar to utility theory, this foundation requires that the Archimedian property must be satisfied, which dictates that the annihilation condition is not satisfied. Since the AHP's ratings are derived from a linear weighted sum, it only implements a compensating trade-off.

AHP can only consider discrete alternatives; no continuous variations can be incorporated. Additionally, all alternatives are compared to the lowest performing alternative; AHP includes no ability to indicate that one (or more) alternatives are completely unacceptable, or have violated one or more constraint(s). However, AHP is one of the few methodologies that can incorporate hierarchical objective criteria.

Finally, "Quality Function Deployment," or QFD [2, 34] begins by listing the customer requirements for a design on one axis of a chart and the performance metrics for the design on the other axis. Ratings are performed by using a transformation to convert symbols to numerical equivalents, and summing. This summation has the difficulties of the additive metrics discussed above.

Probability Methods. In Taguchi's method [13, 36, 41, 75, 86, 90, 91], the "quality" of a design variable set $\vec{d}$ is defined by the expected variation of a single performance variable p from a target value τ due to uncontrolled stochastic noise. Thus, Taguchi's method finds the mean of a single performance variable (variation of p from τ), not preference over many design variables and performance variables [59].

Experimental design techniques can also be used to determine experimental points in the noise space. See [8, 12, 40] for a discussion of factorial methods for determining experimental points. Alternatively, Monte Carlo simulation [30] could be used for more accuracy.

Probabilistic optimization methods can be used to evaluate a solution when noise is present, by evaluating the expected value of the objective function, as presented by [85, Chapter 13]. The discussion on optimization methods above applies to probabilistic optimization methods as well.

Necessity Methods. Necessity determines the probability (α) of a design operating successfully despite uncontrolled variations in one or more parameters over a range (confidence interval). This approach can account for the worst case of random disturbances. [98] and [100] have developed a "Labeled Interval Calculus" that consists of interval mathematics with associated "only" or "every" labels. [99] have shown that a set-based approach to design decision-making (using intervals) can facilitate concurrent engineering.

Noise can be rated by the expected value, or by the worst case scenario (using necessity). [85, Appendix C] and [81] reject using confidence intervals. Siddall argues that one has difficulty determining the density function pdf and the confidence level α. Savage argues against the use of confidence information since he feels one is still obliged to choose the expected value. However, confidence intervals are widely used in practice [33, 49], as is modeling to consider the worst case noise [19].

Fuzzy Design Methods. Fuzzy design methods combine many of the valuable attributes of the methods described above. They are formal (computable) methods for representing and manipulating design imprecision (uncertainty in choosing among alternatives) using the mathematics of fuzzy sets. Several groups are applying fuzzy methods to engineering design problems [105, 58, 53, 114, 115].

Imprecision is represented by a range, and a function defined on this range (μ_d), to describe the desirability of or preference for particular values, and to incorporate the designer's experience and judgement into the design evaluation. Non-parametric attributes (such as material choice, color, style, *etc.*) as well as real-valued attributes (such as physical dimensions, material properties, cost, *etc.*) can be used.

In the Method of Imprecision (MoI) [105, 106, 63], constraints can be similarly imprecise, permitting the customer to specify preferences over a range of values, rather than a crisp constraint that may be moved by negotiation later in the design process. Because the method was developed specifically for engineering design, the trade-off combination functions meet the restrictions shown in Table 5.1 [58]. A choice of two basic combination functions is available to aggregate the preferences for the attributes of the design: the (non-compensating) *min* and a (compensating) product of powers.

Because the MoI does not require all attributes to be aggregated into one evaluation metric, evaluations of the various aspects of a design can be made in a hierarchy. For example, safety margin might be traded-off in a non-compensating way among several parts of the design that are subject to loading, and weight and cost might be traded-off in a compensating way. The results of those two trade-offs then might be traded-off with a non-compensating combination function. Because importance weighting can be readily applied, the relative weight of each aspect of the decision can be incorporated into the hierarchy [48].

Finally, stochastic uncertainty (such as uncontrolled manufacturing variations) and possibilistic uncertainty and necessity (such as post-manufacturing tuning adjustments) can be incorporated into the design decision-making by utilizing well known expectation calculations [62].

Utility theory and the MoI are strongly similar when there is only one goal, and a compensating strategy is used [60]. When goals are traded-off in a non-compensating manner, and without considering importance weightings, the MoI reduces to a convolution of the constraints and goals used in fuzzy sets for decision-making [9].

The Method of Imprecision

The following sections will present a brief tutorial on how imprecision is used to facilitate decision-making in engineering design using the MoI.

Definitions and Notation. *Design variables* are denoted d_i, and the valid design variable values within the *design variable space* (DVS) form a subset $\mathcal{X}$. The set of valid values for d_i is denoted $\mathcal{X}_i$. The preference that a designer has for values of d_i, the ith design variable, is represented by a preference function on $\mathcal{X}_i$, called the *design preference*: $\mu_{d_i}(d_i)$.

Performance variables are denoted p_j. For each performance variable p_j there must be a mapping f_j such that $p_j = f_j(\vec{d})$. The mappings f_j can be any calculation or procedure to measure the performance of a design, including closed-form equations (*e.g.*, for stress, weight, speed, cost, *etc.*), iterative solutions, heuristic methods, "black box" calculations, testing of prototypes, or consumer evaluations. The subset of valid performance variable values $\mathcal{Y}$ is mapped from $\mathcal{X}$ and the set of valid values for p_j is denoted $\mathcal{Y}_j$. The *performance variable space* (PVS) is the dependent set of performances evaluated for each design in the DVS. In order to compare design alternatives, design preferences are mapped onto the PVS via the extension principle [111], discussed below.

Specifications and requirements also embody design imprecision, even though most are written as if they were crisp, *e.g.*, "This device must have a range of at least 250 km." Such a requirement implies that given two designs arbitrarily close together, one with a range of 250 km and one just below, the first would be acceptable but not the second, as shown by the dashed line in Figure 5.2. Specifications and requirements in the real world are commonly fuzzy. Often the designer must ask questions to distinguish the underlying fuzzy constraint so that the final design will satisfy the customer's actual requirements even though it may violate the crisp constraint initially given. The fuzziness of constraints and the fuzziness of preliminary design variables are both forms of design imprecision and can be represented in exactly the same way. The customer's preference (requirements) for values of p_j, the jth performance variable, is represented by a preference function called the *functional requirement*: $\mu_{p_j}(p_j)$. The solid line in Figure 5.2 shows the fuzzy functional requirement.

Trade-Off Strategies. The combined preference of the designer and customer for a particular design $\vec{d}$ is represented by an overall preference $\mu_o(\vec{d})$, which is a function of the design preferences $\mu_{d_i}(d_i)$, and the functional requirements $\mu_{p_j}(p_j)$:

$$\mu_o(\vec{d}) = \mathcal{P}\left[\mu_{d_1}(d_1), \ldots, \mu_{d_n}(d_n), \mu_{p_1}(f_1(\vec{d})), \ldots, \mu_{p_q}(f_q(\vec{d}))\right]. \qquad (5.1)$$

Two combination functions ($\mathcal{P}$) that satisfy the restrictions of Table 5.1 have been identified: *min* and a product of powers. Figure 5.3 shows the overall

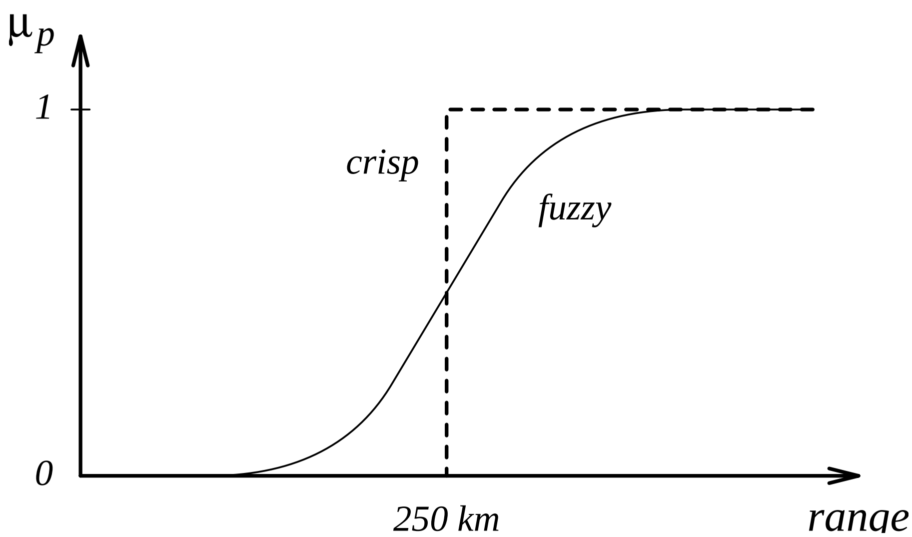

Figure 5.2 Example imprecise functional requirement

preference μ_o obtained by combining a design preference μ_d and a functional requirement μ_p. Both a compensating trade-off ($\mu_o = \sqrt{\mu_d \, \mu_p}$) and a non-compensating trade-off ($\mu_o = \min[\mu_d, \mu_p]$) are shown. Note that the compensating trade-off results in an overall preference μ_o that is greater than or equal to μ_o for the non-compensating trade-off. Where unequal preferences are traded-off, higher preferences compensate for lower preferences, raising the combined preference above the minimum. Importance weightings can further shift combined preferences, as discussed in [58].

Quantifying Imprecision. Utility and risk-aversion are quantified in utility theory via the lottery method [43]. Unfortunately no such formal method exists for eliciting preference. However, limits of acceptability for variable values, whether communicated formally or established informally by experience, are familiar to engineers in industry [99]. Such acceptable intervals correspond to intervals over which preference is greater than zero. This suggests that rather than determine the preference μ_d at each value of d, as shown in Figure 5.1, it may be more natural to determine the intervals in d, called α-cuts, over which μ_d equals or exceeds certain preference values α.

The use of intervals encourages the passing of set-based design information between engineering groups early in the design process [99], and permits the early release of possible sets of design data from one engineering group to the next in advance of precise design information. This approach has many advantages over the traditional “point-by-point” design iteration. The MoI can

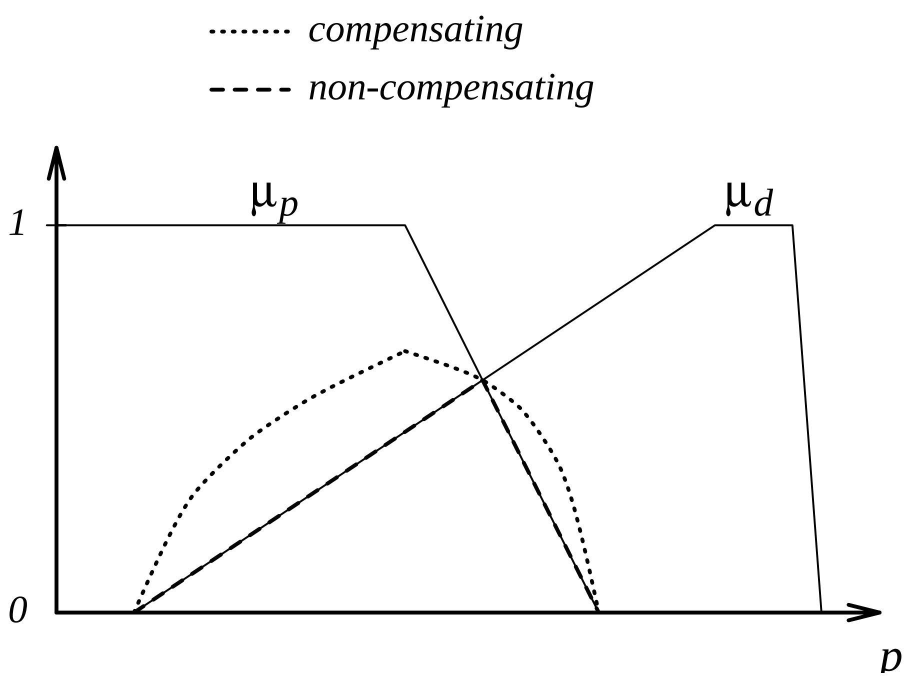

Figure 5.3 $\mu_o = \mathcal{P}(\mu_d, \mu_p)$ for compensating and non-compensating trade-offs

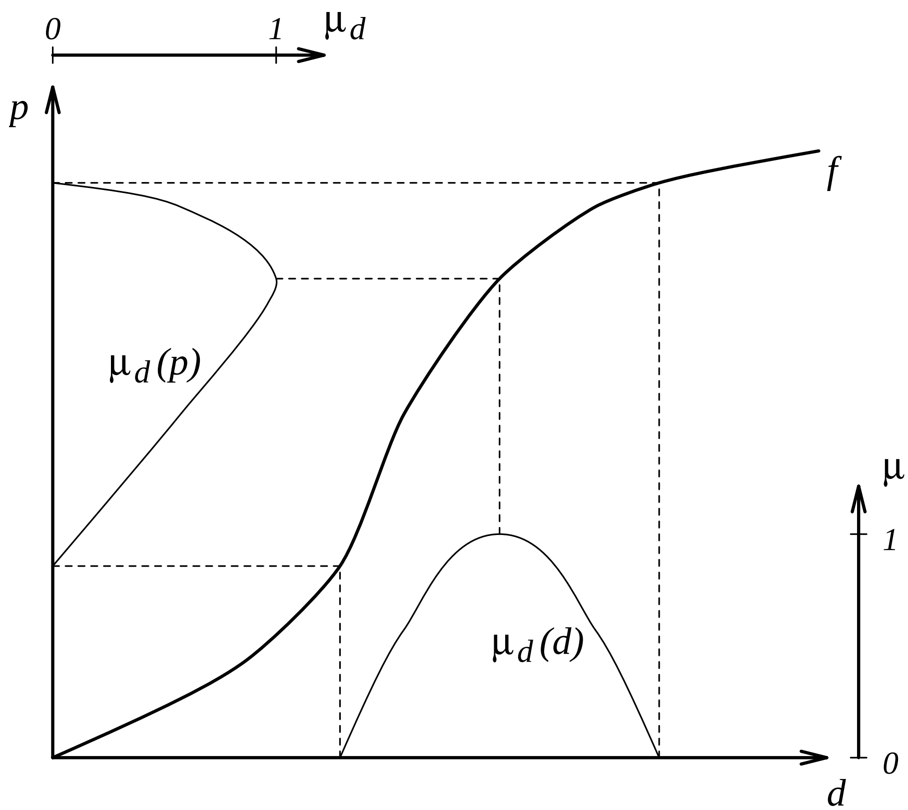

Figure 5.4 Zadeh's extension principle

extend set-based concurrent design by providing preference information over the possible range of design data.

Imprecision Calculations. After specifying design preferences μ_{d_i} on $\mathcal{X}_i$ and functional requirements μ_{p_j} on $\mathcal{Y}_j$, and identifying a design trade-off strategy, the next step is to determine the induced values of μ_{d_i} on $\mathcal{Y}$ (design preferences mapped onto the performances), given by the extension principle [111]:

$$\mu_d(\vec{p}) = \sup_{\vec{d}:\vec{p}=\vec{f}(\vec{d})} \left[\mu_d(\vec{d})\right] \tag{5.2}$$

A simple one-dimensional example of Zadeh's extension principle is shown in Figure 5.4. The performance p achieved for each value of the design variable d is given by the function f, which is a curve in this simple example.[4] The corresponding $\mu_d(d)$ can be mapped onto p, producing $\mu_d(p)$: the design pref-

[4]Note that here f is non-linear. Non-monotonic and discrete functions can also be used [107, 64].

erence mapped onto the performance space (as illustrated by the dashed lines in Figure 5.4). For more realistic design problems, each p will be a function of many d's, and each function f will be a hyper-surface.

An algorithm to compute Zadeh's extension principle (and thus to calculate $\mu_d(\vec{p})$) is the *Level Interval Algorithm* (LIA), first proposed by [18] as the "Fuzzy Weighted Average" algorithm and also called the "Vertex Method", and extended by [107, 64].

Once the imprecision on each design variable ($\mu_d(\vec{d})$) is induced onto the PVS, the induced preferences are combined with the functional requirements ($\mu_p(\vec{p})$) to obtain an overall preference ($\mu_o(\vec{p})$). The point (or points) with the highest preference correspond to the performance of the overall most preferred design(s). The design problem is to find the corresponding set of design variables ($\mu_d(\vec{d^*})$) that produce the maximum overall preference (μ_o^*). In the typical engineering design case, where the inverse mapping ($\vec{f}^{-1} : \mathcal{Y} \rightarrow \mathcal{X}$) doesn't exist, $\mu_o(\vec{d})$ can still be obtained point by point [46].

Example

An industrial example of cost estimation for aircraft engine design, utilizing the Engine Development Cost Estimator developed by General Electric Aircraft Engines [47, 46], shows how a crisp design cost estimator can be integrated with the MoI. This permits imprecise cost estimates to be developed when only imprecise design data is available. The original example included a formal, and imprecise, functional requirement (μ_p). The example has recently been extended to include more informal functional requirements.

Figure 5.5 shows the results of the MoI applied to the engine development cost estimator, using a compensating strategy. Two different design options were simultaneously explored, and are both shown on the figure. Option 1 is to develop the new engine from an existing turbojet design by the addition of a front fan with matching shaft and low pressure turbine. Option 2 is to modify an existing, but dated, turbofan design. The two curves are the designers' preferences (for a range of each design variable) induced onto the performance space (here: cost). Each point on the curves in Figure 5.5 corresponds to (at least) one design.

In comparing the cost specifications to the calculated imprecise design costs, if the customer has imposed a strict limit on the development cost, it may be applied by choosing the point with highest preference below this limit. If a formal, and imprecise, functional requirement for cost is available, the customer's preference and the designer's preference (induced onto the cost variable) can be traded-off, as described above. However, if the customer can only provide "cheaper is better" as an informal cost specification, an informal trade-off can still be made. Let $\vec{d}_{ref}$ be the design with the least cost among the highest

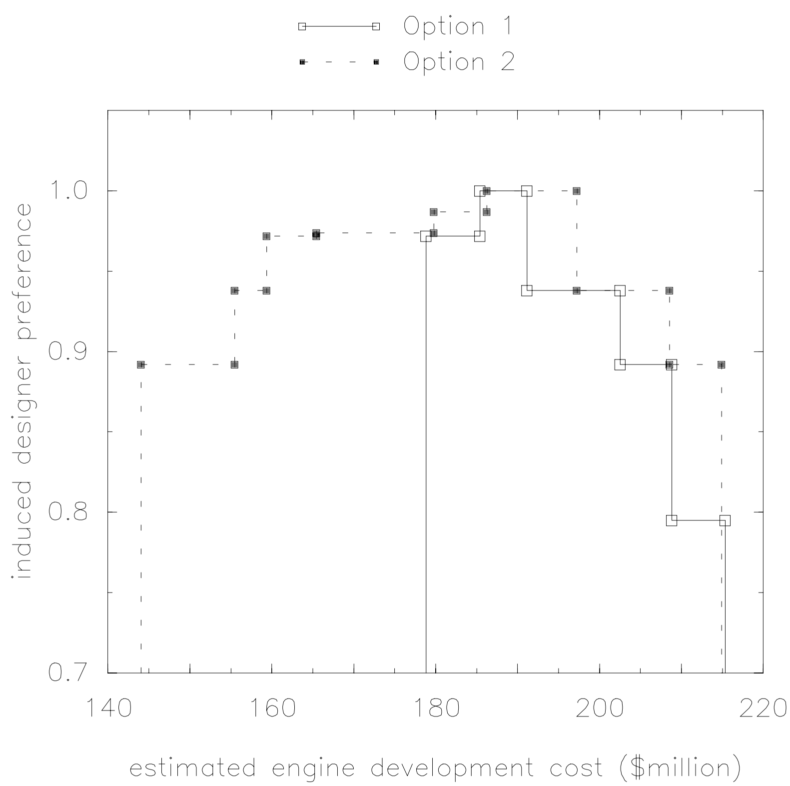

Figure 5.5 $\mu_d(p)$ for a compensating strategy.

preference designs (in Figure 5.5, $\vec{d}_{ref}$ is the left-most point at $\mu_d = 1.0$ for each option). A design $\vec{d}$ with lower cost is compared to $\vec{d}_{ref}$ by examining the slope of the line through both designs: the larger the slope, the greater the trade-off. The customer or designer should choose a design on a line with a slope that represents an acceptable trade-off.

Future Work

Promising areas for future work in further developing methods for representing and manipulating imprecision in engineering design include: developing a consistent (and formal) method for eliciting preference from designers and customers, and refinement of the weighting methods. Noise (meaning probabilistic uncertainty) has been thoroughly included in the MoI [62, 61], but an application to a commercially relevant industrial problem has not yet been made. The MoI was developed to permit hierarchical design decision-making, but a scheme to represent the hierarchy is lacking. Incorporating preference to create fuzzy-set based concurrent design methods appears to have significant promise, and is presently an active area of research. One area of future work that seems especially interesting is the notion of a fuzzy solid modeling system. Imprecise physical dimensions can be represented mathematically, but a useful display of this information has not yet been developed.

Conclusion

Imprecision and uncertainty occur throughout the engineering design process. Many methods for incorporating uncertainty (*e.g.*, utility theory, probability methods, Taguchi's method, *etc*.) are in common use and are reviewed here, but methods to represent imprecision in engineering design are few. The Method of Imprecision (MoI) is a formal method for incorporating the natural level of imprecision that occurs throughout the engineering design process, and can include: many incommensurate aspects of a design, imprecise constraints, compensating and non-compensating trade-offs, hierarchical trade-offs, and importance weightings. Uncontrolled variations (noise) can also be incorporated so that the design with the greatest overall preference and most robustness to the noise can be found.

Furthermore, by encouraging the designer and customer to specify preferences on design and performance variables, design communication will evolve from individual "point" designs to (fuzzy) sets of designs. Since a range of possible design variable values can be released to down-stream design processes earlier than a completed individual design, the MoI can facilitate (fuzzy) set-based concurrent design.

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References

[1] ADBY, P., AND DEMPSTER, M. *Introduction to Optimization Methods*. Chapman and Hall, London, 1974.

[2] AKAO, Y. *Quality Function Deployment: Integrating Customer Requirements into Product Design*. Productivity Press, Cambridge, 1990.

[3] ALGER, J., AND HAYS, C. *Creative Synthesis in Design*. Prentice Hall, Englewood Cliffs, 1964.

[4] ARORA, J. S. *Introduction to Optimum Design*. McGraw-Hill Book Company, New York, NY, 1989.

[5] ASHLEY, S. Engineous explores the design space. *Mechanical Engineering* (Feb. 1992), 49–52.

[6] ASIMOW, M. *Introduction to Design*. Prentice-Hall, Inc., Englewood Cliffs, N. J., 1962.

[7] BAAS, S., AND KWAKERNAAK, H. Rating and ranking of multiple-aspect alternatives using fuzzy sets. *Automatica 13* (1977), 47–48.

[8] BARKER, T. *Quality by Experimental Design*. Marcel Dekker, Inc., New York, 1985.

[9] BELLMAN, R. E., AND ZADEH, L. A. Decision-making in a fuzzy environment. *Management Science 17*, 4 (Dec. 1970), B–141–163.

[10] BENAYOUN, R., DE MONTGOLFIER, J., TERGNY, J., AND LARITCHEV, O. Linear programming with multiple objective functions: Step method (STEM). *Mathematical Programming 1*, 3 (1971), 366–375.

[11] BIEGEL, P., AND PECHT, M. Design trade-offs made easy. *Concurrent Engineering 1*, 3 (May/June 1991), 29–40.

[12] BOX, G. E. *Statistics for Experimenters*. J. Wiley and Sons, New York, 1978.

[13] BYRNE, D. M., AND TAGUCHI, S. The Taguchi approach to parameter design. In *Quality Congress Transaction – Anaheim* (May 1986), ASQC, pp. 168–177.

[14] CHEN, S.-J., AND HWANG, C.-L. *Fuzzy Multiple Attribute Decision Making: Methods and Applications*, vol. 375 of *Lecture Notes in Economics and Mathematical Systems*. Springer-Verlag, New York, NY, 1992. In collaboration with Frank P. Hwang.

[15] DIAZ, A. R. Fuzzy set based models in design optimization. In *Advances in Design Automation - 1988* (New York, NY, Sept. 1988), S. S. Rao, Ed., vol. DE-14, ASME, pp. 477–485.

[16] DIAZ, A. R. A strategy for optimal design of hierarchical systems using fuzzy sets. In *The 1989 NSF Engineering Design Research Conference* (College of Engineering, University of Massachusetts, Amherst, June 1989), J. R. Dixon, Ed., NSF, pp. 537–547.

[17] DLESK, D. C., AND LIEBMAN, J. S. Multiple objective engineering optimization. *Engineering Optimization 6* (1983), 161–175.

[18] DONG, W. M., AND WONG, F. S. Fuzzy weighted averages and implementation of the extension principle. *Fuzzy Sets and Systems 21*, 2 (Feb. 1987), 183–199.

[19] DOYLE, J., FRANCIS, B., AND TANNENBAUM, A. *Feedback Control Theory*. MacMillan Publishing Co., 1990.

[20] DUBOIS, D., AND PRADE, H. *Fuzzy Sets and Systems: Theory and Applications*. Academic Press, New York, 1980.

[21] DUBOIS, D., AND PRADE, H. A class of fuzzy measures based on triangular norms. *International Journal of General Systems 8* (1982), 43–61.

[22] EFSTATHIOU, J., AND RAJKOVIČ. Multiattribute decisionmaking using a fuzzy heuristic approach. *IEEE Transactions on Systems, Man and Cybernetics 9*, 6 (1979), 326–333.

[23] ESCHENAUER, H., KOSKI, J., AND OSYCZKA, A., Eds. *Multicriteria Design Optimization*. Springer-Verlag, Berlin, 1990.

[24] FISHBURN, P. *Foundations of Expected Utility*. D. Reidel, 1982.

[25] FREIHEIT, T., AND RAO, S. S. A modified game theory approach to multi-objective optimization. In *Advances in Design Automation - 1988* (New York, NY, Sept. 1988), S. S. Rao, Ed., vol. DE-14, ASME, pp. 107–114.

[26] FRENCH, S. *Decision Theory: An Introduction to the Mathematics of Rationality*. Ellis Horwood, Chichester, 1988.

[27] GAVIN, JR., J. G. Fly me to the Moon: An interview with Joseph G. Gavin, Jr. *Technology Review 97*, 5 (July 1994), 61–68.

[28] GEOFFRION, A., DYER, J., AND FEINBERG, A. An interactive approach for multicriterion optimization, with an application to the operation of an academic department. *Management Science 19*, 4 (1972), 357–368.

[29] GOLDEN, B., WASIL, E., AND HARKER, P., Eds. *The Analytic Hierarchy Process: Applications and Studies*. Springer-Verlag, New York, NY, 1989.

[30] HAMMERSLEY, J., AND HANDSCOMB, D. *Monte Carlo Methods*. Methuen, London, 1964.

[31] HARKER, P. T. The art and science of decision making: The Analytic Hierarchy Process. In *The Analytic Hierarchy Process: Applications and Studies*, B. Golden, E. Wasil, and P. Harker, Eds. Springer-Verlag, New York, NY, 1989, pp. 3–36.

[32] HARRINGTON, E. The desirability function. *Industrial Quality Control* (1965), 494–498.

[33] HARRY, M. J. The nature of six sigma quality. Tech. rep., Government Electronics Group, Motorola, Inc., 1991.

[34] HAUSER, J., AND CLAUSING, D. The House of Quality. *Harvard Business Review 66*, 3 (May/June 1988), 63–73.

[35] HOLMES, M. F. Machine dynamics, The need for greater productivity. In *Research Needs in Mechanical Systems* (New York, NY, 1984), K. N. Reid, Ed., ASME, pp. 140–159.

[36] HUNTER, J. S. Statistical design applied to product design. *Journal of Quality Technology 17*, 4 (Oct. 1985), 210–221.

[37] IVERSEN, G. R. *Bayesian Statistical Inference*. Series no. 07-043. Sage Publications, Beverly Hills and London, 1984.

[38] JAIN, R. Decisionmaking in the presence of fuzzy variables. *IEEE Transactions on Systems, Man, and Cybernetics* (1976), 698–703.

[39] JAIN, R. A procedure for multiple-aspect decision making using fuzzy sets. *International Journal of System Science 8*, 1 (1977), 1–7.

[40] JOHN, P. W. *Statistical Design and Analysis of Experiments*. Macmillan Co., 1971.

[41] KACKAR, R. N. Off-line quality control, parameter design, and the Taguchi approach. *Journal of Quality Technology 17*, 4 (Oct. 1985).

[42] KAUFMANN, A., AND GUPTA, M. M. *Fuzzy Mathematical Models in Engineering and Management Science*. North Holland, Amsterdam, 1988.

[43] KEENEY, R., AND RAIFFA, H. *Decisions with Multiple Objectives: Preferences and Value Tradeoffs*. Cambridge University Press, Cambridge, U.K., 1993.

[44] KLIR, G. J., AND FOLGER, T. A. *Fuzzy Sets, Uncertainty, and Information*. Prentice Hall, Englewood Cliffs, New Jersey, 1988.

[45] KORHONEN, P., AND LAASKO, J. A visual interactive method for solving the multiple criteria problem. *European Journal of Operations Research* (1985).

[46] LAW, W. S., AND ANTONSSON, E. K. Implementing the Method of Imprecision: An Engineering Design Example. In *Proceedings of the*

Third IEEE International Conference on Fuzzy Systems (FUZZ-IEEE '94) (June 1994), vol. 1, IEEE, pp. 358–363. Invited paper.

[47] LAW, W. S., AND ANTONSSON, E. K. Including Imprecision in Engineering Design Calculations. In *Design Theory and Methodology – DTM '94* (Sept. 1994), vol. DE-68, ASME, pp. 109–114.

[48] LAW, W. S., AND ANTONSSON, E. K. Hierarchical Imprecise Design with Weights. In *Proceedings of the Fourth IEEE International Conference on Fuzzy Systems (FUZZ-IEEE/IFES'95)* (Mar. 1995), vol. 1, IEEE, pp. 383–388.

[49] LUCAS, A. Motorola's kaizen brigade. *Design News 48*, 3 (Feb. 1992), 98–101.

[50] LUHANDJULA, M. Fuzzy optimization: An appraisal. *Fuzzy Sets and Systems 30* (1989), 257–282.

[51] MENGER, K. Statistical metrics. *Proceedings of the National Academy of Sciences 28* (1942), 535–537.

[52] MILNOR, J. Games against nature. In *Decision Processes* (New York, NY, 1957), R. Thrall et al., Eds., J. Wiley and Sons, pp. 49–59.

[53] MÜLLER, K., AND THÄRIGEN, M. Applications of fuzzy hierarchies and fuzzy MADM methods to innovative system design. In *Proceedings of the Third IEEE International Conference on Fuzzy Systems* (June 1994), vol. 1, IEEE, pp. 364–367.

[54] NEGOIŢĂ, C. V., AND RALESCU, D. A. *Applications of Fuzzy Sets to Systems Analysis*. Halsted Press, New York, 1975.

[55] NISHIWAKI, Y. Fuzzy optimization of radiation protection and nuclear safety. In *Combining Fuzzy Imprecision with Probabilistic Uncertainty in Decision Making* (New York, 1988), J. Kacprzyk and M. Fedrizzi, Eds., Spring-Verlag, pp. 353–273.

[56] NOVÁK, V. *Fuzzy Sets and Their Applications*. Adam Hilger, Philadelphia, 1989. Published in English by IOP Publishing Ltd.

[57] OSYCSKA, A. *Multi-Criterion Optimization in Engineering with Fortran Examples*. Halstad Press, New York, NY, 1984.

[58] OTTO, K. N., AND ANTONSSON, E. K. Trade-Off Strategies in Engineering Design. *Research in Engineering Design 3*, 2 (1991), 87–104.

[59] OTTO, K. N., AND ANTONSSON, E. K. Extensions to the Taguchi Method of Product Design. *ASME Journal of Mechanical Design 115*, 1 (Mar. 1993), 5–13.

[60] OTTO, K. N., AND ANTONSSON, E. K. The Method of Imprecision Compared to Utility Theory for Design Selection Problems. In *Design Theory and Methodology – DTM '93* (Sept. 1993), ASME, pp. 167–173.

[61] OTTO, K. N., AND ANTONSSON, E. K. Tuning Parameters in Engineering Design. *ASME Journal of Mechanical Design 115*, 1 (Mar. 1993), 14–19.

[62] OTTO, K. N., AND ANTONSSON, E. K. Design Parameter Selection in the Presence of Noise. *Research in Engineering Design 6*, 4 (1994), 234–246.

[63] OTTO, K. N., AND ANTONSSON, E. K. Modeling Imprecision in Product Design. In *Proceedings of the Third IEEE International Conference on Fuzzy Systems (FUZZ-IEEE '94)* (June 1994), vol. 1, IEEE, pp. 346–351. Invited paper.

[64] OTTO, K. N., LEWIS, A. D., AND ANTONSSON, E. K. Approximating α-cuts with the Vertex Method. *Fuzzy Sets and Systems 55*, 1 (Apr. 1993), 43–50.

[65] PAPALAMBROS, P., AND WILDE, D. *Principles of Optimal Design*. Cambridge University Press, New York, NY, 1988.

[66] PIERRE, D. *Optimization Theory with Applications*. Dover Press, New York, NY, 1986.

[67] PUGH, S. *Total Design*. Addison-Wesley, New York, NY, 1990.

[68] RAMASWAMY, R., ULRICH, K., KISHI, N., AND TOMIKASHI, M. Solving parametric design problems requiring configuration choices. In *Advances in Design Automation* (New York, NY, Sept. 1991), L. Stauffer, Ed., ASME, pp. 103–110. Volume DE-31.

[69] RAO, S. S. Description and optimum design of fuzzy mechanical systems. *ASME Journal of Mechanisms, Transmissions, and Automation in Design 109* (Mar. 1987), 126–132.

[70] RAO, S. S., AND DHINGRA, A. K. Integrated optimal design of planar mechanisms using fuzzy theories. In *Advances in Design Automation - 1989* (New York, NY, Sept. 1989), vol. DE-15-2, ASME, pp. 161–168.

[71] RAO, S. S., AND DHINGRA, A. K. Applications of fuzzy theories to multi-objective system optimization. NASA Contractor Report 177573, NASA Ames Research Center, 1991.

[72] RAO, S. S., DHINGRA, A. K., AND KUMAR, V. Nonlinear membership functions in the fuzzy optimization of mechanical and structural systems. Tech. Rep. 90-1175-CP, AIAA, New York, NY, 1990.

[73] RAO, S. S., SUNDARARAJU, K., PRAKASH, B., AND BALAKRISHNA, C. Multiobjective fuzzy optimization techniques for engineering design. *Computers and Structures 42*, 1 (1992), 37–44.

[74] REKLAITIS, G. V., RAVINDRAN, A., AND RAGSDELL, K. M. *Engineering Optimization, Methods and Applications*. Wiley, New York, NY, 1983.

[75] ROSS, P. *Taguchi Techniques for Quality Engineering*. McGraw Hill, New York, NY, 1988.

[76] SAATY, T. L. Exploring the interface between hierarchies, multiple objectives, and fuzzy sets. *Fuzzy Sets and Systems 1* (1978), 57–68.

[77] SAATY, T. L. *The Analytic Hierarchy Process: Planning, Priority Setting, Resource Allocation*. McGraw Hill, New York, NY, 1980.

[78] SAATY, T. L. Axiomatic foundation of the analytic hierarchy process. *Management Science 32* (1986), 841–855.

[79] SALUKVADZE, M. *Vector-Valued Optimization Problems in Control Theory*. Academic Press, New York, NY, 1979.

[80] SANDGREN, E. A multi-objective design tree approach for the optimization of mechanisms. *Mechanisms and Machine Theory 25*, 3 (1990), 257–272.

[81] SAVAGE, J. L. *The Foundations of Statistics*. (Revised Edition, Dover Publication, 1972). Wiley, New York, 1954.

[82] SEINFELD, J., AND MCBRIDE, W. Optimization with multiple performance criteria. *Ind. Eng. Chem. Process Des. Develop. 9*, 1 (1970), 53–57.

[83] SHAFER, S. *A Mathematical Theory of Evidence*. Princeton University Press, 1976.

[84] SIDDALL, J. N. *Analytical Decision Making in Engineering Design*. Prentice-Hall, 1972.

[85] SIDDALL, J. N. *Probabilistic Engineering Design; Principles and Applications*. Marcel Dekker, New York, NY, 1983.

[86] SINGPURWALLA, N. D. Design by decision theory: A unifying perspective on Taguchi's approach to quality engineering. In *NSF Design and Manufacturing Systems Grantees Conference* (Tempe Arizona, 1990), NSF. In the supplement to the Proceedings.

[87] SMETS, P. Belief functions. In *Non-Standard Logics for Automated Reasoning* (1988), P. Smets, E. Mamdani, D. Dubois, and H. Prade, Eds., Academic Press, pp. 254–286.

[88] STEUER, R. *Multiple Criteria Optimization: Theory, Computation, and Application*. J. Wiley, New York, NY, 1986.

[89] SULLIVAN, L. P. Quality function deployment. *Quality Progress 19*, 6 (June 1986), 39–50.

[90] TAGUCHI, G. *Introduction to Quality Engineering*. Asian Productivity Organization, Unipub, White Plains, NY, 1986.

[91] TAGUCHI, G., AND PHADKE, M. Quality engineering through design optimization. *IEEE* (1984), 1106–1113.

[92] THURSTON, D. L. A formal method for subjective design evaluation with multiple attributes. *Research in Engineering Design 3*, 2 (1991), 105–122.

[93] THURSTON, D. L., AND ESSINGTON, S. K. A tool for optimal manufacturing design decisions. *Manufacturing Review 6*, 1 (Mar. 1993), 48–59.

[94] THURSTON, D. L., AND TIAN, Y. A method for integrating utility analysis into an expert system for design evaluation under uncertainty. In *Proceedings of the Seventh Uncertainty in Artificial Intelligence Conference* (San Mateo, CA, 1991), B. D'Ambrosio, Ed., Morgan Kaufmann Publishers, pp. 398–405.

[95] TRIBUS, M. *Rational Descriptions, Decisions, and Designs*. Pergamon Press, New York, NY, 1969.

[96] ULRICH, K. T., AND PEARSON, S. A. Does product design really determine 80% of manufacturing cost? Working Paper MSA 3601-93, MIT, Sloan School of Management, Cambridge, MA, Aug. 1993.

[97] VINCENT, T. Game theory as a design tool. *ASME Journal of Mechanisms, Transmissions, and Automation in Design 105* (June 1983), 165–170.

[98] WARD, A. C. *A Theory of Quantitative Inference for Artifact Sets, Applied to a Mechanical Design Compiler*. PhD thesis, MIT, 1989.

[99] WARD, A. C., LIKER, J. K., SOBEK, D. K., AND CRISTIANO, J. J. Set-based concurrent engineering and Toyota. In *Design Theory and Methodology – DTM '94* (Sept. 1994), vol. DE68, ASME, pp. 79–90.

[100] WARD, A. C., LOZANO-PÉREZ, T., AND SEERING, W. P. Extending the constraint propagation of intervals. *Artificial Intelligence in Engineering Design and Manufacturing 4*, 1 (1990), 47–54.

[101] WARD, A. C., AND SEERING, W. P. The performance of a mechanical design compiler. memo 1084, MIT Artificial Intelligence Laboratory, 1989.

[102] WARD, A. C., AND SEERING, W. P. Quantitative inference in a mechanical design compiler. memo 1062, MIT Artificial Intelligence Laboratory, 1989.

[103] WHITNEY, D. E. Manufacturing by design. *Harvard Business Review 66*, 4 (July 1988), 83–91.

[104] WINKLER, R. L. *An Introduction to Bayesian Inference and Decision*. Holt, Rinehart and Winston Inc., 1972.

[105] WOOD, K. L., AND ANTONSSON, E. K. Computations with Imprecise Parameters in Engineering Design: Background and Theory. *ASME*

Journal of Mechanisms, Transmissions, and Automation in Design 111, 4 (Dec. 1989), 616–625.

[106] WOOD, K. L., AND ANTONSSON, E. K. Modeling Imprecision and Uncertainty in Preliminary Engineering Design. *Mechanism and Machine Theory 25*, 3 (Feb. 1990), 305–324. Invited paper.

[107] WOOD, K. L., OTTO, K. N., AND ANTONSSON, E. K. Engineering Design Calculations with Fuzzy Parameters. *Fuzzy Sets and Systems 52*, 1 (Nov. 1992), 1–20.

[108] WOODSON, T. *Introduction to Engineering Design*. McGraw-Hill, New York, NY, 1966.

[109] YAGER, R. Decision making with fuzzy sets. *Decision Sciences 6*, 3 (1975), 590–600.

[110] YAGER, R. Fuzzy decision making including unequal objectives. *Fuzzy Sets and Systems 1* (1978), 87–95.

[111] ZADEH, L. A. Fuzzy sets. *Information and Control 8* (1965), 338–353.

[112] ZADEH, L. A. Fuzzy sets as a basis for a theory of possibility. *Fuzzy Sets and Systems 1* (1978), 3–28.

[113] ZIMMERMANN, H.-J. *Fuzzy Set Theory - and Its Applications*. Management Science/Operations Research. Kluwer-Nijhoff Publishing, Boston, MA, 1985.

[114] ZIMMERMANN, H.-J., AND SEBASTIAN, H.-J. Optimization and fuzziness in problems of design and configuration. In *Proceedings of the Second IEEE International Conference on Fuzzy Systems* (San Francisco, CA, June 1993), IEEE, Pergamon Press, pp. 1237–1240.

[115] ZIMMERMANN, H.-J., AND SEBASTIAN, H.-J. Fuzzy design - Integration of fuzzy theory with knowledge-based system-design. In *Proceedings of the Third IEEE International Conference on Fuzzy Systems* (June 1994), vol. 1, IEEE, pp. 352–357.

Chapter 6

AGGREGATION FUNCTIONS FOR ENGINEERING DESIGN TRADE-OFFS

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Abstract The choice of an aggregation function is a common problem in Multi Attribute Decision Making (MADM) systems. The *Method of Imprecision* (M$_o$I) is a formal theory for the manipulation of preliminary design information that represents preferences among design alternatives with the mathematics of fuzzy sets. The M$_o$I formulates the preliminary design problem as a MADM problem. To date, two aggregation functions have been developed for the M$_o$I, one representing a compensating strategy and one a non-compensating strategy. Much of the prior fuzzy sets research on aggregation functions has been inappropriate for application to engineering design. In this paper, the selection of an aggregation function for MADM schemes is discussed within the context of the M$_o$I. The general restrictions on design-appropriate aggregation functions are outlined, and a family of functions, modeling a range of trade-off strategies, is presented. The results are illustrated with an example.

Keywords: Fuzzy design; Engineering; Aggregation functions; Averaging operators; Decision making; Multicriteria Analysis

Introduction

Preliminary design decisions are among the most important in engineering. It is when the details of a design are unknown and the design description is still *imprecise*, that the most costly and important decisions are made [19]. A rigorous treatment of design decision-making is necessary for the construction of computer tools to aid the designer in that decision-making. One such tool is the *Method of Imprecision* (M$_o$I) [20], a formal method for preliminary design decision-making that uses the mathematics of fuzzy sets to represent and manipulate imprecise design information. The M$_o$I casts the design decision-making problem as a Multi Attribute Decision Making (MADM) or Multi Objective Decision Making (MODM) problem [4, 24].

The choice of an aggregation function is a crucial part of any MADM or MODM scheme. The M$_o$I has attempted to approach this choice axiomatically. The axioms of the M$_o$I restrict the choice of candidate aggregation functions so as to reflect the natural structure of the design decision-making process [10, 11]. Within the axiomatic framework of the M$_o$I, a designer can combine preliminary design information using different trade-off strategies. In particular, two aggregation functions have been used for multi-attribute decision making, one which provides a compensation between the criteria, and one which does not compensate.

Many different aggregation functions for fuzzy sets have been proposed and studied. Much fuzzy set research has focused on *t-norms* and *t-conorms* [5]. T-norms are bounded above by the *min* function, and are the appropriate model for extensions of the logical AND to fuzzy sets, while t-conorms are bounded below by the *max* function and are an extension of the logical OR. Both classes of functions have been studied in detail, and research is on-going (*e.g.*, [14]). MADM schemes have applied a wide range of t-norms and other operators to decision problems. While averaging operators, which fall between *min* and *max*, have been acknowledged for some time [5, 22, 23], comparatively little study has been devoted to these connectives. Averaging operators are not appropriate for binary logic, but they are well suited to engineering design decisions: indeed, the axioms of the M$_o$I require its aggregation functions to fall between *min* and *max* [10]. This paper will offer a more thorough treatment of averaging operators than has been presented previously.

This paper discusses the problem of choosing an aggregation function for multiple criteria decision making. The results are presented within the context of the M$_o$I, but they have general applicability in MADM and MODM approaches. The axioms of the M$_o$I and the reasons for their use in modeling engineering design decisions will be presented. In the context of these axioms, potential aggregation functions will be discussed. The M$_o$I presently uses two different aggregation functions to model two different design trade-off strategies. This paper will explore the range of possible aggregation functions suitable for such decision-making. A parameterized family of functions that satisfies all the axioms for design and that models a continuum of strategies between the two existing strategies of the M$_o$I will be presented. The possibility of suitable aggregation functions outside this range will be discussed, and an example of an application will be given.

The organization of the paper is as follows. The first section presents the issue of uncertainty in engineering design and introduces the M$_o$I. The second section discusses the axioms of the M$_o$I in more detail. The third section covers some of the prior research on fuzzy MADM systems. In the fourth section, a parameterized family of aggregation functions is presented. The fifth section contains some philosophical remarks on the implications of this paper. Finally, the sixth section illustrates the application of the result with an example.

1. Uncertainty in Engineering Design

In preliminary engineering design, the final values of *design variables* are uncertain. As this uncertainty is not probabilistic, but will be resolved by further refinement and specification later in the design process, it is appropriately modeled using the mathematics of fuzzy sets [20, 21]. The Method of Imprecision is a formal system for the representation and manipulation of such imprecise design data. When a design is finalized, standard analysis tools (*e.g.*, finite element analysis, *etc.*) serve to calculate the performance of a design. These standard analysis tools do not operate on the imprecise information that is available during preliminary design, and designers rely on experience and intuition in this early stage. In the M$_{o}$I, designers represent their preferences for different values of a design variable using fuzzy sets: each value of a design variable is assigned a preference between zero (totally unacceptable) and one (completely acceptable). Design variables can take on discrete, continuous, or linguistic values. The designer's preferences create a fuzzy set that could be called "Values of design variable d preferred by the designer"; the membership in this set of a particular value of d is the designer's preference for that value. In this way, the designers' judgment and experience are formally incorporated into the preliminary problem. The designers' preferences, expressed on the design variables, are mapped to the performance space where they become preferences on the *performance variables* [8]. Preferences are also specified directly on performance variables by designers or others, and represent the functional requirements for the design. Whether calculated or expressed directly, these performance variables are likewise represented with fuzzy sets. Since a design's performance is usually described by several different performance variables, reconciliation of competing aspects of a design's performance forms an important part of the M$_{o}$I.

The general problem is thus a Multiple Attribute Decision Making (MADM) problem: the designer is to choose the highest performing design configuration from an available design space, when each design is to be judged by several, perhaps competing, performance criteria (variables). These performance variables may have different levels of importance, or *weights*. The MADM problem is one of the aggregation of weighted fuzzy sets. The goal is to choose an *aggregation function* that properly models the concerns of a designer evaluating a design.

A few points of notation: preferences are denoted by μ, indexed as μ_i, and their attendant weights are denoted by ω_i. The overall preference μ_o is calculated using an aggregation function $\mathcal{P}$:

$$\mu_o = \mathcal{P}((\mu_1, \omega_1), \cdots, (\mu_n, \omega_n))$$

If it is convenient to ignore the weights, as in the common case of equal weights for all objectives, the function can be written simply

$$\mu_o = \mathcal{P}(\mu_1, \cdots, \mu_n)$$

The M$_\text{o}$I has previously used two different aggregation functions to model two different situations in decision-making in design. When the overall preference for the performance of a design is limited by the attribute with the lowest performance, the decision-making problem is said to be *non-compensating*, and the aggregation function used is the simple minimum. In this case, weights are immaterial, and are not included in the calculations. When good performance on one attribute is perceived to partially compensate for lower performance on another, the problem is called *compensating*, and the geometric weighted mean or product of powers has been used.[1] Which aggregation functions are appropriate for design decision-making is the subject of this paper.

2. The Axioms of the M$_\text{o}$I

In any MADM system, it is desirable that the aggregation functions used be justifiable models for decision-making behavior. The choice of an aggregation function may be justified in several ways. Empirical tests, such as those conducted by other researchers [15] can help determine which aggregation functions best model human decision-making in various contexts. (The study mentioned supports the aggregation functions used by the M$_\text{o}$I.) Computational simplicity is often used as a basis for the choice of an aggregation function, however inappropriate that may be. The development of the M$_\text{o}$I has been to appeal to intuitive notions of rational human behavior [16], and to formalize this rationality in a set of axioms that the aggregation functions must follow. The axioms of the M$_\text{o}$I (see Table 6.1) [10] are a formal description of restrictions on any aggregation function for (rational) engineering design.

The axioms of monotonicity, commutativity and continuity are common to many multi-attribute decision making schemes, and they are uncontroversial. Also uncontroversial is the idea that an attribute with zero weight should contribute nothing to the overall performance. Self-scaling weights are convenient

[1] Both of these aggregations are analogous to Pareto-optimal solutions in game theory [9, 18], and the product of powers is analogous to a Nash solution. However, neither correspondence is mathematically precise, since preferences are not equivalent to utilities.

Monotonicity: $\mathcal{P}((\mu_1,\omega_1),\cdots,(\mu_n,\omega_n)) \leq \mathcal{P}((\mu_1,\omega_1),\cdots,(\mu_n',\omega_n))$ for $\mu_n \leq \mu_n'$ $\mathcal{P}((\mu_1,\omega_1),\cdots,(\mu_n,\omega_n)) \leq \mathcal{P}((\mu_1,\omega_1),\cdots,(\mu_n,\omega_n'))$ for $\omega_n \leq \omega_n'$; $\mu_i < \mu_n \ \forall i < n$
Commutativity: $\mathcal{P}((\mu_1,\omega_1),\cdots,(\mu_i,\omega_i),\cdots,(\mu_j,\omega_j),\cdots,(\mu_n,\omega_n)) =$ $\mathcal{P}((\mu_1,\omega_1),\cdots,(\mu_j,\omega_j),\cdots,(\mu_i,\omega_i),\cdots,(\mu_n,\omega_n)) \ \forall i,j$
Continuity: $\mathcal{P}((\mu_1,\omega_1),\cdots,(\mu_k,\omega_k),\cdots,(\mu_n,\omega_n)) =$ $\lim_{\mu_k'\to\mu_k} \mathcal{P}((\mu_1,\omega_1),\cdots,(\mu_k',\omega_k),\cdots,(\mu_n,\omega_n)) \quad \forall k$ $\mathcal{P}((\mu_1,\omega_1),\cdots,(\mu_k,\omega_k),\cdots,(\mu_n,\omega_n)) =$ $\lim_{\omega_k'\to\omega_k} \mathcal{P}((\mu_1,\omega_1),\cdots,(\mu_k,\omega_k'),\cdots,(\mu_n,\omega_n)) \quad \forall k$
Idempotency: $\mathcal{P}((\mu,\omega_1),\cdots,(\mu,\omega_n)) = \mu$ for $\omega_1,\cdots,\omega_n \geq 0$; $\omega_1+\cdots+\omega_n > 0$
Annihilation: $\mathcal{P}((\mu_1,\omega_1),\cdots,(0,\omega),\cdots,(\mu_n,\omega_n)) = 0$ for $\omega \neq 0$
Self-scaling weights: $\mathcal{P}((\mu_1,\omega_1 t),\cdots,(\mu_n,\omega_n t)) = \mathcal{P}((\mu_1,\omega_1),\cdots,(\mu_n,\omega_n))$ $\forall\, \omega_1,\cdots,\omega_n \geq 0$; $\omega_1+\cdots+\omega_n, t > 0$
Zero weights: $\mathcal{P}((\mu_1,\omega_1),\cdots,(\mu_k,0),\cdots,(\mu_n,\omega_n))$ $= \mathcal{P}((\mu_1,\omega_1),\cdots,(\mu_{k-1},\omega_{k-1}),(\mu_{k+1},\omega_{k+1}),\cdots,(\mu_n,\omega_n))$

Table 6.1 Axioms of the MoI

for the hierarchical combination of an arbitrary number of attributes. Two of the axioms, however, idempotency and annihilation, are fundamental to the M_oI and are particularly relevant here.

The idempotency axiom appeals to a notion of rational behavior. It states that if several variables with identical preferences are combined, the overall preference must be the same as the (identical) preferences on the individual variables. Idempotency reflects the constraint that the overall preference for a design should never exceed the preference of the highest-ranked attribute, nor fall below the preference of the lowest-ranked attribute. Idempotency and monotonicity together lead to the requirement that $min \leq \mathcal{P} \leq max$.

The annihilation axiom is specific to engineering design, and others have argued its validity [3, 12, 17]. It states that if the preference for any one attribute of the design sinks to zero (unacceptable) then the overall preference for the design is zero. For example, given a fixed material, the tensile strength limit cannot be exceeded no matter the reduction in the design's cost or weight. This is in contrast to a decision-making situation in which all performances can be converted into monetary units; in the latter case, two goals can always be traded, or bought, off.

One axiom that is *not* necessary for design-compatible aggregation functions is an axiom of strict monotonicity, and such a requirement would be incompatible with annihilation. The non-compensating function *min* is an example of a function that fulfills all of the axioms of the M_oI and is not strictly monotonic.

The axioms of idempotency and annihilation set the M_oI apart from other multi-attribute decision making systems. The reader is referred to [10] for a more detailed discussion of the motivation for this particular set of axioms. This paper posits the axioms of the M_oI as reasonable for design, and asks what can be said about the functions that satisfy these axioms. Such functions will be known as *design-appropriate*. This paper does not directly address those decision-making problems, outside of the field of engineering design, for which these axioms do not supply a reasonable model, though many of the ideas discussed here will be relevant to other non-design MADM schemes.

3. Fuzzy Multi Attribute Decision Making

In their recent book on the subject, Chen and Hwang [4] identify 18 fuzzy MADM methods, which they systematically classify into eight categories: simple additive weighting methods, the Analytic Hierarchy Process (AHP), the Conjunction/Disjunction method, MAUF, the General MADM method, the outranking method, maximin, and their own proposed MADM method. In their survey, Chen and Hwang do not draw the distinction observed by Zimmermann [24] between continuous Multi Objective Decision Making (MODM) problems and discrete Multi Attribute Decision Making (MADM) problems. This paper shall follow Chen and Hwang and use MADM to refer to the general problem, whether continuous or discrete.

Several of the methods surveyed by Chen and Hwang are similar to the M_oI. In addition, the application of utility theory [7] to decision problems bears some similarity to the M_oI and to the methods listed above. The possible application of utility theory to engineering design has been considered previously, and shown to be problematic [12]. Matrix methods such as QFD [6] and Pugh charts [13] also support decision making by simple additive aggregation over several requirements.

Aggregation operators are important in all MADM methods, from the most formal to the most casual. The arithmetic mean or weighted sum is popular in matrix methods and elsewhere, as it is simple to calculate. The *min* enjoys considerable popularity as well. Chen and Hwang provide an overview of commonly used aggregation operators; the *min* and the product operators presently in use in the M_oI appear in their list, as do weighted sums. However, the general weighted means discussed in this paper do not.

4. Weighted Means

Fuzzy set researchers have productively applied the study of functional equations [1] to explore t-norms and t-conorms. This section applies the same general approach to design-appropriate aggregation functions: an intuitively reasonable set of axioms is translated into a set of functional equations, and these equations are then solved.

A promising class of functions is the class of weighted means. The properties that define weighted means are listed in Table 6.2. While weighted means are defined here as functions of two arguments, they can be extended to several arguments.

The properties of the weighted mean include all of the properties of design-appropriate aggregation functions with the exception of annihilation; a comparison of these properties with the axioms of the M_oI shows that **any weighted mean that satisfies annihilation is design-appropriate.** The properties of the weighted mean also include conditions that are not explicitly design axioms; nevertheless, these conditions are consistent with the axioms of the M_oI. The bisymmetry condition is a surrogate for commutativity and associativity, and assures that $\mathcal{P}$ can be consistently defined for more than two arguments. Weighted means are strictly monotonic, which is a stronger condition than the monotonicity of the design axioms. **Any strictly monotonic design-appropriate aggregation function must be a weighted mean.** There are design-appropriate functions that are not weighted means, since they are monotonic but fail to satisfy strict monotonicity. Such operators are often

Idempotency: $\mathcal{P}((\mu, \omega_1), (\mu, \omega_2)) = \mu \quad \forall\, \mu, \omega_1, \omega_2$
Internality: $\exists\, \mu_a < \mu_b$ such that $\forall\, \omega_1, \omega_2 > 0$ $\mu_a = \mathcal{P}((\mu_a, 1), (\mu_b, 0)) < \mathcal{P}((\mu_a, \omega_1), (\mu_b, \omega_2)) < \mathcal{P}((\mu_a, 0)(\mu_b, 1)) = \mu_b$
Homogeneity of weights: $\mathcal{P}((\mu_a, \omega_1 t), (\mu_b, \omega_2 t)) = \mathcal{P}((\mu_a, \omega_1), (\mu_b, \omega_2))$ $\forall\, \omega_1, \omega_2 \geq 0;\ \omega_1 + \omega_2, t > 0$
Bisymmetry: $\mathcal{P}[(\mathcal{P}[(\mu_1, \omega_1), (\mu_2, \omega_2)], \omega_1 + \omega_2), (\mathcal{P}[(\mu_3, \omega_3), (\mu_4, \omega_4)], \omega_3 + \omega_4)]$ $= \mathcal{P}[(\mathcal{P}[(\mu_1, \omega_1), (\mu_3, \omega_3)], \omega_1 + \omega_3), (\mathcal{P}[(\mu_2, \omega_2), (\mu_4, \omega_4)], \omega_2 + \omega_4)]$
Increasing in weights: $\mathcal{P}((\mu_a, \omega_1), (\mu_b, \omega_2)) < \mathcal{P}((\mu_a, \omega_1), (\mu_b, \omega_3))$ for $\omega_2 < \omega_3 \quad (\mu_a < \mu_b)$
Increasing in variables: $\mathcal{P}((\mu_1, \omega_1), (\mu_2, \omega_2)) < \mathcal{P}((\mu_1, \omega_1), (\mu_3, \omega_2))$ for $\mu_2 < \mu_3, \quad \omega_2 \neq 0$

Table 6.2 Properties of the Weighted Mean

conditional rather than algebraic; the *min* is but one example. The class of weighted means will not encompass any of these aggregation functions that are only weakly monotonic. However, we shall see that the weak monotonic operators presently used for design can be approximated arbitrarily closely by strictly monotonic operators.

The structure of the class of weighted means is described completely in the following theorem, proven in [1]:

Theorem 1 *The properties of the weighted mean are necessary and sufficient for the function $\mathcal{P}((\mu_1, \omega_1), (\mu_2, \omega_2))$ to be of the form*

$$\mathcal{P}((\mu_1, \omega_1), (\mu_2, \omega_2)) = f\left(\frac{\omega_1 f^{-1}(\mu_1) + \omega_2 f^{-1}(\mu_2)}{\omega_1 + \omega_2}\right)$$

where $\mu_a \leq \mu_1, \mu_2 \leq \mu_b$; $\omega_1, \omega_2 \geq 0$; $\omega_1 + \omega_2 > 0$ and f is a strictly monotonic, continuous function with inverse f^{-1}.

It follows that any strictly monotonic design-compatible function must have a generating function f. For example, $f(t) = e^t$ (with $\mu_a = 1$ and $\mu_b = e$) generates the familiar weighted product of powers, denoted $\mathcal{P}_\Pi$ and also known as the geometric mean:

$$\mathcal{P}_\Pi((\mu_1, \omega_1), (\mu_2, \omega_2)) = ({\mu_1}^{\omega_1} {\mu_2}^{\omega_2})^{\frac{1}{\omega_1 + \omega_2}}$$

This function only satisfies the properties of the weighted mean for $\mu_i > 0$, but it satisfies all of the design axioms, including annihilation, on the closed interval $[0, 1]$.

A parameterized family of equations of particular interest for design is generated by the functions $f(t) = t^{\frac{1}{s}}$, where s is a real number. The aggregation function so generated is

$$\mathcal{P}_s((\mu_1, \omega_1), (\mu_2, \omega_2)) = \left(\frac{\omega_1 {\mu_1}^s + \omega_2 {\mu_2}^s}{\omega_1 + \omega_2}\right)^{\frac{1}{s}}$$

Note that a weighted mean satisfies annihilation if and only if $f^{-1}(0)$ is unbounded for that function. For $s < 0$, $f^{-1}(t) = t^s$ is unbounded at $t = 0$ and $\mathcal{P}_s$ satisfies annihilation. Similarly, $\mathcal{P}_\Pi$ satisfies annihilation, as $f^{-1}(t) = \ln(t)$ is unbounded at $t = 0$. Figure 6.1 shows the behavior of $\mathcal{P}_s$ for several negative values of the parameter s, and for equal weights ($\omega_1 = \omega_2$). In this graph, $\mu_2 = 0.5$ is fixed and μ_1 varies from 0 to 1 along the x-axis. It is graphically evident, and easily shown analytically, that $\mathcal{P}_0$ is identical to the weighted product of powers $\mathcal{P}_\Pi$. Thus the generating function for the geometric mean is not unique. Furthermore, as s tends to $-\infty$, $\mathcal{P}_s$ tends to $\min(\mu_1, \mu_2)$, regardless of the weights.

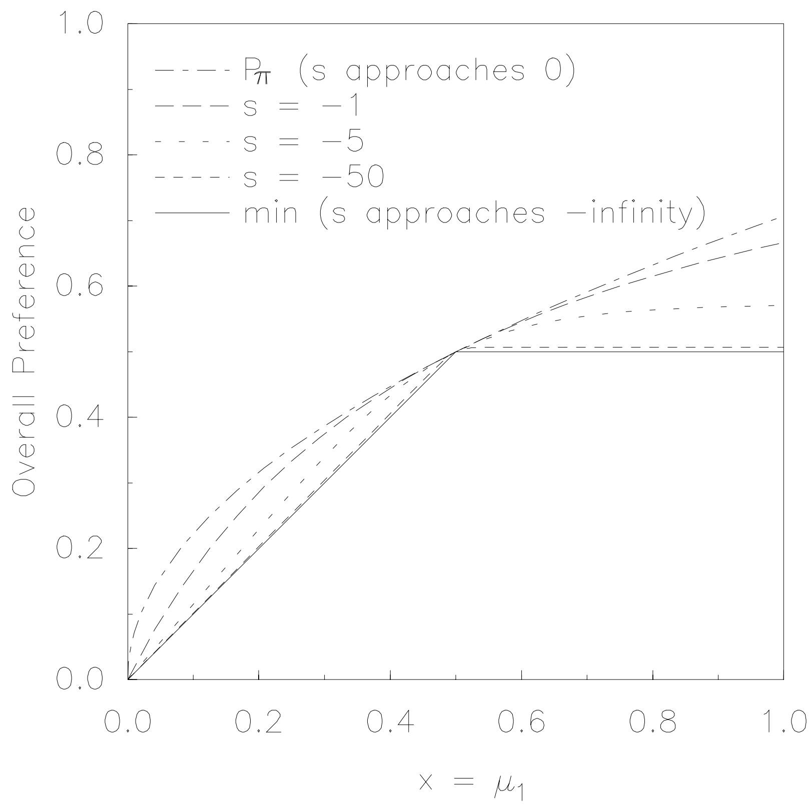

Figure 6.1 Functions between *min* and $\mathcal{P}_\Pi$

Claim 1 *$\mathcal{P}_0$ is identical to the weighted product of powers $\mathcal{P}_\Pi$:*

$$\lim_{s\to 0} \mathcal{P}_s((\mu_1,\omega_1),(\mu_2,\omega_2)) = \mathcal{P}_\Pi((\mu_1,\omega_1),(\mu_2,\omega_2))$$

Proof 1

$$\begin{aligned}
&\lim_{s\to 0} \mathcal{P}_s((\mu_1,\omega_1),(\mu_2,\omega_2)) \\
&= \lim_{s\to 0} \left(\frac{\omega_1{\mu_1}^s + \omega_2{\mu_2}^s}{\omega_1+\omega_2}\right)^{\frac{1}{s}} \\
&= \exp^{\lim_{s\to 0} \ln\left(\frac{\omega_1{\mu_1}^s + \omega_2{\mu_2}^s}{\omega_1+\omega_2}\right)^{\frac{1}{s}}} \\
&= \exp^{\lim_{s\to 0} \frac{1}{s}\ln\left(\frac{\omega_1{\mu_1}^s + \omega_2{\mu_2}^s}{\omega_1+\omega_2}\right)}
\end{aligned} \tag{6.1}$$

Note that, by the definition of the derivative,

$$\begin{aligned}
&\lim_{s\to 0} \frac{1}{s}\ln\left(\frac{\omega_1{\mu_1}^s + \omega_2{\mu_2}^s}{\omega_1+\omega_2}\right) \\
&= \frac{d}{ds}\left[\ln\left(\frac{\omega_1{\mu_1}^s + \omega_2{\mu_2}^s}{\omega_1+\omega_2}\right)\right]_{s=0} \\
&= \frac{\omega_1 \ln\mu_1 + \omega_2\ln\mu_2}{\omega_1+\omega_2}
\end{aligned}$$

Thus, proceeding from (1), it follows that

$$\begin{aligned}
&\lim_{s\to 0} \mathcal{P}_s((\mu_1,\omega_1),(\mu_2,\omega_2)) \\
&= \exp^{\frac{\omega_1\ln\mu_1+\omega_2\ln\mu_2}{\omega_1+\omega_2}} \\
&= \left({\mu_1}^{\omega_1}{\mu_2}^{\omega_2}\right)^{\frac{1}{\omega_1+\omega_2}} \\
&= \mathcal{P}_\Pi((\mu_1,\omega_1),(\mu_2,\omega_2))
\end{aligned}$$

which proves the claim.

In light of the preceding claim, the geometric mean $\mathcal{P}_\Pi$ will also be referred to as $\mathcal{P}_0$.

Claim 2 *As $s \to -\infty$, $\mathcal{P}_s$ tends to min:*

$$\lim_{s\to -\infty} \mathcal{P}_s((\mu_1,\omega_1),(\mu_2,\omega_2)) = \min(\mu_1,\mu_2)$$

Proof 2

$$\begin{aligned}
&\lim_{s\to-\infty}\left(\frac{\omega_1{\mu_1}^s+\omega_2{\mu_2}^s}{\omega_1+\omega_2}\right)^{\frac{1}{s}}\\
&= \lim_{s\to-\infty}\left(\frac{\omega_1}{\omega_1+\omega_2}\right)^{\frac{1}{s}}\left({\mu_1}^s+\frac{\omega_2}{\omega_1}{\mu_2}^s\right)^{\frac{1}{s}}\\
&= \lim_{t\to+\infty}\left(\frac{1}{{\mu_1}^t}+\frac{\omega_2}{\omega_1{\mu_2}^t}\right)^{\frac{1}{-t}}\\
&= \lim_{t\to+\infty}\left(\frac{{\mu_2}^t}{\left(\frac{\mu_2}{\mu_1}\right)^t+\frac{\omega_2}{\omega_1}}\right)^{\frac{1}{t}}\\
&= \lim_{t\to+\infty}\frac{\mu_2}{\left(\left(\frac{\mu_2}{\mu_1}\right)^t+\frac{\omega_2}{\omega_1}\right)^{\frac{1}{t}}}
\end{aligned}$$

Note that if $\mu_2 \le \mu_1$, then the denominator tends to one and the limit is μ_2; if $\mu_1 < \mu_2$, then the denominator tends to $\frac{\mu_2}{\mu_1}$ and the limit is μ_1. This proves the claim.

Thus the two functions originally proposed as aggregation functions for the $M_o I$, the *min* and the product of powers, turn out to be two limiting cases of a parameterized family of weighted means. Each $\mathcal{P}_s$, $s \le 0$, models a point on a continuum of trade-off strategies between the original non-compensating and compensating functions.

5. Application to design

In multi-attribute decision making, aggregation functions should provide a useful, justifiable model of the design decision process. The parameterized family $\mathcal{P}_s$ provides a continuum of weighted means between the two existing functions of the $M_o I$. While this family of functions is useful for design decision-making, it is not exhaustive. Other generating functions give rise to other aggregation functions that satisfy all the axioms of the $M_o I$ but behave differently from any of the $\mathcal{P}_s$.

The *min* is the least compensating possible design-appropriate function. No design aggregation function can take on values less than the *min* at any point in a design space. The *min* function defines a boundary not only of a certain family of weighted means, but also of design-compatible functions in general. The product of powers $\mathcal{P}_\Pi$ is a pivotal example among weighted means, but it is not so clear that it is maximal among design-compatible functions. Indeed, a maximal design-compatible function is problematic: such a function $\mathcal{P}_{\max}$ would satisfy

$$\mathcal{P}_{\max}(0, \mu) = 0 \quad \forall\, \mu$$

but also satisfy

$$\mathcal{P}_{\max}(\epsilon, \mu) = \mu \quad \forall\, \mu > \epsilon > 0$$

for all non-zero weights. $\mathcal{P}_{\max}$ so defined takes on the largest possible value while satisfying both idempotency and annihilation, but it fails another design axiom: it is discontinuous at zero. It is clear that there is no maximal design function corresponding to the *min*.

It was noted above that the arithmetic mean is an aggregation function commonly used in multi-attribute decision making. Yet the arithmetic mean does not satisfy all of the axioms of the M$_{\text{O}}$I, as it fails annihilation. The arithmetic mean is the aggregation function $\mathcal{P}_1$, and allows goals to compensate more strongly than the geometric mean. Indeed, $\mathcal{P}_s$, for $s > 0$, always fails annihilation, and the level of compensation between goals increases with s all the way to $\mathcal{P}_{+\infty} =$ *max*. If the arithmetic mean is only chosen for computational simplicity, then its use must be questioned.

The family of weighted means that formed a continuum between the *min* and $\mathcal{P}_{\Pi}$ was found by varying the parameter s in $\mathcal{P}_s$ between $-\infty$ and 0. It is interesting to examine what happens if the parameter is varied between 0 and $+\infty$. In this case a family of aggregation functions between $\mathcal{P}_0 = \mathcal{P}_{\Pi}$ and $\mathcal{P}_{\infty} =$ *max* is generated. These functions do not satisfy annihilation, so they do not appear to be appropriate for design. As long as no preference approaches zero, however, these functions satisfy all the axioms of the M$_{\text{O}}$I. Figure 6.2 shows $\mathcal{P}_s$ plotted for several positive values of s. As $s \to \infty$, $\mathcal{P}_s \to$ *max*. The "multi-linear" function of utility theory is also shown, though it fails idempotency as well as annihilation.

In practice, the M$_{\text{O}}$I is always implemented using discrete functions. This is partly an artifact of computer implementation, but more fundamentally arises from the fact that some changes in preference are too small to be distinguished. A designer does not actually specify a continuous preference function on the interval $[0, 1]$, but rather gives the values for each variable on several different α-cuts [2]. For example, for a particular design variable, the designer may specify which values correspond to $\mu = 0$, $\mu = 0.25$, $\mu = 0.5$, $\mu = 0.75$, and $\mu = 1$.

The discontinuous manner in which preferences are specified provides some justification for allowing the use of $\mathcal{P}_s$ with $s > 0$, or even the *max* operator, as an aggregation function when all of the preferences achieve some level. This seems to model the design process, as well: when all attributes are performing to some acceptable standard, a designer may choose to allow the highest pref-

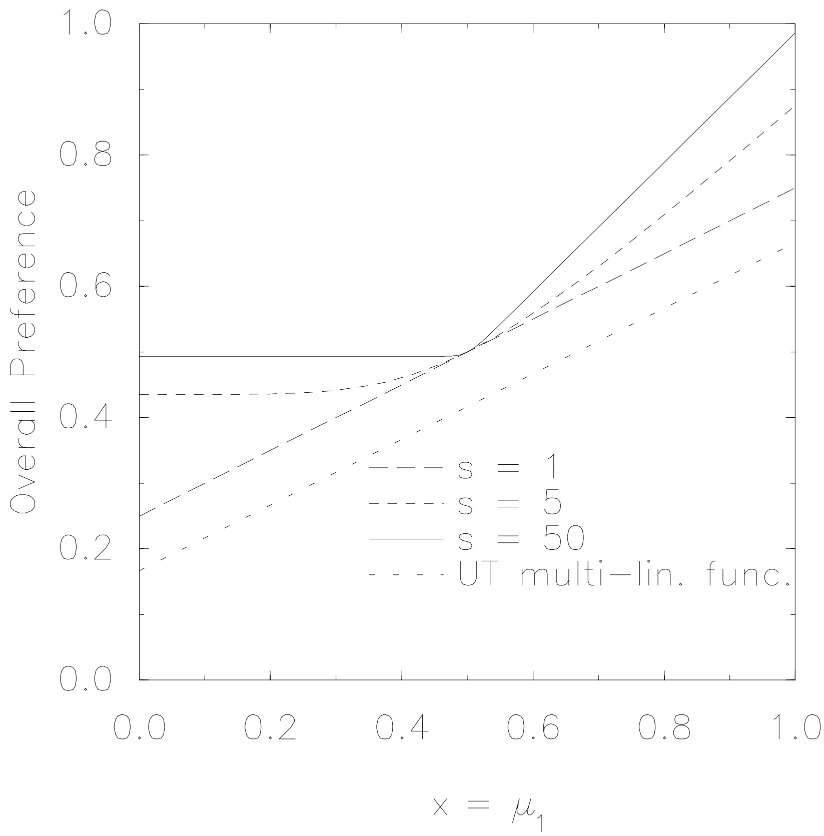

Figure 6.2 Functions that exceed $\mathcal{P}_{\Pi}$

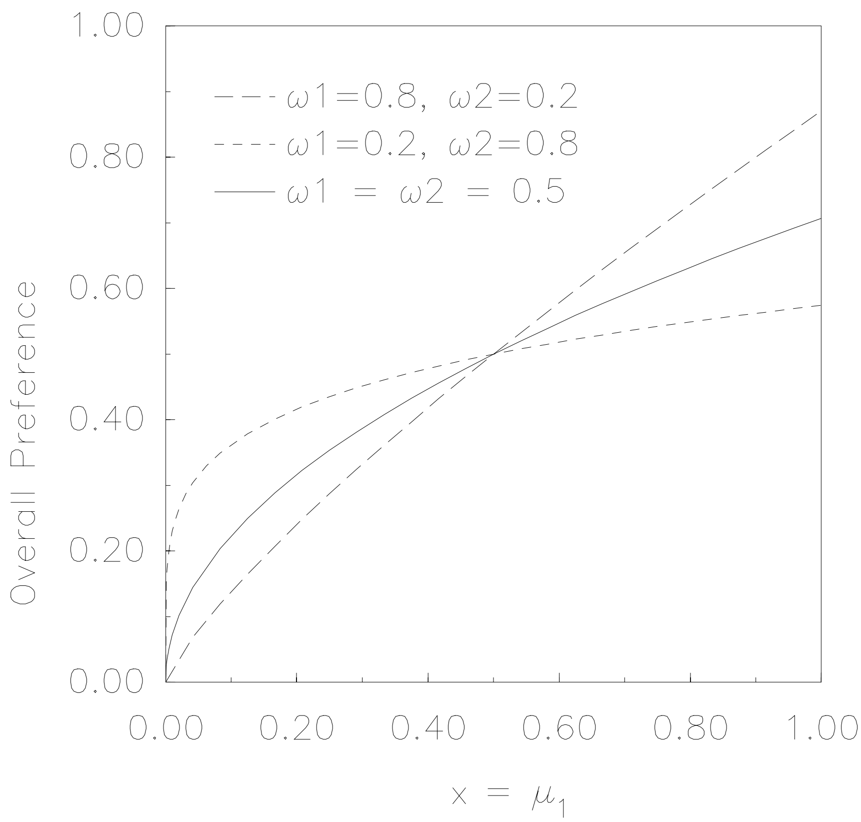

Figure 6.3 The product of powers, with various weights

erences great importance. Unacceptable performance on a single objective, however, can scuttle the design, so annihilation is still satisfied.

Theorem 1 provides a technical justification for the use of the family of operators between $\mathcal{P}_0$ and $\mathcal{P}_{+\infty}$. If it can be assumed that preferences less than some small ϵ, say 0.1, are not relevant to the designer, then the theorem indicates that there is a continuous aggregation function that satisfies $\mathcal{P}(0, y) = 0$ and

$$\mathcal{P}(x, y) = \mathcal{P}_s(x, y) \text{ for } x, y \geq \epsilon$$

The theorem guarantees that there is a formal operator that models this level of compensation without violating continuity or any other axiom of the MoI.

Thus trade-offs for all cases of compensation, from none ($s = -\infty$) to fully compensating ($s = 0$) to supercompensating ($s > 0$) are accomodated by the weighted means in Theorem 1.

When weights are unequal, even the relatively non-compensating functions between *min* and $\mathcal{P}_\Pi$ can compensate strongly in some regimes. Figure 6.3 shows $\mathcal{P}_\Pi$ with $\mu_2 = 0.5$ fixed and $\mu_1 = x$ as in the other plots, with three different weighting schemes. Notice that when ω_1 and μ_1 are both small, this function offers an annihilating, but still compensatory trade-off. The influence of weights on $\mathcal{P}_\Pi$ could also be exploited to construct a strongly compensating function.

6. Example

To illustrate the family of aggregation operators outlined in this paper, consider example 12-10 from Prof. Zimmermann's textbook [24]. This example is originally presented in the text as a MODM problem, and solved as a fuzzy linear programming problem with continuous variables. In the expression of the problem, however, the variables are quantities of two products to be produced, and it would be natural to assume that they can only take on integer values. Thus the problem can be thought of as a (discrete) MADM problem.

The example involves a company that produces two products, which yield different returns in profit and balance of trade. (Product 1 yields \$2 profit but requires \$1 in imports; product 2 can be exported for \$2 revenue but makes only \$1 profit.) The problem is to decide on a "best" production schedule to achieve high profits and a favorable balance of trade. The production schedule is subject to capacity constraints and is modeled by Prof. Zimmermann as follows:

$$\text{“maximize”} \qquad z(x) = \begin{pmatrix} -1 & 2 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

subject to

$$\begin{aligned} -x_1 + 3x_2 &\leq 21 \\ x_1 + 3x_2 &\leq 27 \end{aligned}$$

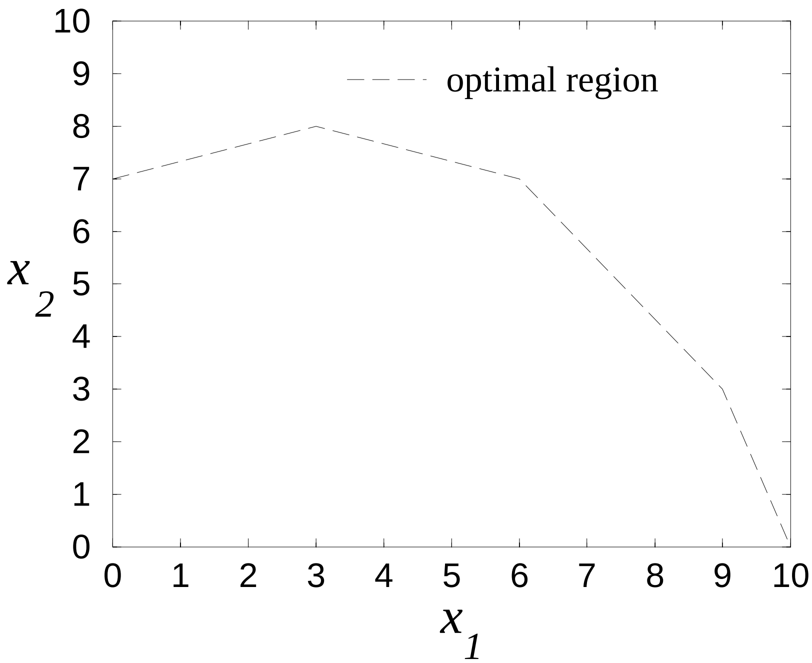

Figure 6.4 Decision Space with Optimal Region

$$\begin{aligned} 4x_1 + 3x_2 &\leq 45 \\ 3x_1 + x_2 &\leq 30 \\ x_1, x_2 &\geq 0 \end{aligned}$$

Prof. Zimmermann shows a plot of the decision space with a region of optimal values, similar to the one shown in Figure 6.4. There are seven undominated points in the decision space, which are shown in Table 6.3.

It is clear that this problem is not simply an exercise in mathematical programming. Three important questions remain. First, the decision maker must specify what it means to achieve the two individual goals (high profit, favorable balance of trade). Second, the relative importance of the goals must be addressed. Third, to what extent should high performance with respect to one goal be allowed to compensate for low performance elsewhere?

The example is solved in the textbook by an application of fuzzy sets that is substantively similar to that used by the $\mathrm{M_oI}$. The first step is to determine a level of satisfaction for each of the two goals, in essence to create the two fuzzy sets "Decisions that satisfy the profit goal" and "Decisions that satisfy the balance of trade goal." What the $\mathrm{M_oI}$ would call *preference* the textbook refers to as *level of satisfaction*. The preference or satisfaction for the performance

(x_1, x_2)	z_1	z_2
(0,7)	14	7
(3,8)	13	14
(4,7)	10	15
(5,7)	9	17
(6,7)	8	19
(8,4)	0	20
(9,3)	-3	21

Table 6.3 Undominated points in the decision space.

on balance of trade increases linearly from $\mu_1(x) = 0$ at $z_1(x) = -3$ to $\mu_1(x) = 1$ at $z_1(x) = 14$. The preference for profit increases linearly from $\mu_2(x) = 0$ at $z_2(x) = 7$ to $\mu_2(x) = 1$ at $z_2(x) = 21$. These preferences are generated in the textbook with reference to the values listed in Table 6.3; in the general application of the $\mathrm{M_oI}$, the memberships μ_i may be specified using a different process. In any event, the given fuzzy sets "Decisions that satisfy the objectives", with memberships ranging from 0 to 1 throughout the decision space, are valid preference functions for the $\mathrm{M_oI}$.

The problem of relative importance of the two goals does not come into play in this problem, and it can be assumed that the two goals are equally weighted. The aggregation functions presented in this paper can model preferential weighting of an arbitrary number of goals. However, the choice of an aggregation function, which encapsulates the decision of how much high performance on on attribute is to compensate for low performance on the other, remains.

If the decision problem is treated as a fuzzy linear programming problem, as in the text, the aggregation function used is the *min*. The *min* is "natural" for ease of computation, but not necessarily natural for the decision. When the problem is solved using the *min* operator, as shown in Figure 6.5, the maximum degree of "overall satisfaction" is given by the point $x_0 = (5.03, 7.32)$, with $\mu_o = 0.74$. Among the integer choices available, $x_0 = (5, 7)$ is the best, with $\mu_o = 0.71$. This corresponds to the solution given by the $\mathrm{M_oI}$ when it is determined that the problem is non-compensating, *i.e.*, the overall performance is limited by the lowest performance of all attributes.

In many situations, the overall performance of a design, or the general attractiveness of a decision, is not limited by the lowest performance among the attributes. For each problem, there is a level of compensation that is appropriate. The selection of the appropriate level of aggregation is one question; how to model it is another. This paper focuses on the second question and defers

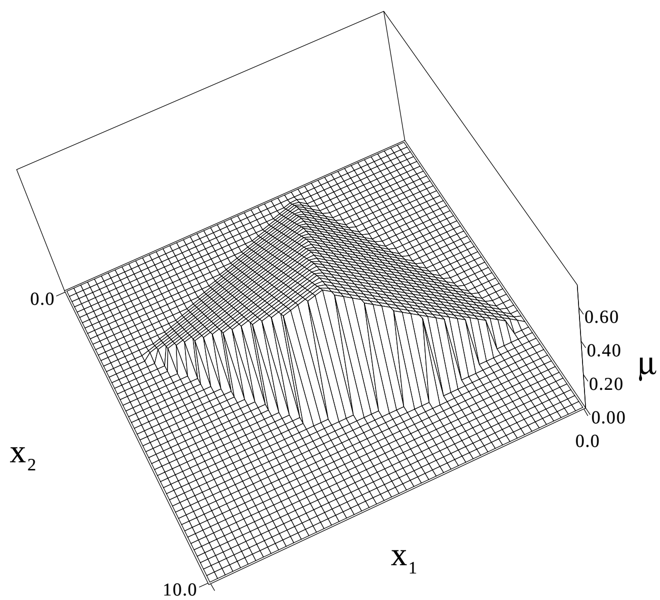

Figure 6.5 Decision surface with minimum operator

the discussion of the first. However, it shall be seen that different levels of compensation lead to different decisions.

The MoI has used a weighted product of powers for problems that demand a higher level of compensation than is afforded by the minimum function. The point with highest combined preference is $x_1 = (5.70, 7.10)$ with $\mu_o = 0.75$, as shown in Figure 6.6. The point of highest preference is not far from the point of highest preference achieved with the *min* operator. However, if one considers only the discrete points with integer values (if the company cannot manufacture 5.70 of a product), the highest performing point is $x_1 = (6, 7)$ (with $\mu_o = 0.74$). A different level of compensation among goals leads to a different decision.

The application of the entire family of of aggregation functions $\mathcal{P}_s$ to this example problem shows that there are at least three optimal points, each suitable over a range of values of the parameter s, and thus over a range of levels of compensation of goals. In addition, the point (1,7), though dominated by the point (3,8), approaches it asymptotically in preference as the level of compensation approaches the maximum. Figure 6.7 shows the overall preferences for all of these "optimal points" calculated using $\mathcal{P}_s$, with the parameter s ranging from -10 to 10. When $\mathcal{P}_s = \min$ ($s \to -\infty$), the point with the highest pref-

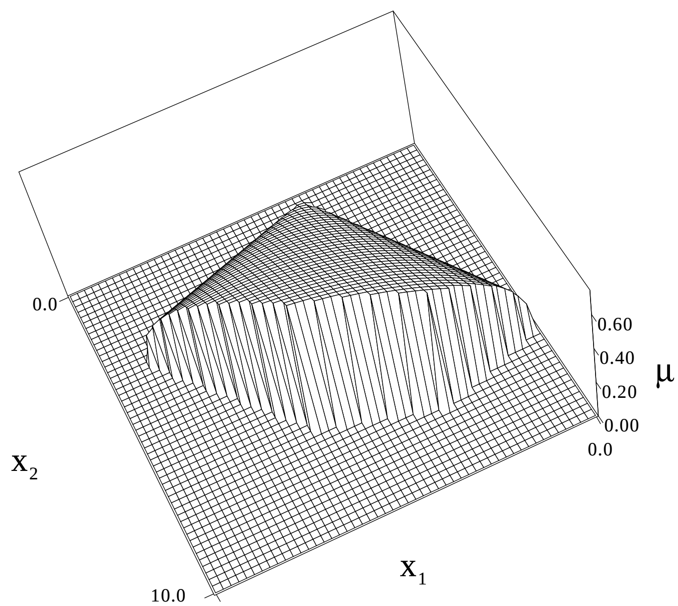

Figure 6.6 Decision surface with product of powers operator

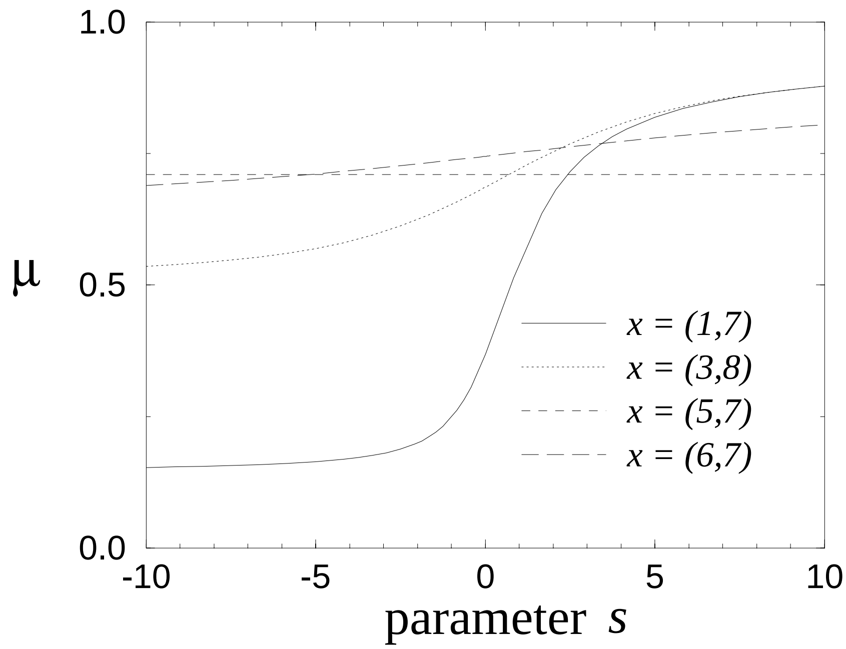

Figure 6.7 Optimal Points Varying with Parameter s

erence is $x = (5, 7)$, the textbook answer. As s grows and $\mathcal{P}_s$ becomes more compensating, the preference for other points grows stronger. As s crosses -5.25, $x = (6, 7)$ overtakes $x = (5, 7)$ and remains the most preferred point through approximately $s = 2.25$. This region includes two common aggregation functions: the weighted product of powers or geometric mean ($s = 0$), and the classical weighted sum or arithmetic mean ($s = 1$). Values of s greater than 2.25, corresponding to even stronger levels of compensation, lead to an overall preference for the point (3,8), and as $s \to +\infty$, the preference for the dominated point $x = (1, 7)$ approaches the preference for $x = (3, 8)$ asymptotically. The limits $s = 10$ and $s = -10$ were chosen to show the full range of behavior of this problem; values of s outside this range are certainly permissible.

The functions discussed in this paper provide models for a continuum of trade-offs ranging from the non-compensating *min* to the compensating $\mathcal{P}_\Pi$ all the way to the *max* operator. Instead of two aggregation functions, there is a parameterized family of functions ranging from the *min* to $\mathcal{P}_\Pi$, and another from $\mathcal{P}_\Pi$ to the *max*.

Conclusion

This paper has discussed the selection of an aggregation function for Multi Attribute Decision Making. The problem was investigated within the context of the Method of Imprecision, a formal system, based on the mathematics of fuzzy sets, for the representation and manipulation of imprecise design information through the specification of preferences on design and performance variables. The M$_o$I casts the preliminary design decision problem as a MADM problem, and uses different aggregation functions to formally model different trade-off strategies. The class of functions appropriate for the aggregation of these preferences has been explored in this paper. While they are directly applicable to decision making in engineering design, the results of this paper are also relevant to other MADM schemes.

This paper has presented a complete characterization of aggregation functions that satisfy the axioms of the M$_o$I. The class of functional equations known as quasi-linear weighted means was shown to be crucial. It was demonstrated that any strictly monotonic design-compatible aggregation function is generated by a generating function as detailed in Theorem 1. The conditional operators in use, while not weighted means, were shown to be limits of sequences of such functions. A parameterized family of functions was detailed, spanning two continua of possible design strategies, one between the non-compensating *min* and the compensating $\mathcal{P}_\Pi$, the two original aggregation functions of the M$_o$I, and one between $\mathcal{P}_\Pi$ and the *max*.

Many MADM systems use aggregation functions, such as the arithmetic mean, that compensate between goals more aggressively than the existing functions of the M$_o$I. These highly compensating functions may seem to be in conflict with the axioms of the M$_o$I. This paper has assessed the possibility of using these common aggregation functions in design decision-making problems.

There are an infinite number of aggregation operators that are suitable for engineering design. In particular, the parameterized family $\mathcal{P}_s$ is a range of functions that models a broad spectrum of design decision-making situations, with the parameter s indicating the degree of compensation permitted among performance criteria. The appropriate choice of the parameter s is problem-specific and is a matter for further investigation.

The use of an aggregation function in any MADM system may be justified on empirical as well as rational grounds. It is not within the scope of this paper to provide empirical studies of designers' decision making behavior. The development here has focused on a rational basis for the choice of an aggregation function. More empirical studies are needed to confirm in practice the results in this paper.

Acknowledgments

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References

[1] ACZÉL, J. *Lectures on Functional Equations and their Applications*. Academic Press, New York, 1966.

[2] ANTONSSON, E. K., AND OTTO, K. N. Imprecision in Engineering Design. *ASME Journal of Mechanical Design 117(B) (Special Combined Issue of the Transactions of the ASME commemorating the 50th anniversary of the Design Engineering Division of the ASME.)* (June 1995), 25–32. Invited paper.

[3] BIEGEL, P., AND PECHT, M. Design trade-offs made easy. *Concurrent Engineering 1*, 3 (May/June 1991), 29–40.

[4] CHEN, S.-J., AND HWANG, C.-L. *Fuzzy Multiple Attribute Decision Making: Methods and Applications*, vol. 375 of *Lecture Notes in Economics and Mathematical Systems*. Springer-Verlag, New York, NY, 1992. In collaboration with Frank P. Hwang.

[5] DUBOIS, D., AND PRADE, H. A review of fuzzy set aggregation connectives. *Information Sciences 36* (1985), 85–121.

[6] HAUSER, J., AND CLAUSING, D. The House of Quality. *Harvard Business Review 66*, 3 (May/June 1988), 63–73.

[7] KEENEY, R., AND RAIFFA, H. *Decisions with Multiple Objectives: Preferences and Value Tradeoffs*. Cambridge University Press, Cambridge, U.K., 1993.

[8] LAW, W. S., AND ANTONSSON, E. K. Hierarchical Imprecise Design with Weights. In *Proceedings of the Fourth IEEE International Conference on Fuzzy Systems (FUZZ-IEEE/IFES'95)* (Mar. 1995), vol. 1, IEEE, pp. 383–388.

[9] LUCE, R., AND RAIFFA, H. *Games and Decisions*. J. Wiley and Sons, New York, NY, 1957. Dover Edition, 1985.

[10] OTTO, K. N. *A Formal Representational Theory for Engineering Design*. PhD thesis, California Institute of Technology, Pasadena, CA, June 1992.

[11] OTTO, K. N., AND ANTONSSON, E. K. Trade-Off Strategies in Engineering Design. *Research in Engineering Design 3*, 2 (1991), 87–104.

[12] OTTO, K. N., AND ANTONSSON, E. K. The Method of Imprecision Compared to Utility Theory for Design Selection Problems. In *Design Theory and Methodology – DTM '93* (Sept. 1993), ASME, pp. 167–173.

[13] PUGH, S. *Total Design*. Addison-Wesley, New York, NY, 1990.

[14] ROYCHOWDHURY, S., AND WANG, B. H. Composite generalization of dombi class and a new family of t-operators using additive-product connective generator. *Fuzzy Sets and Systems 66*, 3 (Sept. 1994), 329–346.

[15] THOLE, U., ZIMMERMAN, H.-J., AND ZYSNO, P. On the suitability of minimum and product operators for the intersection of fuzzy sets. *Fuzzy Sets and Systems 2*, 2 (1979), 167–180.

[16] TRIBUS, M. *Rational Descriptions, Decisions, and Designs*. Pergamon Press, New York, NY, 1969.

[17] VINCENT, T. Game theory as a design tool. *ASME Journal of Mechanisms, Transmissions, and Automation in Design 105* (June 1983), 165–170.

[18] VON NEUMANN, J., AND MORGENSTERN, O. *Theory of Games and Economic Behavior*, 2nd ed. Princeton University Press, Princeton, NJ, 1953.

[19] WHITNEY, D. E. Manufacturing by design. *Harvard Business Review 66*, 4 (July 1988), 83–91.

[20] WOOD, K. L., AND ANTONSSON, E. K. Computations with Imprecise Parameters in Engineering Design: Background and Theory. *ASME Journal of Mechanisms, Transmissions, and Automation in Design 111*, 4 (Dec. 1989), 616–625.

[21] WOOD, K. L., ANTONSSON, E. K., AND BECK, J. L. Comparing Fuzzy and Probability Calculus for Representing Imprecision in Preliminary Engineering Design. In *Design Theory and Methodology - 1989* (New York, Sept. 1989), W. P. Seering, Ed., ASME, pp. 99–105. Presented at the First International ASME *Design Theory and Methodology Conference* (DTM '89), September 18-19, 1989, Montréal, Canada.

[22] YAGER, R. Decision making with fuzzy sets. *Decision Sciences 6*, 3 (1975), 590–600.

[23] YAGER, R. Fuzzy decision making including unequal objectives. *Fuzzy Sets and Systems 1* (1978), 87–95.

[24] ZIMMERMANN, H.-J. *Fuzzy Set Theory - and Its Applications*. Management Science/Operations Research. Kluwer-Nijhoff Publishing, Boston, MA, 1985.

Chapter 7

FORMALISMS FOR NEGOTIATION IN ENGINEERING DESIGN

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Abstract Engineering projects often undergo several design iterations before being completed. Information received from other groups working on a project (analysis, manufacturing, marketing, sales) will often necessitate changes in a design. The interaction between different groups associated with a design project often takes the form of informal "negotiation." This form of interaction commonly arises when engineering information is imprecise. The Method of Imprecision (M_oI) is a formal method for the representation and manipulation of preliminary and imprecise design information. It provides a mechanism for the formalization of these informal negotiations. The nature and scope of informal negotiation in engineering is explored and discussed, and application of the M_oI is illustrated with an example.

Introduction

Engineering design projects in industry commonly involve many different work groups or individuals. In addition to a division of labor by subsystems, there is a division of labor by engineering task. For example, the solid modeling and finite element analysis of an artifact may be handled by different engineers. Production engineers form a group with its own set of concerns, reflected in the considerable literature on the importance of design for manufacture (see [5] for a good bibliography). Finally, design engineers are ultimately responsible to their customers. The customers' concerns are typically represented by management or marketing, who form yet another group with an interest in the design process. The groups within an organization that participate in a project, who are not necessarily all engineers, will be referred to here as *working groups*.

Interest in the issues that arise when so many different groups share the design task has led other researchers to explore concurrent engineering. [16], in particular, have pointed out the importance of the communication of set-based information between downstream and upstream processes. In this paper, we deal with situations of *negotiation* between working groups that arise in the course of a shared design task. Negotiation occurs whenever one working group desires a change in work done by another working group. Often, a defining feature of a negotiation situation is that the two groups have different views of the design object or process. It is possible, however, for situations that are appropriately modeled as negotiation to arise where there is a single, agreed-upon view of the design. In general, this negotiation is highly informal.

Other researchers, particularly in the artificial intelligence community, have seen conflict and negotiation between agents as a crucial part of the design process. Several systems have been developed that attempt to model, incorporate, or handle conflict in design [2, 3, 6, 9, 13]. These projects will be discussed below.

This paper was motivated by work done by the authors in collaboration with a US auto manufacturer and a material manufacturing research center. We found that the informal negotiations described above are common, so common that informal negotiation appears nearly an automatic part of any design process. In most cases, it is not even recognized as negotiation, yet in some instances the culture of negotiation is so highly developed that parties "come to the bargaining table" with exaggerated estimates because they expect to be "bargained down."

Why is this sort of negotiation so prevalent? One reason is the fundamental imprecision inherent in engineering design. Final designs are exact (including manufacturing tolerances), yet such precision is present only at the end of the design process. Engineers (and managers) routinely conduct analyses and make decisions with imprecise quantities. As a design progresses, information becomes more precise, and it may become necessary to revise earlier estimates. Yet the preliminary decisions account for an overwhelming fraction of the total cost of a design, with some studies citing figures up to 70% [17].

Despite the ubiquity of imprecision in engineering design, there are few tools for dealing with imprecise information. One such tool is the *Method of Imprecision* ($\mathrm{M_oI}$) [18], a formal method for representing and manipulating uncertainty in engineering design employing the mathematics of fuzzy sets. It has been shown that the $\mathrm{M_oI}$ can be used to combine design information using a variety of different trade-off strategies [11, 15]. Other references have developed the $\mathrm{M_oI}$ for industrial application [8].

Current work undertakes to use the structure of the MoI to formalize the presently informal negotiation process. The mechanisms employed by the MoI for the representation of uncertain or imprecise information are particularly well-suited to the formal representation of the negotiation process. By formalizing the negotiation process, design teams can promote a more complete exchange of information and have a mechanism to trace the history of a design through its iterations. The existence of a formal negotiation tool may facilitate the inclusion of important performance goal and market information, thus allowing the incorporation of more relevant information into the early design stages.

This paper discusses the context and importance of design negotiation in industry, and demonstrates the application of the MoI to place negotiations on a formal basis.

Examples of Negotiation in Design

The following examples are not exhaustive, but they indicate the wide range of design negotiation situations:

Unreachable target performance values

One example of design negotiation occurs when an engineer or engineering group is given the task of designing a product to a target performance specification. When the product is a newer model of an existing product, the target is often an incremental improvement over last year's model. As an example, consider an automobile chassis, where an existing model has a torsional stiffness of 12,000 ft.-lbs./degree and this year's requirement is to exceed that by 10%. If the engineers are unable to reach the fixed target easily, they will return to the manager who set the task and begin a negotiation process (indeed, this meeting may be scheduled long before any potential problems are known). The engineers may ask for more resources, for a relaxation of other targets, or for a compromise on the original target. Targets are almost never immovable, and managers are commonly willing to negotiate.

Here, negotiation serves to address an inadequacy in the original description of the problem: the ostensibly exact (or *crisp*) requirement is in fact fuzzy, and through negotiation the two groups (in this case, chassis designers and their managers) explore the nature of the "constraints." To formalize this negotiation and reduce pre-distorted bargaining positions, it is important to be able to represent the inherent fuzziness in the constraint. The MoI uses fuzzy sets to represent such imprecision, as will be shown further below.

Trade-offs between facets of performance

An additional layer of complication is added when several target performances are considered at once; here, negotiation can occur even when all specifications are met. In fact, there are usually at least two specifications, since cost of engineering and production resources is almost always a factor. In the example of the chassis design, the designers' position may be to offer the manager a choice between a 6% improvement at a production cost slightly lower than the present model's, or an 11% improvement at a substantially higher cost. To this, the managers may well counter that the new target is 8% improvement, as cheaply as possible.

The trade-off between cost and performance is one of many conflicts that are resolved through negotiation. A typical project will have an array of performance targets. The chassis example mentioned above will also have bending stiffness, weight, noise, and vibrational targets in addition to the torsional stiffness. The overall performance of the design depends on the individual performances, but the exact nature of the dependence varies greatly with the particular problem. The negotiation process is a means by which the true measure (and compromise) of overall performance is uncovered. A method to formalize negotiation may provide quicker and more complete information about the overall performance relationship.

Conflicts between design and manufacturing

The problem of design for manufacturability has been addressed by others [5], but it has not previously been noted that conflicts between design and manufacturing are often resolved through a negotiation process. Sometimes the issue is the rejection of an unmanufacturable design by the production engineer. In many cases, a production engineer suggests changes that will make manufacture simpler, and negotiates with the design engineer for a compromise that will give the most satisfactory overall performance when production cost and reliability are taken into account. In the most optimistic case, a manufacturing group may suggest changes that improve the overall design performance. In a worst-case scenario, poor or no negotiation can lead to spectacular failures. The infamous 1981 collapse of the Hyatt Regency walkway in Kansas City, Missouri, the deadliest engineering disaster in U.S. history, has been attributed to miscommunication between designers and fabricators [14]. A formalism for negotiation can help to facilitate the resolution of these conflicts, and can at the same time provide an unambiguous record of decisions that are made at each step of the design process.

Conflicts between engineering groups

When different working groups have responsibility for different subsystems, or for different aspects, of a design, the requirements of one group may conflict with the requirements of another. Stiffeners added to improve the structural rigidity of a frame might eliminate space that the fuel system group was counting on for the fuel tank. While in mature designs a structural part may well be described by a volume envelope and a few immutable points of contact, there are many situations in which the interaction between parts is not so rigidly described. Even when constraints are imposed in an attempt to avoid conflict between working groups, points of intersection between subsystems are often negotiated.

The incorporation of unquantifiable performances

Many design problems include performance criteria that are difficult, if not impossible, to measure, yet these criteria can be so important as to drive a design. Aesthetic and emotional concerns are certainly of great importance in the auto industry [4], and they also play a surprisingly significant role in other fields, from heavy machinery to military aircraft.

Style, beauty, appearance of solidity, color, image, are all examples of immeasurable attributes that can play a substantial role in the desirability of an engineered object. The fact that they are not easily quantified can lead to either underestimation or overestimation of their role in a design. An engineer designing for more concrete performance specifications may ignore them altogether, yet that same engineer may be working within strict geometrical constraints dictated by a stylist's vision.

Immeasurable performances present the greatest challenge in the formalization of design negotiation as presented in this paper. Still, steps can be taken to formalize this part of the design process, and this formalization can lead to a clearer picture of true overall design requirements.

These examples are meant to convey the nature of the problem of informal negotiation in engineering design. The following section describes a formal system to conduct negotiation more rigorously.

Prior Research on Negotiation

Other researchers have also recognized the importance of negotiation in design. Some take the point of view that any contradiction that arises in the design process is a conflict to be resolved by negotiation. This paper takes a less inclusive view of negotiation, but points out that the formal representation of any trade-off is realized in the same way as a negotiation between two parties.

Researchers in artificial intelligence have noted that conflict is an integral part of the design process. A few are mentioned here: [9] present a more comprehensive bibliography of current research and a thoughtful list of potential sources of conflict in addition to their own work on a design support environment called Schemebuilder. [2] have approached conflict from a utility theory point of view; their work is perhaps the most comparable to the research direction outlined in this paper, but they have focused on a computer implementation of the decision and have limited themselves to a linear weighted sum model and fairly restrictive representations for goals. The work accomplished previously with the MoI, and the applications proposed in this paper, offer more possibilities for the modeling of the design but less automation of the decisions. Some interesting work has been done on a design support system using Pareto optimality by [13]. The system does not calculate optimal solutions, but rather tracks a history of design decisions and automatically notifies agents when it seems that a better design might be overlooked.

These approaches to managing conflict in design, and others from the artificial intelligence community [3, 6], have concentrated on environments that model the design process itself, with the idea that such a model will be applicable in any design situation, thus approaching the design problem from above. The act of negotiation is seen as an entity to be modeled. The research discussed here approaches the design problem from below, where the crucial problem is to model the imprecision inherent in design information, and to use that model of design imprecision to guide negotiation.

Imprecision in the Design Process

The following is a brief review of the Method of Imprecision and some necessary definitions. The reader is referred to [1, 10, 12], and [18] for more details.

The MoI formalizes design decision making in the presence of uncertainty by the specification of *preferences* on *design* and *performance variables*. Variables are sometimes referred to as *parameters*. Design variables are denoted d_i, where i ranges from 1 to n. The set of design variable values under consideration for d_i is denoted $\mathcal{X}_i$. The preference that a designer has for values of d_i, the ith design variable, is represented by a preference function on $\mathcal{X}_i$, termed the *design preference*:

$$\mu_{d_i}(d_i) : \mathcal{X}_i \rightarrow [0, 1]$$

A preference of 1 denotes a completely satisfactory value of the variable, a preference of 0 denotes an unacceptable value, and values in between represent intermediate levels of satisfaction. By treating the preferences μ as a membership function, the MoI can employ the mathematics of fuzzy sets [7, 19] to perform calculations on the uncertain variables.

Performance variables are denoted p_j, with j ranging from 1 to q, and the set of possible values of p_j is denoted $\mathcal{Y}_j$. The customer's[1] preference for values of p_j, the jth performance variable, is called a *functional requirement* and is represented by a preference function on $\mathcal{Y}_j$:

$$\mu_{p_j}(p_j) : \mathcal{Y}_j \rightarrow [0, 1]$$

The set of all performance variables can be represented as a vector $\vec{p} \in \mathcal{Y}$, and the set of all design variables as $\vec{d} \in \mathcal{X}$. Each performance variable p_j is defined by a mapping f_j such that $p_j = f_j(\vec{d})$. The mappings f_j can be any calculation or procedure to measure the performance of a design, including closed-form equations, iterative and heuristic methods, "black box" functions, experiments, and consumer evaluations. When performance and design variables are not all of equal importance, each can be assigned a *weight*, ω_{p_j} or ω_{d_i}.

The individual preferences on variables are combined into an overall preference μ_o by the use of an *aggregation function* $\mathcal{P}$:

$$\mu_o = \mathcal{P}\left(\mu_{d_1}, \omega_{d_1}, ..., \mu_{d_n}, \omega_{d_n}, \mu_{p_1}, \omega_{p_1}, ..., \mu_{p_q}, \omega_{p_q}\right).$$

This overall preference is a measure of the overall performance of the design when all criteria are considered. The design problem is thus to identify design configurations that maximize μ_o, i.e., designs $\vec{d}^*$ such that:

$$\mu_o(\vec{d}^*) = \mu_o^* = \sup\{\mu_o(\vec{d}) \mid \vec{d} \in \mathcal{X}\}.$$

The aggregation function $\mathcal{P}$ reflects the design or trade-off strategy, which indicates to what degree competing attributes of the design should be traded-off against each other [11, 15]. The appropriate design strategy is dictated by the design problem. A design problem will in general require a hierarchy of different trade-off strategies which successively aggregate design attributes.

Formalizing Negotiation

The $\mathrm{M_oI}$ has been applied previously as a decision support tool for self-contained design problems. Its extension to facilitate negotiation between working groups in engineering design entails a broader perspective on the design problem.

The first step in any application of the $\mathrm{M_oI}$ is the identification of design and performance variables. Variables may be continuous or discrete, numerical or represented by linguistic terms. The model used by the $\mathrm{M_oI}$ will employ

[1] The functional requirement μ_{p_j} is called the customer's preference for values of p_j, even if it is the designer who estimates μ_{p_j}.

the most important variables. An advantage of formalizing decisions is that it allows the timely incorporation of information that hitherto was not considered until after the end of a design iteration (when redesign is often the only option).

The second step is the specification of preferences over the values of the variables. By treating the specified preferences on these variables as membership functions, the M_oI can use the mathematics of fuzzy sets to make calculations on these imprecise variables. The imprecision that arises here is *uncertainty* as to the (best) final value—this imprecision will be lessened as the design progresses until the final design is precisely described. The specification of preference on performance variables logically precedes preferences on design variables. Preferences on performance variables represent targets, and are, at least at the outset, independent requirements. Preferences on design variables are likely to change if there is a change in the functional requirements.

The performance variables embody the design task: a designer must create plans for a device that will satisfy given values of a number of performance measures. Some performance variables arise because of a particular choice of design solution. For example, in the course of performing its specified task (providing rotational power), an internal combustion engine generates heat, so heat dissipation becomes a performance parameter, although it is incidental to the primary function of the automobile, which is to carry a load from one point to another. Responsibility for providing preferences on the performance variables will depend on the structure of the company. Ultimately, it is the customer who has preferences for performance, though the customer is often not polled directly. (This paper suggests, but does not explore, the possibilities offered by the M_oI in market research.) Typically, market analysts and management teams are responsible for providing detailed information as to customers' desires. Simply formalizing the preferences can resolve many confusions. The fuzzy preferences contain much more information than a simple list of target values. They show a full range of acceptable values, and the relative desirability of values over that range. Together with weighting and trade-off strategy information, they can tell how much a change in one performance variable is worth in terms of another variable. Details of the specification of preference have been presented previously [10].

Design engineers then specify preferences on the design variables. These design preferences embody everything that is not explicitly represented in the calculation of the performances. The design preferences may thus contain the engineers' intuition about such concerns as manufacturability; in the example detailed below, style is taken into account as a design preference.

Preferences alone provide only a portion of the relevant information. Variables are assigned weights that reflect their relative importance. Trade-off combination strategies [11, 15] must be determined, indicating to what extent superior performance in one aspect is to compensate for lower performance elsewhere. There can be more than one trade-off strategy in a design with many variables: for example, one might allow significant trade-off between cost and weight, but insist on considering safety independently. Even in cases where two variables can trade-off strongly, an unacceptable preference ($\mu = 0$) in one variable can never be overcome. The preferences, weights, and trade-off strategy are used to calculate overall preference for the designs in the design space [11, 15].

A complete specification of preferences on all performance variables would eliminate the need for negotiation whose sole purpose is to clarify the functional requirements. Design engineers are generally qualified to calculate the values of the performance variables p_j; what they lack is complete information as to which performance vectors $\vec{p}$ are to be preferred if one performance variable comes in above or below target. It is not just in situations where targets are unreachable that negotiation ensues; negotiation can also be required because an engineering group finds one target particularly easy to meet and wants to know where to spread around the extra slack that variable provides. If all design variables were simply quantifiable, as are cost, weight, stiffness, and so forth, the task of formalization would be more straightforward. The incorporation of unquantifiable performances presents a particular difficulty. Managers or customers may find it difficult to describe their global preferences for these unquantifiable variables, and a design may have to be seen for a level of satisfaction to be determined. This difficulty can be partially surmounted by the use of linguistic variables, which fuzzy mathematics handles naturally [20].

In situations in which it is unreasonable to expect one group to be expert in another's field, such as the cases mentioned earlier of conflicts between design and manufacturing, or between different design groups, the M_oI can provide a common language for the resolution of contradictions. Two working groups with radically different concerns can use the formalism of preference as a starting point, so that the issues that one group addresses can be taken into account by another group. More detailed formalisms, such as those to represent particular manufacturing issues, will need to be developed in knowledge-specific contexts. The formalism of preference by the M_oI provides a common language for discourse.

At each stage or iteration of the design process, the formalization of decision-making documents the process. In many cases, the decision will include a redesign, with perhaps a modification of preferences on performance variables. In the redesign, with greater information and more analysis, preferences on design variables are likely to change as well. As working groups familiarize themselves with the method, the results of the informal negotiation can be compared to the formal answer produced by the analysis of the M_oI. With greater familiarity, the calculations of the M_oI serve as decision support. One of the

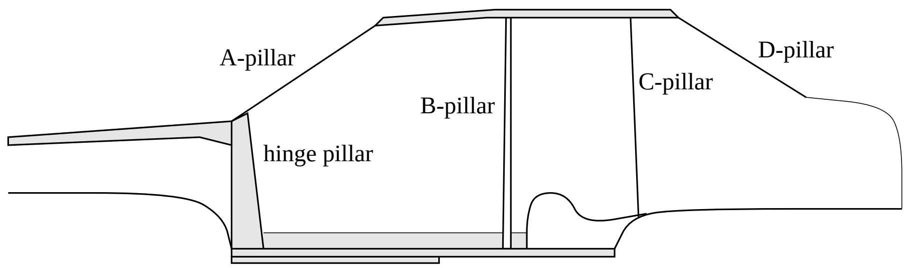

Figure 7.1 Schematic of the car body

many advantages of using the MoI is the clear record of decisions and rationales that it provides.

Example

A simple example of an automotive design problem will help to illustrate the method. Vehicle body design is concerned with tens of design variables and many performance parameters from noise and vibrational response to stiffness to manufacturing cost to style. For this illustrative example, we will assume that there are only three relevant design variables: pillar gauge, roof and floor plate thickness, and fore-aft location of the B-pillar (Figure 7.1). There are three relevant performance parameters: weight (assumed linear with material cost), bending stiffness, and style.

In the example under discussion, as is often the case in the automobile industry, style drives the design. Suppose that the car being designed is not a new design, but an incremental change from, and, it is is hoped, improvement on, a two-door model produced the year before. Here the directive from the stylist is that the new look is lower and sleeker: the roof should be lowered, the clearance between the frame and the ground reduced, and the window opening between the A-pillar and B-pillar should be lengthened. In addition to this small but crucial styling change, the designers are asked to make a 10% improvement over the bending stiffness K_B^{old} of last year's model, with the weight W to stay the same, so that the increase in material cost reflects only inflation. The designers thus have in hand a set of functional requirements consisting of a sketch with some vague explanation, and two targets:

$$K_B^{\text{new}} \geq 1.10 K_B^{\text{old}} \qquad \text{and} \qquad W^{\text{new}} \leq W^{\text{old}}$$

Finite element analysis is used to evaluate candidate designs for performance with respect to the two hard targets, and the attendant solid model will provide sketches for managers or stylists to evaluate the aesthetic impact.

Even in this quite simplified problem, there are already several issues that can be modeled as negotiation. Weight and stiffness will tend to increase together, and the trade-off between these facets of performance is an example of a case where, although there may be only one designer, there is a conflict between aspects of the design that can be formally modeled as a negotiation. If the target of a 10% improvement over the bending stiffness of last year's model turns out to be difficult to achieve, a negotiation between engineers and managers will ensue. The incorporation of the styling requirements is probably the most clear-cut example of the need for negotiation, since the engineers who effect the design will need to consult with the stylists as to the suitability of a completed design. In this illustrative example, we ignore many complications, notably the consideration of manufacturing concerns at the design stage. These additional complications would be addressed formally in a similar manner.

The first step in the formalization of the problem is the expression of more complete preferences $\mu_p(K_B)$ and $\mu_p(W)$ in place of the given hard constraints on the performance variables. Figure 7.2 shows preferences on the two performance variables. The information contained in these plots already provides the possibility for sensible trade-offs between the two using the formalisms of the MoI. Figure 7.3 shows the overall preference for W and K_B in a three-dimensional plot; the vertical axis is the combined preference using a compensating aggregation function:

$$\mu_o = (\mu_p(K_B)\mu_p(W))^{\frac{1}{2}}$$

Thus any two or more candidate designs can be compared by examining the combined preference for the two performances W and K_B.

The problem is more complicated, of course, because a design is not judged on the basis of weight and bending stiffness alone. Even if the design team is able find an "optimal" design in the sense of maximizing the combined performance shown in Figure 7.3, other aspects of the design's performance, such as the style, will need to be taken into account, and it is likely that this will be done through an informal negotiation.

In this case the engineers have a simple measure that can guide them in their work. The stylists have expressed a preference for a wide window opening; this is interpreted as a preference to locate the B-pillar as far aft as possible. This preference is shown in Figure 7.4 (the x-axis represents inches aft of center, with negative numbers being to the fore of center). Since this preference is expressed on a design variable (B-pillar location), the MoI treats it as a design preference. The location of the B-pillar will affect the bending stiffness; this will appear in the finite element calculation. There may well be other preferences for B-pillar location, for example, manufacturing concerns. The computations of the MoI take into account all of this information, with the possibility to assign weights and hierarchies. The overall preference for a design, calculated by the MoI, thus contains the analysis that has been performed (in

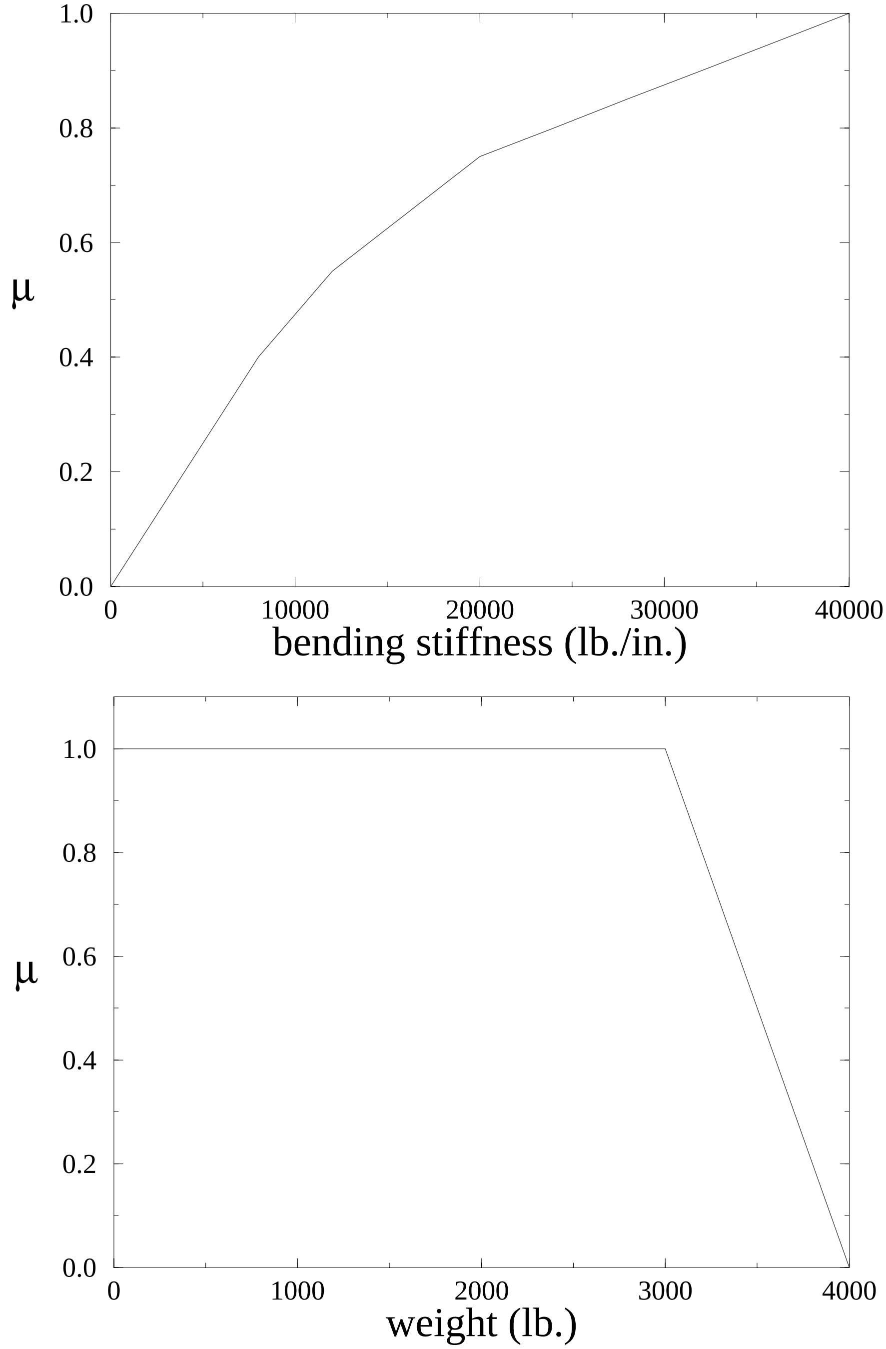

Figure 7.2 Performance preferences $\mu_p(K_B)$ and $\mu_p(W)$

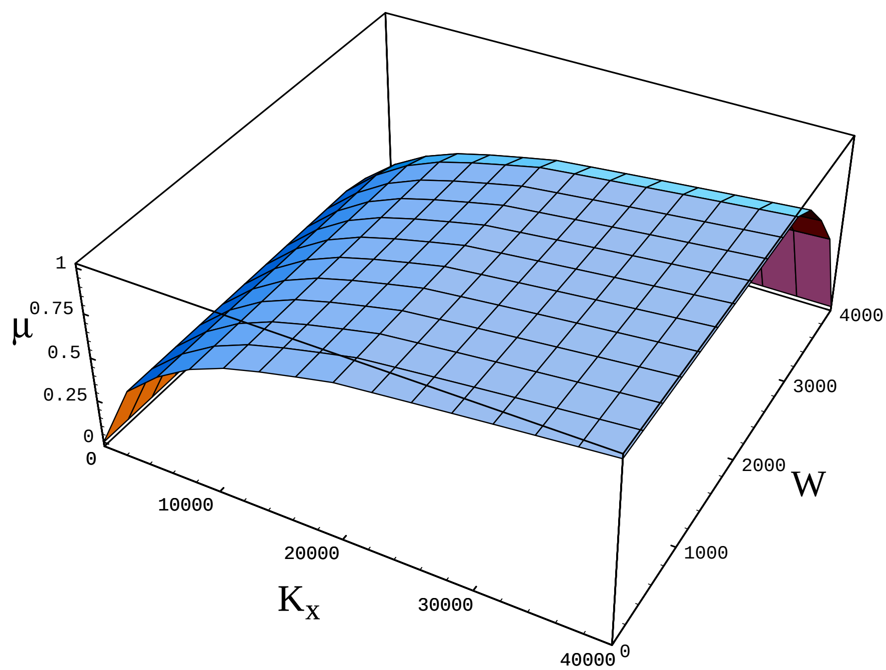

Figure 7.3 Combined preference $\mu_p(K_B, W)$

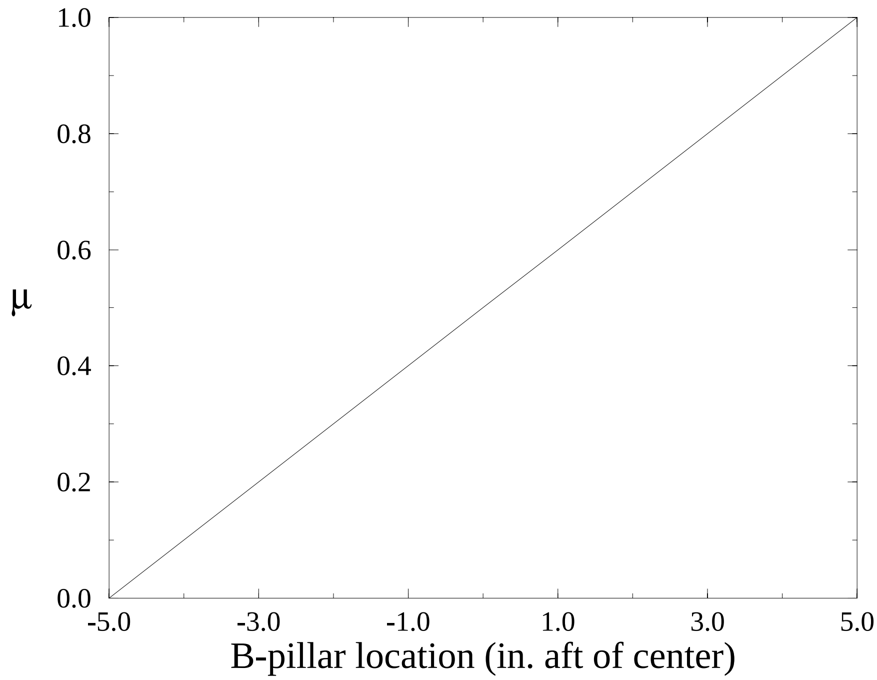

Figure 7.4 Stylists' preference for B-pillar location

this case, the finite element analysis) as well as preferences for the aspects of the design, such as style, that are not calculated by the analysis tools.

It is naturally not possible to represent the effects of all design variables on a single graph, but the effects of one or two design variables (with others held fixed at some nominal values) are easily plotted. Figure 7.5 shows the overall preference μ_o plotted for various B-pillar locations, with the other design variables held fixed. It is interesting to note that the optimal value for weight and bending stiffness alone is for the B-pillar location to be near center. Although bending stiffness is decreasing as the location moves further aft, reconciling the stylists' preference for a wider window requires some compromise in bending stiffness.

Conclusion

The issue of negotiation between engineering groups has been described and discussed. Different negotiation situations that may appear in industrial design settings have been considered. The application of the M_oI in these situations has been illustrated, and the benefits of such formalism have been discussed.

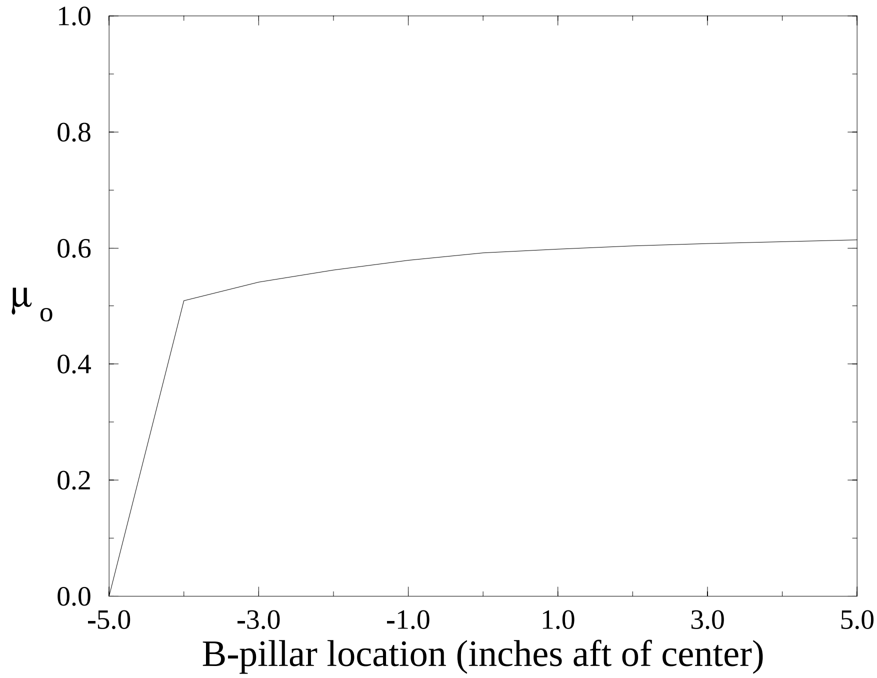

Figure 7.5 Overall preference for different B-pillar locations

The $\mathrm{M_oI}$ has been developed previously for engineering design applications. This paper has suggested the extension of the method to encompass the domains of management and marketing, thus pointing out the possibilities for using the formalism of uncertainty in the entire design process.

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References

[1] ANTONSSON, E. K., AND OTTO, K. N. Imprecision in Engineering Design. *ASME Journal of Mechanical Design 117(B) (Special Combined Issue of the Transactions of the ASME commemorating the 50th anniversary of the Design Engineering Division of the ASME.)* (June 1995), 25–32. Invited paper.

[2] BAHLER, D., DUPONT, C., AND BOWEN, J. Mixed quantitative/qualitative method for evaluating compromise solutions to conflicts in collaborative design. *Artificial Intelligence for Engineering Design, Analysis and Manufacturing 9*, 4 (September 1995), 325–336.

[3] BRAZIER, F. M. T., VAN LANGEN, P. H. G., AND TREUR, J. Modelling conflict management in design: An explicit approach. *Artificial Intelligence for Engineering Design, Analysis and Manufacturing 9*, 4 (September 1995), 353–366.

[4] CLARK, K. B., AND FUJIMOTO, T. *Product Development Performance: Strategy, Organization and Management in the World Auto Industry*. Harvard Business School Press, Boston, MA, 1991.

[5] FINGER, S., AND DIXON, J. R. A review of research in mechanical engineering design. part ii: Representations, analysis and design for the life cycle. *Research in Engineering Design 1*, 2 (1989), 121–137.

[6] HAROUD, D., BOULANGER, S., GELLE, E., AND SMITH, I. Management of conflicts for preliminary engineering design tasks. *Artificial Intelligence for Engineering Design, Analysis and Manufacturing 9*, 4 (September 1995), 313–323.

[7] KLIR, G. J., AND FOLGER, T. A. *Fuzzy Sets, Uncertainty, and Information*. Prentice Hall, Englewood Cliffs, New Jersey, 1988.

[8] LAW, W. S., AND ANTONSSON, E. K. Implementing the Method of Imprecision: An Engineering Design Example. In *Proceedings of the Third IEEE International Conference on Fuzzy Systems (FUZZ-IEEE '94)* (June 1994), vol. 1, IEEE, pp. 358–363. Invited paper.

[9] OH, V., AND SHARPE, J. Conflict management in an interdisciplinary design environment. *Artificial Intelligence for Engineering Design, Analysis and Manufacturing 9*, 4 (September 1995), 247–258.

[10] OTTO, K. N. *A Formal Representational Theory for Engineering Design*. PhD thesis, California Institute of Technology, Pasadena, CA, June 1992.

[11] OTTO, K. N., AND ANTONSSON, E. K. Trade-Off Strategies in Engineering Design. *Research in Engineering Design 3*, 2 (1991), 87–104.

[12] OTTO, K. N., AND ANTONSSON, E. K. Modeling Imprecision in Product Design. In *Proceedings of the Third IEEE International Conference*

on Fuzzy Systems (FUZZ-IEEE '94) (June 1994), vol. 1, IEEE, pp. 346–351. Invited paper.

[13] PETRIE, C. J., WEBSTER, T. A., AND CUTKOSKY, M. R. Using pareto optimality to coordinate distributed agents. *Artificial Intelligence for Engineering Design, Analysis and Manufacturing 9*, 4 (September 1995), 313–323.

[14] RODDIS, W. M. K. Structural failures and engineering ethics. *ASCE Journal of Structural Engineering 119*, 5 (May 1993), 1539–1555.

[15] SCOTT, M. J., AND ANTONSSON, E. K. Aggregation Functions for Engineering Design Trade-offs. In *9th International Conference on Design Theory and Methodology* (Sept. 1995), vol. 2, ASME, pp. 389–396.

[16] WARD, A. C., LIKER, J. K., SOBEK, D. K., AND CRISTIANO, J. J. Set-based concurrent engineering and Toyota. In *Design Theory and Methodology – DTM '94* (Sept. 1994), vol. DE68, ASME, pp. 79–90.

[17] WHITNEY, D. E. Manufacturing by design. *Harvard Business Review 66*, 4 (July 1988), 83–91.

[18] WOOD, K. L., AND ANTONSSON, E. K. Computations with Imprecise Parameters in Engineering Design: Background and Theory. *ASME Journal of Mechanisms, Transmissions, and Automation in Design 111*, 4 (Dec. 1989), 616–625.

[19] ZADEH, L. A. Fuzzy sets. *Information and Control 8* (1965), 338–353.

[20] ZADEH, L. A. The concept of a linguistic variable and its application to approximate reasoning – I. *Information Sciences 8* (1975), 199–249.

Chapter 8

PRELIMINARY VEHICLE STRUCTURE DESIGN APPLICATION

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Abstract The Method of Imprecision, or M_oI, is a formal method for incorporating imprecise information into a design process. This methodology has been exercised on a problem in preliminary vehicle structure design in collaboration with VW Wolfsburg. Results show that the method is useful in trading off multiple conflicting attributes, including styling preferences and engineering requirements.

Keywords:
Industrial Applications of DTM; Vehicle Structure Design; Design Methods and Models; Design Representations; Computational Methods of Design; Fuzzy Sets

Introduction

Preliminary design is inherently imprecise [3, 4, 12, 40], and many preliminary design decisions are made informally. Preliminary design has enormous economic importance, as much of the cost of a design is determined by these (often informal) preliminary decisions [37]. A further complication is the difficulty of communicating imprecise information between different members or groups involved in the design process. Many "interface" decisions are made after design analysis is complete; these *post hoc* decisions can result in costly redesigns.

The Method of Imprecision, or M_oI [39, 1] has been developed to formally incorporate imprecise information into the engineering design process. In the summer of 1997, an application of the M_oI to preliminary vehicle structure design was demonstrated for Volkswagen Wolfsburg. The application serves both to demonstrate the capabilities of the method, and as an introduction to some of the underlying concepts.

A brief introduction to the Method of Imprecision is followed by a description of the demonstration project. The application of the MoI to the problem is then described in detail, and the implications of the results are discussed.

The Method of Imprecision

This introduction is necessarily brief; the application serves as a further tutorial to explicate the ideas reviewed here.

The original work on the MoI [38, 39] formulated the design problem as a decision problem: given a set of candidate designs, identified by vectors $\vec{d}$ of *design variables* in a Design Variable Space (*DVS*), a set of performances, described by vectors $\vec{p}$ of *performance variables* in a Performance Variable Space (*PVS*), and a mapping $f : \vec{d} \mapsto \vec{p}$, choose the candidate design $\vec{d}^*$ which maps to the "best" possible performance $\vec{p}^* = f(\vec{d}^*)$. So stated, there is insufficient information to determine what constitutes a "best" performance, and hence a "best" design. On the one hand, requirements are imprecise, while on the other hand, there is no obvious way to compare different performance variables which are usually not expressed in the same units.

The need to include imprecision in engineering design can be illustrated by a simple example. Figure 8.1 shows a specification for one performance variable (p_j), with the *performance preference* μ_p on the vertical axis. As specifications are commonly written, $p_j \geq 500$ km would be represented by the dashed line (the sharp-edged rectangular step), where $\mu_p = 1$ in the acceptable region, and $\mu_p = 0$ for unacceptable values. However, this crisp specification (or requirement) indicates that two different designs, one with $d_j = 500 - \epsilon$, and another with $d_j = 500 + \epsilon$, would have completely different acceptabilities, no matter how small ϵ becomes. Thus two designs, indistinguishably different in d_j (as $\epsilon \rightarrow 0$), have completely different preferences: one is completely acceptable and one is unacceptable. This situation makes no sense.

Alternatively, the solid line shown in Figure 8.1 indicates a smooth transition of acceptability of performances from unacceptable ($\mu_p = 0$) to acceptable ($\mu_p = 1$), and thus reflects a more realistic specification. The range over which the transition from unacceptable performance to most desired performance takes place will depend on the particular design problem, and may be more or less steep, and smooth or faceted.

Thus the MoI introduces the notion of *preferences*, denoted by $\mu \in [0, 1]$, both to represent the imprecision inherent in the preliminary design problem, and to provide a basis for comparison between different attributes. Performance preferences μ_p on the *PVS* express the requirements more completely

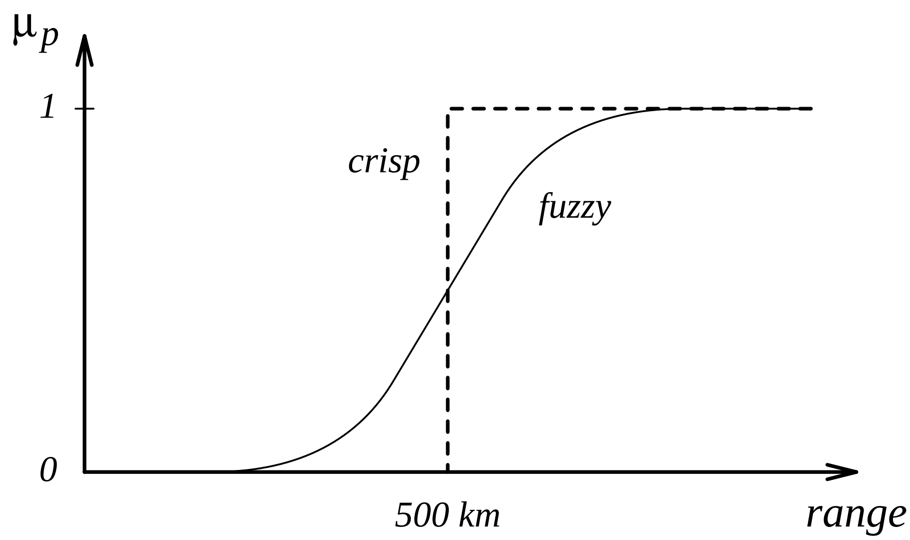

Figure 8.1 Example Imprecise Specification.

than crisp targets. In addition, engineers express *design preferences* μ_d on design variables, allowing the incorporation of performance aspects that are not explicitly calculated by f. Preferences are naturally represented and manipulated using the mathematics of fuzzy sets [42].

The design preferences $\mu_d(\vec{d})$, which are specified on the *DVS*, can be mapped onto the *PVS* by use of the *extension principle* [41]. The various preferences are then combined with an aggregation function $\mathcal{P}$; at first, the $\mathrm{M_oI}$ made use of two different aggregation operators [24], the non-compensating $\mathcal{P}_{\min}(\mu_1, \mu_2) = \min(\mu_1, \mu_2)$ for situations where the overall performance is dictated by the lowest-performing attribute, and the compensating $\mathcal{P}_{\Pi}(\mu_1, \mu_2) = \sqrt{\mu_1 \mu_2}$, when high performance on one attribute is deemed to partly compensate for lower performance on another. Each candidate design $\vec{d}$ thus has an associated overall preference:

$$\mu_o(\vec{d}) = \mathcal{P}\left(\mu_d(\vec{d}), \mu_p(f(\vec{d}))\right)$$

(where $\mu_d(\vec{d})$ and $\mu_p(\vec{p})$ are themselves aggregations of their constituent preferences), and candidate designs can be compared on the basis of this overall preference.

Further research on the $\mathrm{M_oI}$ developed techniques for including noise [26] and adjustments, or *tuning parameters* [25], in the imprecision calculations, and placed an axiomatic framework on the calculations [23]. Implementation of the M$_o$I continued with the development of a computational tool [19]. The applicability of the method was seen to be limited by large computational requirements, so the inclusion of Design of Experiments (DOE) approximations [21] and other computational innovations [18] followed. The need for more than two aggregation functions to model different trade-off levels was recognized, and a family of aggregations introduced [32, 33]. The M$_o$I is reviewed in more detail, and compared to other methods, in [1].

Other researchers have applied fuzzy methods to design optimization problems [7, 8, 28, 29, 6]. Related research includes: chemical process synthesis [9]; fuzzy constraint propagation applied to manufacturing [10]; fuzzy scheduling [11]; application of fuzzy methods to windturbine design [13]; multiobjective scheduling [14]; management of uncertain knowledge in engineering design [15]; engineering design optimization [16]; imprecise calculations in engineering design [5]; evaluation of design alternatives [17]; fuzzy MADM methods in system design [22]; fuzzy evaluations [27]; multiobjective fuzzy optimization [30, 34]; fuzzy ratings and utility analysis in preliminary design evaluation of multiple attributes [35]; scheduling system design [36]; and fuzzy multi-criteria decision making [44, 45, 43]. In addition, there is increasing interest in related work in evolutionary algorithms [46], including the combination of evolutionary algorithms with the M$_o$I [31].

Preliminary Vehicle Structure Design

The general vehicle structure design problem is the engineering of a *body-in-white*, which consists of the (usually metal) frame to which components and exterior panels are fastened. While there are interesting alternative solutions such as space frames and monocoques, this paper is concerned with the welded metal structure typical of passenger automobiles of the present day (see Figures 8.2 and 8.3). The vehicle structure engineers must design a body-in-white that meets certain measurable engineering targets such as stiffnesses, stress levels under load, and weight. In addition, they must satisfy many performance targets associated with less easily measured concepts such as style, manufacturability, and requirements of other engineering groups involved in the design process. These unmeasured peformances are handled informally, often by negotiation between groups working on the same vehicle. The M$_o$I was developed to allow for a formal approach to the incorporation of this imprecise information.

In order to avoid any difficulties involving confidential information, it was decided that an older model vehicle would provide an effective demonstration of the method. To this end a 1980 VW Rabbit (see Figure 8.2) was acquired. The vehicle was stripped to the structural body-in-white, and torsional and bending stiffnesses were measured. The intact body-in-white was found to

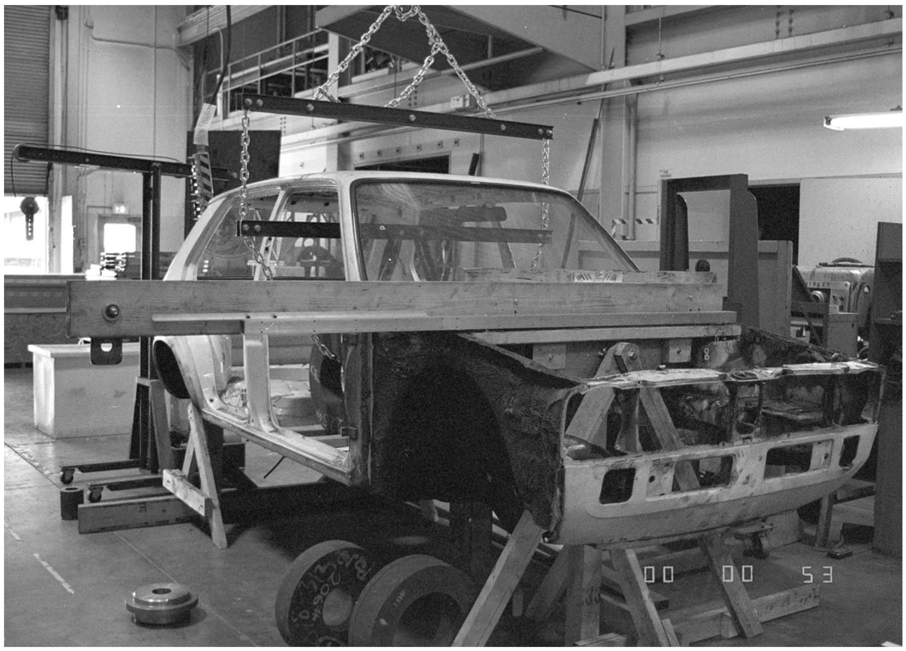

Figure 8.2 1980 VW Rabbit in Stiffness Testing

have a torsional stiffness of approximately 4900 N-m/degree and a bending stiffness of approximately 2500 N/mm. Tables of data from some of the load tests are shown in the Appendix. In addition, geometric data were gathered and used to create a solid model (Figure 8.3). The solid model and the structural stiffness information together were used to create and calibrate a finite element model (Figure 8.4).

The finite element model was parameterized with five[1] design variables:

1. A-pillar thickness (mm)
2. B-pillar thickness (mm)
3. floor pan thickness (mm)
4. floor rail thickness (mm)
5. B-pillar location (mm aft of a nominal point chosen by stylists)

and the performance was assessed with three measures:

[1]The demonstration here was conducted using a subset of the design variables; the method can be applied directly to a larger set of variables.

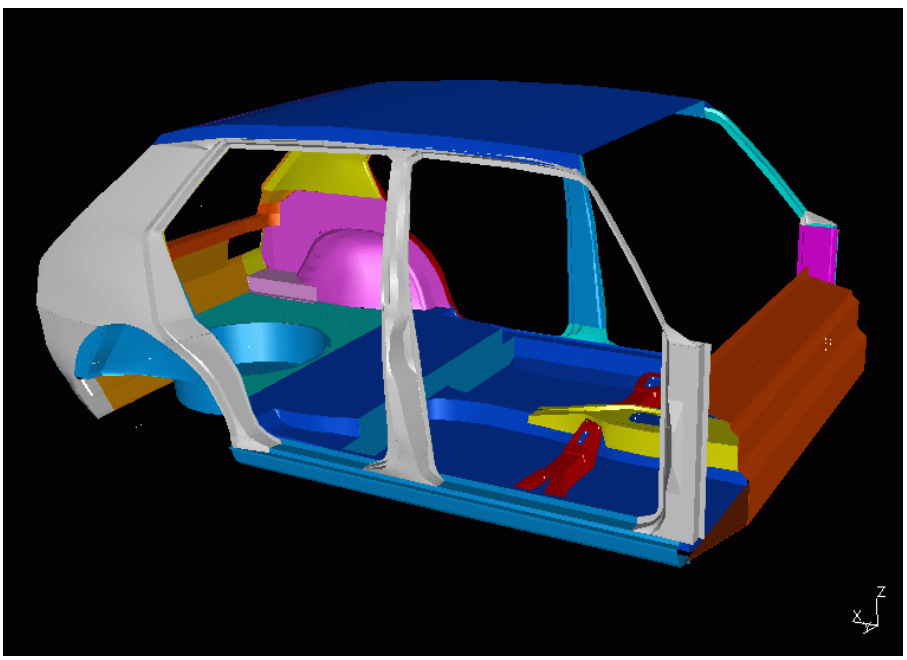

Figure 8.3 Geometric Model of Body-In-White in SDRC I-Deas

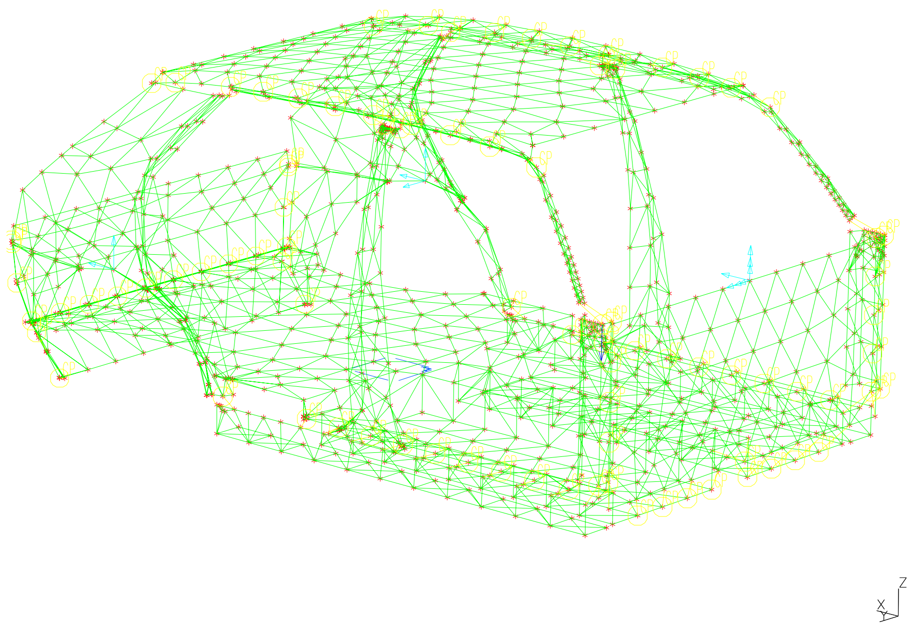

Figure 8.4 Finite Element Model of Body-In-White

1. Bending stiffness (N/mm)
2. Torsional stiffness (N-m/deg)
3. Weight (kg)

The stated design problem was to achieve 10% improvements over the reference model in the three measured performances. In addition, it was understood that that the design must not be difficult to manufacture, and that this year's model should have a somewhat longer and sleeker look.

Applying The M$_{O}$I To Include Imprecise Information

While standard optimization methods could be used to determine the highest achievable bending stiffness, the highest achievable torsional stiffness, or the lowest achievable weight for this analysis model, such an optimization would not tell the designer which designs are the most promising when other relevant considerations are taken into account. On the one hand, there is a necessary trade-off between the stiffnesses and the weight; it is impossible to optimize both simultaneously. Additionally, there is other (imprecise) information to consider when making the decision, such as manufacturing and styling concerns. The application of the M$_{O}$I to this problem involves constructing a differ-

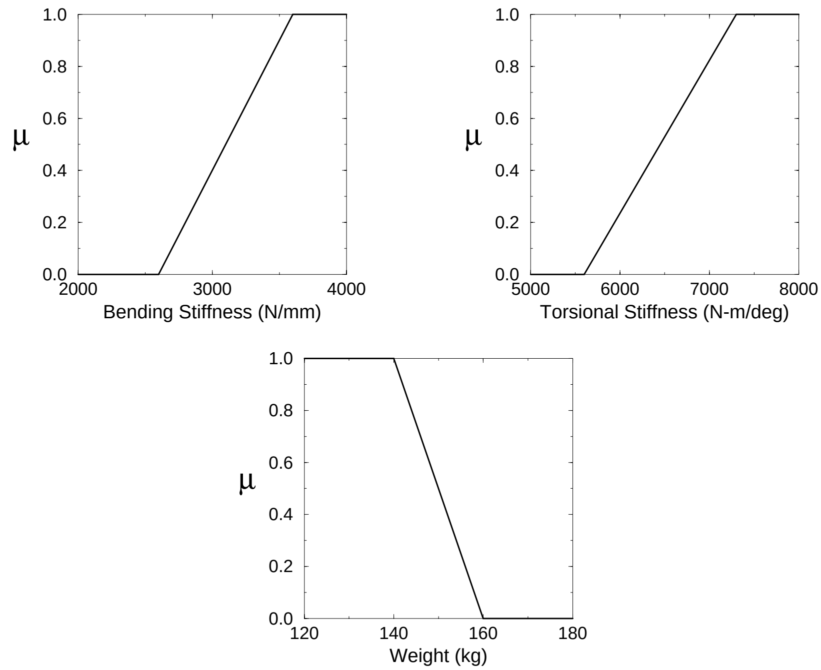

Figure 8.5 Imprecise Performance Requirements

ent "optimization" problem that includes the imprecise information that would be left to the negotiation stage in traditional design.

The calculated performance requirements on bending stiffness, torsional stiffness, and weight were originally expressed as targets of 10% improvements over the reference model. As was discussed above, this is unrealistically, and indeed unproductively, precise. In place of these hard targets, imprecise performance requirements were specified with a linear interpolation between two points. In the implementation of the $\mathrm{M_oI}$, it is common to name the customer as the source of the performance preferences; in fact, it is more likely to be a manager, perhaps informed by market research, serving as the customer's proxy. To specify these imprecise requirements, the manager must answer two simple questions: "What is the lowest performance you can live with (where is $\mu = 0$)? What performance would satisfy you completely (where is $\mu = 1$)?" These bounds are clearly dependent on a number of factors, including the target market and the performance of competitors' products; we have found that engineering managers can answer these two questions with little more effort than is needed to settle on the initial crisp target. Figure 8.5 shows the imprecise requirements on stiffnesses and weight.

To include requirements on manufacturing, availability, style, and other things which are not calculated in the finite element analysis, designer preferences are specified on the design variables. As with the imprecise performance requirements, they range from $\mu = 0$ at the unacceptable limit to $\mu = 1$ at the most preferred. A preference is defined on each of the five design variables, as shown in Figure 8.6. Each preference is representative of imprecise information that can be incorporated using the $\text{M}_\text{o}\text{I}$:

1. The sheet steel for stamping the A-pillar is only available in certain increments, so this plot is discrete rather than continuous. The manufacturing engineer has a higher preference for thinner sheets, since they are easier to form; this is a design preference for manufacturability.

2. The B-pillar thickness is continuous and more complicated than the linear performance preferences. This preference does *not* indicate that the physical B-pillar might be 1.113 or 1.114 mm thick; rather it means that the designer knows that the finite element model is simplified, and that a high number for B-pillar thickness means that more reinforcing features will need to be added to the B-pillar. The designer would like to keep the B-pillar as simple as possible.

3. The floor pan thickness is preferred thicker by the designer for ease of attachments and for durability.

4. The floor rail thickness preference is an example of a sourcing, or availability preference; it states that some thicknesses are more easily obtained than others.

5. The design preference for B-pillar location comes from the stylists, and captures the directive for a longer, sleeker look for this year's model. It has been specified differently from the other design preferences, using α-*cuts* [1], so that the stylists have given a range of perfectly acceptable values, a range of barely acceptable values, and a range of values that fall in the middle. This method of specifying preferences can have computational advantages.

In addition to these preferences, each attribute is assigned a weight indicating its relative importance, and the way in which attributes trade-off against each other must also be specified. In this test example, it was determined that bending and torsional stiffness traded-off in a non-compensating manner — the lowest preference is maximized. Together they traded-off with weight in a compensating manner, so that high performance on stiffness could partly make up for low performance on weight, and vice versa. The designer preferences all traded-off in a compensating manner as well, with the styling preference

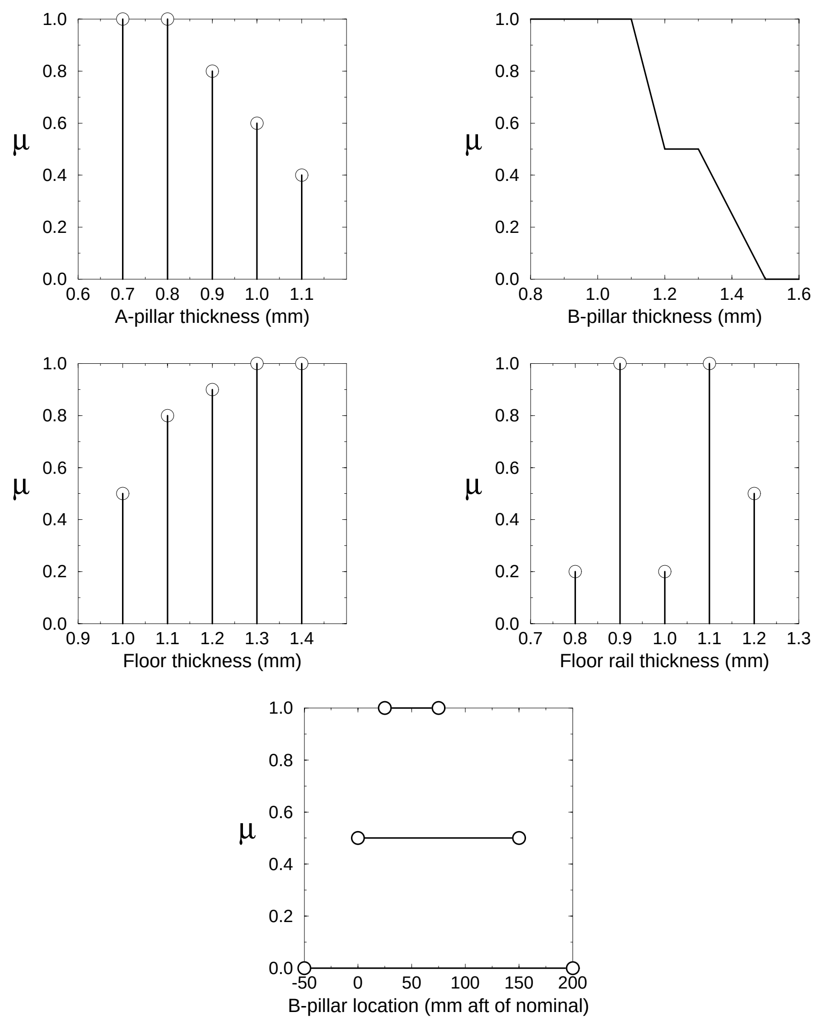

Figure 8.6 Designer Preferences

assigned a relatively high weight to reflect the importance of styling considerations in automobile design. The preferences for the computed performance variables are also weighted heavily. The correct determination of weights and trade-off strategies is crucial to the method, and a full range of strategies [33], of which the original compensating and non-compensating strategies are only two examples, is available.

Through the application of the MoI, the design problem has been reformulated to be the maximization of the overall preference:

$$
\begin{aligned}
\mu_o(\vec{d}) &= \mathcal{P}\left[\mu_d(\vec{d}), \mu_p(f(\vec{d}))\right] \\
&= \mathcal{P}_\Pi\left[\mathcal{P}_\Pi\left(\mu_p(f_3(\vec{d})), \mathcal{P}_{\min}\left[\mu_p(f_1(\vec{d})), \mu_p(f_2(\vec{d}))\right]\right),\right. \\
&\qquad \left.\mathcal{P}_\Pi\left(\mathcal{P}_\Pi\left[\mu_d(d_1), \mu_d(d_2), \mu_d(d_3), \mu_d(d_4)\right], \mu_d(d_5)\right)\right]
\end{aligned}
$$

The computation of $\mu_o(\vec{d})$ for a single design point $\vec{d}$ is limited by the finite element stiffness calculation, which takes about a minute on a Sun Ultra1-170MHz workstation; the calculations of weight and preference aggregation are of negligible cost, regardless of the trade-off strategies employed. Even in this relatively modest problem, where there are only five design dimensions, an exhaustive calculation of preferences over the design space is prohibitively expensive. The MoI exploits the structure of the problem to speed the search for preferred solutions: the internal calculations linearize where possible, effectively reducing the dimension of the search space, and employ Powell's method to locate internal extrema [20].

Results

The design problem, including all imprecise information, was solved in two different ways. First, in order to demonstrate the method, the finite element analysis was run 3125 ($= 5^5$) times to provide a coarse but complete check of the entire design space. The point of peak overall preference of $\mu_o = 0.50$ was found at $\vec{d} = (1.0, 0.9, 0.9, 1.0, 50)$, where the design preferences μ_d are $(0.6, 1.0, 1.0, 0.5, 1.0)$; the stiffnesses and weight at this point were $\vec{p} = (2832, 5836, 147)$, with preferences $(0.23, 0.14, 0.62)$. The maximum achievable stiffnesses are 3365 N/mm ($\mu_p = 0.77$) in bending and 6029 N-m/degree ($\mu_p = 0.25$) in torsion, but the corresponding weight of 170 kg is unacceptable. Similarly, a weight of 144 kg ($\mu_p = 0.78$) is achievable, but stiffnesses drop to 2803 N/mm ($\mu_p = 0.20$) and 5730 N-m/degree ($\mu_p = 0.08$). The combined overall preference μ_o also takes into account the design preferences on style, manufacturability, and the like.

The power of the method lies not in an ability to find a single overall "best" point, but in the information it contains of how the total combined preference μ_o varies with each of the design variables. Although it is impossible to display all five dimensions varying at once, a tool was written that uses a commercial package (Matlab) to display results interactively. Using the tool, the designer can see the change in preference that would occur by varying each design variable independently from a chosen beginning point. Results can be seen on five simultaneous plots in two dimensions (see Figure 8.7), or on a three-dimensional surface plot (see Figure 8.8) with the remaining design variables set to nominal values. The interpretation of these graphical results is discussed in greater detail below.

Approximations

Naturally, the exhaustive evaluation of points in the design space would not be performed on a real design problem. It was performed here *only* for comparison purposes. An approach that utilizes Design of Experiments (DOE) [2] to approximate the finite element calculations for bending and torsional stiffnesses reached substantially similar results in only 21 runs (or approximately 20 minutes). The average difference (from the exhaustive evaluation) in bending stiffness was approximately 1%, with a maximum difference of less than 4%, while the average and maximum differences for torsional stiffness were both less than 1%.

In some cases, the nonlinearities of the analysis function f will defeat a linear or even polynomial approximation, but in many cases, such as the example presented here, these simple approximations can drastically reduce the required computation. Since precise answers are not required for preliminary design, it is sensible to exploit approximation tools when possible. If more computation can be justified, a more thorough calculation can be made.

Discussion

Engineering analysis usually requires some judgement on the part of the designer. Unless a full-scale exact prototype is to be built and tested, the accuracy of any calculated performance measure depends on the fidelity of the model employed. Even when exact data are available for some attributes, final decisions about a design incorporate other, unmodelled concerns, such as manufacturing and styling.

The MoI constructs a model of the entire decision process, expressing the calculated overall performance $\mu_o(\vec{d})$ as a function of the design variables. It depends on many factors: the function f for calculating measurable performances, the specification of design preferences μ_d and performance preferences μ_p, the weighting of these preferences, and the specification of trade-off aggregations between attributes. A change in any of these will affect the shape of the function μ_o in design space, and thus affect the decision. The analysis f is here relatively expensive to compute, and changes in the finite element

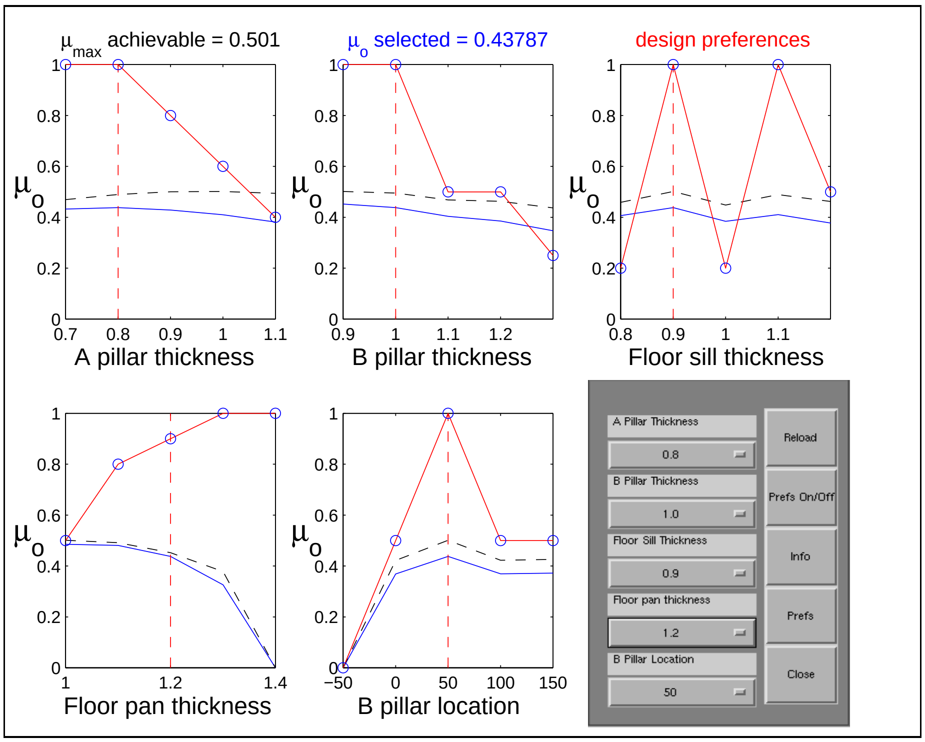

Figure 8.7 Graphical User Interface for Preference Display

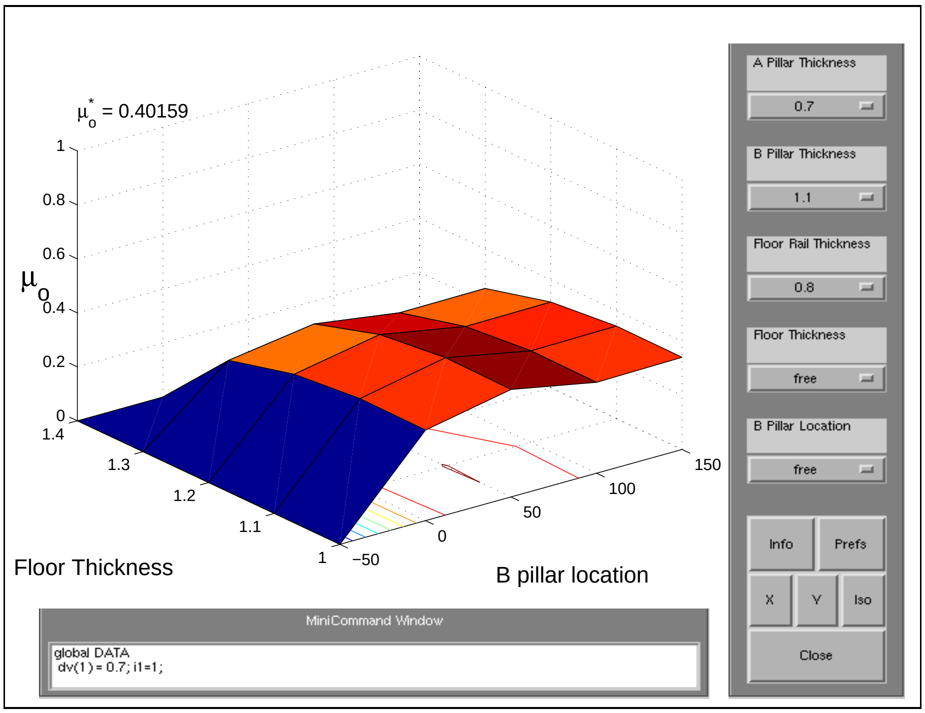

Figure 8.8 3-D Graphical User Interface for Preference Display

model are costly to propagate to overall preference. Changes in the other factors, on the other hand, are easily incorporated, as finite element results $f(\vec{d})$ are stored so that the same design point need never be analyzed more than once. This allows the $\mathrm{M_oI}$ to support an iterative decision process, when the information from the first round of calculations inspires a change in the preference structure.

In the example shown in this paper, the shape of the function $\mu_o(\vec{d})$ is sensitive to changes in the styling preference $\mu_d(d_5)$, which is not surprising, since this preference is accorded a large weight. This and other features of the design problem can be seen in the advanced interface shown in Figure 8.7. The vertical dashed (red) lines indicate the selected values of the design variables. In this figure, $\vec{d} = (0.8, 1, 0.9, 1.2, 50)$, a point which is representative, not optimal. The solid (blue) lines indicate how the overall preference μ_o would change by varying that design variable while holding the other four fixed at their current values. For instance, decreasing the floor pan thickness d_4 will result in a more preferred design. The dashed (black) lines show the maximum achievable μ_o for each value of each design variable. Finally, the solid (red) lines joining the circles are the specified preferences on design variables.

In this example, the desired improvements in performance were achieved by small changes to the sheet-metal thicknesses and B-pillar location. Visually the improved structure would appear quite similar to that shown in Figure 8.3. No change in vehicle structure configuration was required here.

Figure 8.7 shows that the overall preference μ_o varies qualitatively, though not quantitatively, with four of the five design variables; the exception is d_4, floor pan thickness. The variation of the overall preference μ_o with respect to d_4 shows a conflict between the calculated stiffness and weight requirements μ_p and the provisions for attachment and durability captured by the designer preference $\mu_d(d_4)$. This indicates d_4 as a likely candidate for change in a potential redesign. The resolution of the conflict would be achieved by a choice of d_4 that provides the best overall trade-off between the competing attributes. Designers can interact with the preference display to examine trends in the structure of the overall preference.

Conclusions

In preliminary vehicle structure design, as in preliminary engineering design in general, many important decisions are made informally on the basis of imprecise information. Concerns of styling and manufacturability, for instance, can carry great weight in the design process although they are not modelled by any formal analysis. The $\mathrm{M_oI}$ is a tool to formally incorporate such imprecise information into the design process, and thus to make decisions on a sound basis. In a demonstration of the $\mathrm{M_oI}$ prepared for VW Wolfsburg, concerns of manufacturing, styling, parts availability, and design were incorporated with the engineering analysis of the structural stiffness of a VW Rabbit. The results show the usefulness of the method in trading off these conflicting attributes.

Any analysis involving more than two design variables must contend with two difficulties, the exploding need for computation, and the problems of displaying results in several dimensions. The $\mathrm{M_oI}$ uses approximations, when feasible, to address the first difficulty, and an interactive graphical tool for preference display was developed and applied here to address the second.

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References

[1] ANTONSSON, E. K., AND OTTO, K. N. Imprecision in Engineering Design. *ASME Journal of Mechanical Design 117(B) (Special Combined Issue of the Transactions of the ASME commemorating the 50th anniversary of the Design Engineering Division of the ASME.)* (June 1995), 25–32. Invited paper.

[2] BARKER, T. *Quality by Experimental Design*. Marcel Dekker, Inc., New York, 1985.

[3] BECKER, J. M. *A Structural Design Process Philosophy and Methodology*. PhD thesis, University of California, Berkeley, 1973.

[4] BLOCKLEY, D. I. *The Nature of Structural Design and Safety*. Ellis Norwood Limited, Chichester, 1980.

[5] CHEN, J. E., AND OTTO, K. N. A tool for imprecise calculations in engineering design. In *Proceedings of the Fourth IEEE International Conference on Fuzzy Systems (FUZZ-IEEE/IFES'95)* (Mar. 1995), vol. 1, IEEE, pp. 389–394.

[6] DHINGRA, A. K. Multiple objective design optimization. In *Fuzzy Sets in Engineering Design and Configuration*, H.-J. Sebastian and E. K. Antonsson, Eds. Kluwer, 1996, pp. 53–88.

[7] DIAZ, A. R. Fuzzy set based models in design optimization. In *Advances in Design Automation - 1988* (New York, NY, Sept. 1988), S. S. Rao, Ed., vol. DE-14, ASME, pp. 477–485.

[8] DIAZ, A. R. A strategy for optimal design of hierarchical systems using fuzzy sets. In *The 1989 NSF Engineering Design Research Conference* (College of Engineering, University of Massachusetts, Amherst, June 1989), J. R. Dixon, Ed., NSF, pp. 537–547.

[9] DJOUAD, S., FLOQUET, P., DOMENECH, S., AND PIBOULEAU, L. Fuzzy set methods in separation process synthesis. In *Fuzzy Information Engineering: A Guided Tour of Applications*, D. Dubois, H. Prade, and R. R. Yager, Eds. John Wiley & Sons, 1996.

[10] DUBOIS, D. Tuning the cutting edge conditions on a machining transfer line using fuzzy constraint propagation. In *Fuzzy Information Engineering: A Guided Tour of Applications*, D. Dubois, H. Prade, and R. R. Yager, Eds. John Wiley & Sons, 1996.

[11] FARGIER, H. Fuzzy scheduling: Principles and experiments. In *Fuzzy Information Engineering: A Guided Tour of Applications*, D. Dubois, H. Prade, and R. R. Yager, Eds. John Wiley & Sons, 1996.

[12] GAVIN, JR., J. G. Fly me to the Moon: An interview with Joseph G. Gavin, Jr. *Technology Review 97*, 5 (July 1994), 61–68.

[13] GERHART, V. Uncertainty within the design of a wind turbine. Working paper or, RWTH Aachen, Templergraben 64, 52062 Aachen, Germany, 1997.

[14] GRABOT, B., GENESTE, L., AND DUPEUX, A. Tuning of fuzzy rules for multiobjective scheduling. In *Fuzzy Information Engineering: A Guided Tour of Applications*, D. Dubois, H. Prade, and R. R. Yager, Eds. John Wiley & Sons, 1996.

[15] HAMBURG, I., AND HAMBURG, P. Fuzzy logic, A user friendly technology for the management of uncertain knowledge in engineering design process. In *Modelling Uncertain Data* (Berlin, Mar. 1993), H. Bandemer, Ed., Akademie Verlag, pp. 153–157. Mathematical Research series, Volume 68, Lectures from the GAMM-Workshop: Modelling Uncertain Data, held in Freiberg from March 21-24, 1992.

[16] HSU, Y.-L., LIN, Y.-F., AND SUN, T.-L. Engineering design optimization as a fuzzy control process. In *Proceedings of the Fourth IEEE International Conference on Fuzzy Systems (FUZZ-IEEE/IFES'95)* (Mar. 1995), vol. 4, IEEE, pp. 2001–2008.

[17] KNOSALA, R., AND PEDRYCZ, W. Evaluation of design alternatives in mechanical engineering. *Fuzzy Sets and Systems 47*, 3 (1992), 269–280.

[18] LAW, W. S. *Evaluating Imprecision in Engineering Design*. PhD thesis, California Institute of Technology, Pasadena, CA, June 1996.

[19] LAW, W. S., AND ANTONSSON, E. K. Including Imprecision in Engineering Design Calculations. In *Design Theory and Methodology – DTM '94* (Sept. 1994), vol. DE-68, ASME, pp. 109–114.

[20] LAW, W. S., AND ANTONSSON, E. K. Optimization Methods for Calculating Design Imprecision. In *Advances in Design Automation - 1995* (Sept. 1995), vol. 1, ASME, pp. 471–476.

[21] LAW, W. S., AND ANTONSSON, E. K. Multi-dimensional Mapping of Design Imprecision. In *8th International Conference on Design Theory and Methodology* (Aug. 1996), ASME.

[22] MÜLLER, K., AND THÄRIGEN, M. Applications of fuzzy hierarchies and fuzzy MADM methods to innovative system design. In *Proceedings of the Third IEEE International Conference on Fuzzy Systems* (June 1994), vol. 1, IEEE, pp. 364–367.

[23] OTTO, K. N. *A Formal Representational Theory for Engineering Design*. PhD thesis, California Institute of Technology, Pasadena, CA, June 1992.

[24] OTTO, K. N., AND ANTONSSON, E. K. Trade-Off Strategies in Engineering Design. *Research in Engineering Design 3*, 2 (1991), 87–104.

[25] OTTO, K. N., AND ANTONSSON, E. K. Tuning Parameters in Engineering Design. *ASME Journal of Mechanical Design 115*, 1 (Mar. 1993), 14–19.

[26] OTTO, K. N., AND ANTONSSON, E. K. Design Parameter Selection in the Presence of Noise. *Research in Engineering Design 6*, 4 (1994), 234–246.

[27] POSTHOFF, C., AND SCHLOSSER, M. Learning from examples for the construction of fuzzy evaluations. In *Proceedings of the Third IEEE International Conference on Fuzzy Systems* (June 1994), vol. 1, IEEE, pp. 368–371.

[28] RAO, S. S., AND DHINGRA, A. K. Applications of fuzzy theories to multi-objective system optimization. NASA Contractor Report 177573, NASA Ames Research Center, 1991.

[29] RAO, S. S., SUNDARARAJU, K., PRAKASH, B., AND BALAKRISHNA, C. Multiobjective fuzzy optimization techniques for engineering design. *Computers and Structures 42*, 1 (1992), 37–44.

[30] SAKAWA, M., AND KATO, K. An interactive fuzzy satisficing methods for large-scale multiobjective linear programs with fuzzy numbers. In *Proceedings of the Fourth IEEE International Conference on Fuzzy Systems (FUZZ-IEEE/IFES'95)* (Mar. 1995), vol. 3, IEEE, pp. 1155–1162.

[31] SCHLEIFFER, R. Fuzzy engineering design. RWTH Aachen, 1998. Diplomarbeit.

[32] SCOTT, M. J., AND ANTONSSON, E. K. Aggregation Functions for Engineering Design Trade-offs. In *9th International Conference on Design Theory and Methodology* (Sept. 1995), vol. 2, ASME, pp. 389–396.

[33] SCOTT, M. J., AND ANTONSSON, E. K. Aggregation Functions for Engineering Design Trade-offs. *Fuzzy Sets and Systems 99*, 3 (1998), 253–264.

[34] SHIH, C. J., AND WANGSAWIDJAJA, R. A. S. Multiobjective fuzzy optimization with random variables in a mix of fuzzy and probabilistic environment. In *Proceedings of the Fourth IEEE International Conference on Fuzzy Systems (FUZZ-IEEE/IFES'95)* (Mar. 1995), vol. 3, IEEE, pp. 1163–1170.

[35] THURSTON, D. L., AND CARNAHAN, J. Fuzzy ratings and utility analysis in preliminary design evaluation of multiple attributes. *ASME Journal of Mechanical Design 114*, 4 (Dec. 1992), 648–658.

[36] TÜRKŞEN, I. B. Scheduling system design: Three fuzzy theory approaches. In *Fuzzy Information Engineering: A Guided Tour of Applications*, D. Dubois, H. Prade, and R. R. Yager, Eds. John Wiley & Sons, 1996.

[37] WHITNEY, D. E. Manufacturing by design. *Harvard Business Review 66*, 4 (July 1988), 83–91.

[38] WOOD, K. L. *A Method for Representing and Manipulating Uncertainties in Preliminary Engineering Design*. PhD thesis, California Institute of Technology, Pasadena, CA, 1989.

[39] WOOD, K. L., AND ANTONSSON, E. K. Computations with Imprecise Parameters in Engineering Design: Background and Theory. *ASME Journal of Mechanisms, Transmissions, and Automation in Design 111*, 4 (Dec. 1989), 616–625.

[40] YAO, J. T. P., AND FURUTA, H. Probabilistic treatment of fuzzy events in civil engineering. *Probabilistic Engineering Mechanics 1*, 1 (1986), 58–64.

[41] ZADEH, L. A. Fuzzy logic and approximate reasoning. *Synthese 30* (1975), 407–428.

[42] ZIMMERMANN, H.-J. *Fuzzy Set Theory - and Its Applications*. Management Science/Operations Research. Kluwer-Nijhoff Publishing, Boston, MA, 1985.

[43] ZIMMERMANN, H.-J. Media selection and other applications of fuzzy linear programming. In *Fuzzy Information Engineering: A Guided Tour of Applications*, D. Dubois, H. Prade, and R. R. Yager, Eds. John Wiley & Sons, 1996.

[44] ZIMMERMANN, H.-J., AND SEBASTIAN, H.-J. Optimization and fuzziness in problems of design and configuration. In *Proceedings of the Second IEEE International Conference on Fuzzy Systems* (San Francisco, CA, June 1993), IEEE, Pergamon Press, pp. 1237–1240.

[45] ZIMMERMANN, H.-J., AND SEBASTIAN, H.-J. Fuzzy design - Integration of fuzzy theory with knowledge-based system-design. In *Proceedings of the Third IEEE International Conference on Fuzzy Systems* (June 1994), vol. 1, IEEE, pp. 352–357.

[46] ZIMMERMANN, H.-J., AND SEBASTIAN, H.-J. Intelligent system design support by fuzzy-multi-criteria decision making and/or evolutionary algorithms. In *Proceedings of the Fourth IEEE International Conference on Fuzzy Systems (FUZZ-IEEE/IFES'95)* (Mar. 1995), vol. 1, IEEE, pp. 367–374.

A Appendix: Stiffness Test Results

Torsion

Load (N)	Moment (N-m)	Deflection (mm)	Twist (deg)
0.00	0.00	0.00	0.00000
126.99	212.84	1.09	0.04407
275.78	455.03	2.41	0.09736
404.77	667.87	3.48	0.14041
551.55	910.06	4.47	0.18038
680.54	1122.90	5.72	0.23060
845.12	1394.45	6.99	0.28184
974.11	1607.28	8.08	0.32591

Fitting to $y = mx + c$:

$$\begin{aligned} m &= 4960.74 \text{ N-m/deg} \\ c &= -10.16 \text{ N-m} \end{aligned}$$

Fitting to $y = mx + 0$:

$$m = 4917.04\text{N-m/deg}$$

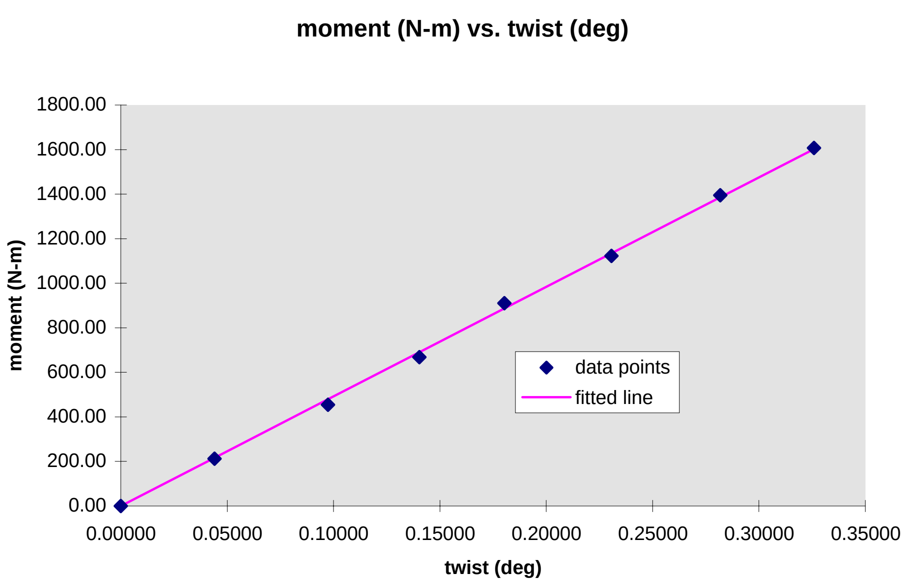

Figure A.1 Load Test, Torsional Stiffness

Bending

Load (N)	Deflection (mm)
0.00	0.00000
284.67	0.101060
551.55	0.17780
836.22	0.33020
2001.60	0.78740
2286.27	0.91440
2837.82	1.16840

Fitting to $y = mx + c$:

$$\begin{aligned} m &= 2426.16\text{N/mm} \\ c &= 51.79\text{N} \end{aligned}$$

Fitting to $y = mx + 0$:

$$m = 2484.80\text{N/mm}$$

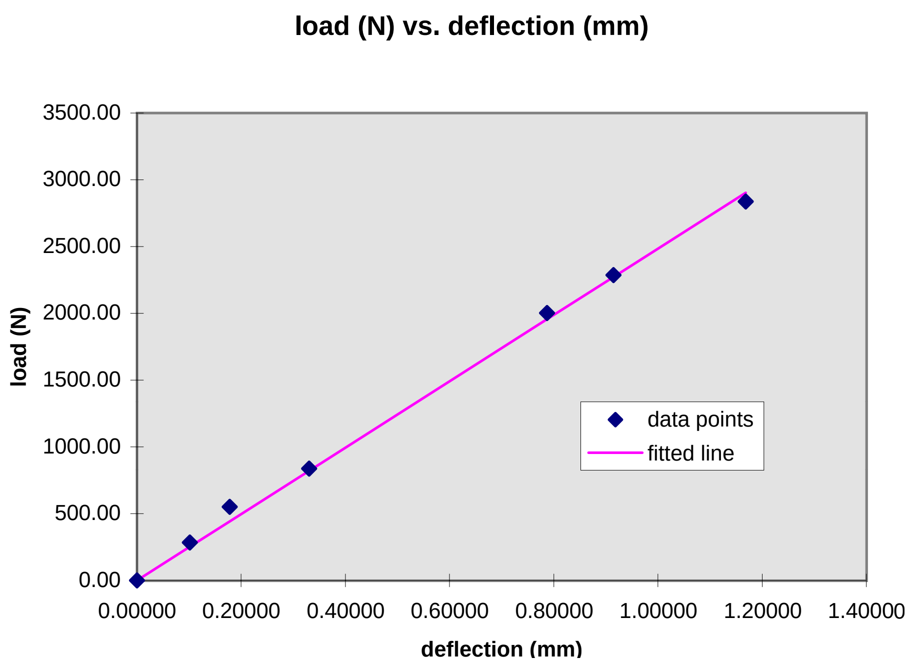

Figure A.2 Load Test, Torsional Stiffness

Chapter 9

ARROW'S THEOREM AND ENGINEERING DESIGN DECISION MAKING

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Abstract This article establishes that Arrow's General Possibility Theorem has only indirect application to engineering design. Arrow's Theorem states that there can be no consistent, equitable method for social choice. Many engineering design decisions are based on the aggregation of preferences. The foundation of many engineering decision methods is the explicit comparison of degrees of preference, a comparison that is not available in the social choice problem. This explicit comparison of preference levels is coupled with the choice of an aggregation method, and some forms of aggregation may be inadequate or inappropriate in engineering design.

Keywords: Design decision-making; Engineering design; Multi-criteria analysis

1. Introduction

Important tasks in engineering design are to generate and refine design alternatives, and then to select a single design, or a set of designs, to fulfill a particular need. In informal terms, the latter problem may be stated, "Find the best alternative(s)." Sometimes the determination of "best" is relatively straightforward and unambiguous, as in: "Find the lightest alternative," or, "Find the stiffest alternative." Sometimes it is less so, as in: "Find the lightest, stiffest alternative." This simple directive is inadequate to choose between one alternative that is stiff but heavy, and another alternative that is light but compliant.

This engineering design decision problem is a problem of decision with multiple criteria, and can be stated as follows:

Given several performance criteria which are to be simultaneously optimized, determine a method for comparing any two design alternatives that depends only the values of the individual criteria for each alternative.

The multiple criteria engineering design decision problem has been addressed by various decision-making systems, such as Quality Function Deployment (QFD) [6], the Analytic Hierarchy Process (AHP) [13], Pugh charts [12], and multi-criteria optimization [11], all of which help guide designers in choosing designs to meet a global performance criterion based on the aggregation of performances on distinct criteria. It is generally assumed that evaluation on the basis of any of the individual criteria is straightforward. In practice, the calculation of the individual criteria may require considerable engineering judgement or resources. It is also assumed that any two alternatives could be compared directly, without resorting to individual criteria. In practice, engineers working on the designs do not make these direct comparisons, but instead work to meet performance specifications. The expertise to assess individual performance criteria is often distributed throughout an engineering enterprise. A method to compare alternatives based on separate criteria is an efficient and feasible alternative to direct comparison.

In addition to the engineering decision methods listed above, there is an extensive literature in decision theory on the topic of multiple criteria decision-making by an individual decision maker (see, for instance, [10], [4], [8]). The idealized decision maker of decision theory need not be an individual person. Rather, it is assumed that there is a single entity that renders decisions; the inner workings of that entity may safely be ignored.

The problem of decision with multiple criteria is similar to the problem of social choice, or group decision-making, in which the rankings of several alternatives by individuals are to be combined into a single, "social," ranking (*e.g.*, selecting candidates in a multi-party election). In social choice, of course, there is no longer a single decision maker, and the goal is to arrive at rational decisions that respect the sovereignty of the individual citizens involved in the decision. In the theory of group decision-making, a well-known objection to the validity of combining separate weak orders into a single social order is Kenneth J. Arrow's General Possibility Theorem [1, 2].

Various engineering design decision methods, such as those mentioned above, depend on the aggregation of several weak orders into a single order. Furthermore, it is common for many individuals to participate in an engineering design decision. Recent discussions in the design research community have raised the question of the applicability of Arrow's Theorem to decision-making in engineering design [7]. The relevance of Arrow's Theorem to the engineering design decision problem depends on whether making a decision in engineering design is sufficiently similar to a social choice problem, or to a decision with a single decision maker but with multiple objectives.

This paper shows how Arrow's Theorem does not apply to the multi-criteria engineering decision problem. It further shows that, despite the common participation of many individuals in engineering design decisions, engineering design is closer to multiple criteria decision-making than it is to social choice. Thus, engineering design decision-making occupies a middle ground between decision with an idealized decision maker and decision by groups of fully autonomous citizens, and on this middle ground Arrow's Theorem has no detrimental consequences.

Also related to multiple criteria analysis is decision under uncertainty, in which the problem is to determine the best alternative when the consequences of each alternative are probability distributions over possible outcomes (*e.g.*, investment decisions). Both multiple criteria analysis and decision under uncertainty usually overlay the weak ordering of alternatives with a numerical scale.

1.1 Preliminaries: Notation and Definitions

Design alternatives are distinguished by uppercase Roman characters: A, B, C, $\ldots$

Design criteria are denoted by lowercase italics: x, y, z, $\ldots$

Individuals or voters are denoted by Arabic numerals: 1, 2, 3, $\ldots$

The most basic concept in the ranking of alternatives is simple comparison, in which there is no association of numbers with alternatives, but only the notion that one alternative A is preferred to another alternative B. A ranking that depends only upon simple comparison is called a *weak order*:

Definition 1 *A* weak order *on a set of alternatives* $\vec{X} = \{\mathrm{A}, \mathrm{B}, \mathrm{C}, \ldots\}$ *is a transitive binary relation* $\succeq$ *such that for any two elements* A *and* B*, either* $\mathrm{A} \succeq \mathrm{B}$ *(*A *is at least as preferable as* B*), or* $\mathrm{B} \succeq \mathrm{A}$ *(*B *is at least as preferable as* A*). Indifference is possible: if* $\mathrm{A} \succeq \mathrm{B}$ *and* $\mathrm{B} \succeq \mathrm{A}$*, then one writes* $\mathrm{A} \sim \mathrm{B}$ *(*A *is indifferent to* B*). If* $\mathrm{A} \succeq \mathrm{B}$ *but* $\mathrm{B} \not\succeq \mathrm{A}$*, then* A *is (strictly) preferred to* B*, written* $\mathrm{A} \succ \mathrm{B}$.

A weak order is an ordinal ranking: it orders the alternatives without assigning numerical values. Any computational method for decision-making requires the further structure of a numerical scale that ranks alternatives. Such a numerical scale is called a *value function*. The familiar $>$ and $\geq$ on the real numbers of the value function correspond to the preference relations $\succ$ and $\succeq$ among alternatives:

Definition 2 *A* value function *is an assignment of real numbers to alternatives that preserves a weak order of acceptability of those alternatives. A value* *function maps a set together with a weak order* $\{\vec{X}, \succeq\}$ *to the real numbers with its usual ordering* $\{\mathbb{R}, \geq\}$. *For a value function* v, $v(\mathrm{A}) \geq v(\mathrm{B})$ *iff.* $\mathrm{A} \succeq \mathrm{B}$, *with equality for indifference.*

While it is always possible to construct a value function from a weak order (see [9]), the mere assignment of a value function does not imply a measure of *degree* of acceptability. A value function is of greater use in a computational decision method if the numerical scale can be used to compare levels of acceptability. The weak order $\mathrm{A} \succeq \mathrm{B} \succeq \mathrm{C}$ is captured both by v_1, where $v_1(\mathrm{A}) = 10$, $v_1(\mathrm{B}) = 9$, and $v_1(\mathrm{C}) = 1$, and by v_2, where $v_2(\mathrm{A}) = 10$, $v_2(\mathrm{B}) = 2$, and $v_2(\mathrm{C}) = 1$, but the relative preference for A, B, and C implied by the two value functions is quite different. It shall be shown below that the correct specification of the numerical scale is crucial to the satisfactory resolution of both the multi-criteria decision problem and the problem of decision under uncertainty.

2. Arrow's Impossibility Theorem and its implications for the aggregation of preference

Kenneth J. Arrow's General Possibility Theorem, now commonly known as Arrow's Impossibility Theorem or simply Arrow's Theorem, is an important and powerful result in the theory of social choice. For that reason, and because a thorough understanding of Arrow's Theorem will facilitate a comparison between the social choice and multi-criteria decision problems, the Impossibility Theorem will be presented here. The treatment refers mainly to [1] and [2].

2.1 The motivating paradox

The context of Arrow's work is politico-economic. Political scientists are interested in determining a "fair" method of reconciling the potentially conflicting interests of individuals in a society. Economists seek the most "satisfactory" distribution of a set of commodities throughout a society. The similarities between the two problems are evident, and indeed both can be formalized in the same way; the notions of "fair" and "satisfactory" are explored through this formalization.

The *majority method of decision-making* is one possible answer to the loosely formulated question of fair social choice, and one that is sufficiently obvious that a contradiction that arises from its employment motivates the Impossibility Theorem. For an odd number of people and two options to choose among, a simple vote is guaranteed to satisfy the most people. But when there are three alternatives, a paradox arises: say that there are three voters, one who orders the options $\mathrm{A} \succeq \mathrm{B} \succeq \mathrm{C}$, another who holds $\mathrm{B} \succeq \mathrm{C} \succeq \mathrm{A}$, and a third who holds $\mathrm{C} \succeq \mathrm{A} \succeq \mathrm{B}$. These preferences are shown in Table 9.1. All three

Table 9.1 Weak orders of three voters

	1st	2nd	3rd
Voter 1	A	B	C
Voter 2	B	C	A
Voter 3	C	A	B

voters have rationally ordered preferences, but a pairwise vote shows that as a group, these three prefer A to B, and prefer B to C, yet also prefer C to A. The resulting social order is not rational, and provides no basis on which to make a decision. This paradox is called the *failure to ensure the transitivity of the majority method*, or the *paradox of voting* [1, p. 2]. It is in the context of this paradox that economists and political scientists explore the limits of "fair" and "satisfactory" social choice: is there any procedure for aggregating social preferences that can avoid this paradox? Arrow's Impossibility Theorem shows that, given a particular set of axioms that define fair and satisfactory, there is no procedure that can (always) fulfill them all. The formal proof proceeds from the description of the problem with a set of axioms.

2.2 Axioms for the social choice problem

By introducing axioms to define any decision problem, two ends are accomplished. Primarily, the problem is modelled so that conclusions about the problem can be derived mathematically. Results are certain with respect to the axiomatic model; their certainty with respect to real problems depends on the validity of the axioms. Additionally, axioms cast such vague descriptions as "fair" and "satisfactory" in precise terms.

The axioms which model the social choice problem make a formal statement of requirements for any procedure that purports to solve the social choice problem, in particular two high-level requirements: consistency of the result, or rationality, and autonomy, or sovereignty, of the individual voters whose preferences are to be combined.

The social choice problem considers decision cases where all options are known, mutually exclusive, and ordered by individuals, and where the task is to produce a single social order yielding the greatest overall benefit while respecting the equal worth of each individual (as with the idealized decision maker, an "individual" is a single decision-making entity, not necessarily an individual person). Note that in the social choice context, each weak order corresponds to the wishes of an autonomous individual; in multiple criteria decision, each order corresponds to a single criterion. In engineering design, there may be many people involved, but decisions still depend upon the aggregation of engineering *criteria*. To formalize a desirable decision situation for social choice, Arrow introduces five axioms [1, pp. 24-30]:

Axiom 1 (unrestricted domain) *Each individual is free to order the alternatives in any way.*

Restricting Axiom 1 is one way to address the paradox, and methods that guarantee the transitivity of the majority method can be ranked by how severely they restrict this freedom. It is not at all obvious that Axiom 1 is reasonable for design decisions, as will be discussed below.

Axiom 2 (positive response) *If a set of orders ranks* A *before* B *and a second set of orders is identical except that individuals who ranked* B *before* A *are entitled to switch, then* A *is before* B *in the second set of orders.*

Axiom 2 is an ordinal version of monotonicity, and is a consistency or rationality requirement.

Axiom 3 (independence of irrelevant alternatives) *If* A *is before* B *in a social order, then* A *is still before* B *if a third alternative* C *is ignored or added.*

Note that Axiom 3 is violated in the motivating paradox, where the relative rankings of A and B are influenced by the addition of the alternative C. Thus, this axiom implicitly enforces the transitivity of the social order.

Axiom 4 (not imposed) *An order is called* imposed *if some* A *is before some* B *in all possible social orders. The social choice problem must not be imposed.*

As Axiom 1 states that individuals can hold any preferences they like, Axiom 4 says that they have a reasonable expectation that their preferences are not excluded from being chosen as the social order. This is a fairness requirement.

Axiom 5 (not dictatorial) *An order is called* dictatorial *if there is one individual whose decisions dictate the social order. This is likewise not allowed.*

Perhaps the simplest way to guarantee consistency of results is to violate Axiom 5: by choosing an individual whose preferences exactly determine the social order, the other four axioms are trivially satisfied. Such a solution is intuitively unfair in a social choice context, and is not allowed. While there is no need in engineering to respect all attributes equally, it is wasteful at best to evaluate attributes that have no impact on the decision. Dictatorship by one evaluation *criterion* is not a rational solution for engineering design. Some engineering cultures may appear to have a dictator in the form of a single decision maker, perhaps a manager with ultimate responsibility for all decisions; however, decisions will still be made by considering several criteria. Rather than violating Axiom 5, this changes the problem to decision with multiple criteria and a single decision maker.

In addition to these five axioms, there are implicit assumptions. Some are merely technical: there are at least three alternatives, and there is an odd and finite number of alternatives. One assumption, however, is substantive and crucial. The social order must be chosen based only on the weak orders supplied by the individuals; it is not permitted to ask for any further information to determine strength of preference, as any comparison between individuals is held to be meaningless.

2.3 The resulting contradiction

Arrow's Theorem shows that a social choice function satisfying all five conditions is an impossibility:

Theorem 1 *Any social choice function satisfying Axioms 1–3 must be either imposed or dictatorial.*

The reader is referred to [1, pp. 51-59] or [2, pp. 20-21] for details of the proof, but the basic line of reasoning is as follows. A *decisive* set of individuals for A over B is a set who guarantee that A will be preferred to B whenever they unanimously agree so; any decisive set must contain a smaller decisive set; there is always a decisive set; any set that is decisive for A over B is decisive for A over anything else and for anything else over B, and thus for all A over all B; thus there must be a dictator. The only way to avoid this dictatorship is to impose some preferences, violating Axiom 4.

Thus, the paradox of the intransitivity of the majority method is a manifestation of a difficulty so deeply embedded in the social choice problem that it cannot be resolved without compromising the defining axioms. It is an intuition-building exercise to take the three orders shown in Table 9.1, and attempt to combine them using a general procedure. It quickly becomes clear that transitivity can be insured by dictatorship. Arrow's Theorem proves essentially that transitivity can only be insured by dictatorship.

2.4 Ways around the contradiction

Arrow's Theorem shows that there is no method of aggregating social choice that is guaranteed to satisfy all five axioms. Are all socio-political systems then fundamentally irrational? Or are there systems that, implicitly or explicitly, operate with restricted versions of one or more of the axioms, and thus avoid gross inconsistency?

Arrow and others have attempted to resolve the paradox by weakening the first condition, arguing that in real political, economic, and even moral[1] systems participants tacitly agree to structure their choices in a "logical" way, *i.e.*,

[1]Arrow goes so far as to quote Kant [1, pp. 81–82]

in a way that keeps contradictions from arising. Thus Arrow introduces the notion of *single-peakedness* as a way around the dilemma. A single-peaked set of alternatives is ordered on some (one-dimensional) external scale, so that each individual is free to choose a favorite alternative, but then must hold descending regard for the other alternatives to the two sides of his first choice. The example of a political spectrum is given: each voter has a preferred, or ideal party, and each step away from the ideal party, whether to the left or to the right, is an ever less desirable alternative. This condition says nothing about comparison between parties to the left and parties to the right of ideal. If a condition of single-peakedness is substituted for the axiom of unrestricted domain, then the Impossibility Theorem no longer holds. An abstracted parliamentary system thus avoids the difficulties of Arrow's Theorem. Of course, in a two-party system there is no contradiction, as the two-alternative situation is not paradoxical.

In general, the difficulty of the Impossibility Theorem can be overcome by restricting the freedom of individuals participating in the process by structuring their preferences in some way. Ranking all alternatives on an external scale as discussed above is one form of structure; allowing limited veto power is another.

2.5 Numerical scales, or comparing strength of feeling

Numerical scales for comparison of different attributes are key to the resolution of the multi-criteria decision problem. Their introduction into the social choice problem does not obviate the problems raised by Arrow's Theorem, but a discussion of why they do not solve the social choice problem will facilitate a later comparison with the multi-criteria decision problem. Consider the example of the motivating paradox discussed above, with the individual voters' weak orders from Table 9.1. A majority pairwise vote to combine these three weak orders leads to an intransitive, and thus untenable answer. Consider adding a numerical measure of strength of feeling to the problem, by supposing that each voter is given 10 points (or votes) to distribute among the three alternatives. A representative combination with a weighted sum is shown here:

Table 2: Three voter's preferences

	A	B	C
Voter 1	6	3	1
Voter 2	1	7	2
Voter 3	2	0	8
Total	9	10	11

In this case, there is no ambiguity: B is clearly preferred to A, and C is clearly preferred to both A and B.

This solution, however, is accidental. If Voter 3, for instance, holds the slightly different preferences shown here, preferences which are still consistent with the weak order in Table 9.1, there is no longer a clear choice between the three alternatives:

Table 3: Three voter's alternate preferences

	A	B	C
Voter 1	6	3	1
Voter 2	1	7	2
Voter 3	3	0	7
Total	10	10	10

Indeed, any possible ordering of A, B, and C, including indifferences, can be achieved with numerical preferences consistent with the weak orders in Table 9.1.

It is unclear whether the procedure of allotting ten votes to each individual violates Axiom 3, the axiom of independence of irrelevant alternatives. Say one of the three options is ignored. Then the comparison between the remaining two is the same if the vote distributions remain identical. If, on the other hand, the full ten votes must now be distributed over only two alternatives, then the final order can change. This procedure does not address the difficulty raised by Arrow's Theorem, where the *only* information allowed in the formation of the social order is the set of individual orders. Also, since any possible social order may result from this method of combination, every individual has a strong incentive to assign all ten points to their most preferred option, thus returning the problem to the precise statement of Arrow's Theorem.

Nor is the difficulty overcome by using different arithmetic (by letting each individual rank each alternative on a scale of 1 to 10, say, or by aggregating with something other than a weighted sum). A numerical scale cannot work in the social choice problem. Because interpersonal comparison is not allowed, the scale can only be assigned arbitrarily. While a particular scale may appear to address an inconsistency for a particular problem, it is merely a matter of arithmetic to recast the problem so that it is directly subject to Arrow's Theorem. The inability of numerical scales to address the social choice problem is discussed further in the Appendix.

3. Decision with multiple criteria

The engineering design decision problem is not a social choice problem, but instead is a decision with multiple criteria, that is: rank a number of alternatives, each of which is ranked separately by several ranking criteria. Although this problem appears superficially similar to the social choice problem, since it seeks to combine several individual rankings into one, it is a distinct problem. Two differences are:

- In the social choice problem, all orderings are accorded equal worth. In the multiple criteria problem, it is desirable to be able to assign importance weightings to criteria. While it is natural to accord all human voters equal worth, there is no obvious reason to require equal weighting of the different engineering criteria that describe a device or system.

  This difference may disappear if the weighted problem can be recast as an unweighted problem with more individuals.

- The social choice problem permits no interpersonal comparison of preferences, and is thus limited to the discussion of weak orders. The heart of the multi-criteria engineering problem is the inter-attribute comparison of preferences. When considering many design goals, it is crucial to understand their relative importance and the way in which they interact. Again, what is natural to require when modelling the sovereignty of individual citizens is not necessarily applicable to separate engineering design criteria.

  This difference makes decision with multiple criteria structurally different from social choice, and has deep implications for the applicability of the Impossibility Theorem to engineering design decision-making.

Even the informal motivating paradox for the Impossibility Theorem (where a majority vote ranked A before B before C before A) loses much of its power if cast in the framework of multi-criteria decision-making. Consider the analogous example of a design or a product that is to be judged on the basis of three criteria: x, y, and z. It is certainly plausible to assume that the designer may be faced with a choice of three candidate designs, A, B, and C, such that A is better than B is better than C with respect to criterion x, B is better than C is better than A with respect to y, and C is better than A is better than B with respect to z. The analogous "paradox" here is that giving x, y, and z one vote each as a method to determine the best design yields no obvious answer. In other words, if all that is known about a design is a weak order among alternatives for each of the three criteria x, y, and z, then there is not enough information to decide upon an overall best design. This "paradox" is resolved in the multi-criteria problem by more careful consideration of preferences for x, y, and z, and consideration of how those preferences interact. Note that even if this decision is made by a group of three individuals, each responsible for one criterion, there is still no paradox; the involvement of more than one person does not by itself make group decision-making.

A crucial assumption was made at the outset about the engineering design decision problem. In general, in a real design situation, there *is* a rational weak order among A, B, and C.[2] Furthermore, that order could in principle be found directly. The "paradox" is merely that additional information beyond the weak orders on x, y, and z is required to recognize the overall order. The question asked here is whether and when it is possible to find consistent, rational techniques to discover the ranking among A, B, and C. If so, it will necessarily be with a slightly different set of assumptions from the ones of the Impossibility Theorem, assumptions more appropriate for engineering decision-making than for social choice.

3.1 Comparison of axioms for the two decision problems

A careful examination of the axioms is necessary before considering the Impossibility Theorem in the context of the design decision problem. When combining engineering criteria (the multi-criteria problem), rather than individual orderings (the social choice problem), the axioms of positive response, independence of irrelevant alternatives, and inadmissibility of dictatorial solutions still appear to hold. However, it is not obvious that domains must be unrestricted or that orders must not be imposed.

The social choice problem must respect individuals by affording them the freedom to order alternatives as they choose; in a design situation, cultural, customer, or managerial structure is almost always imposed on the orders. For instance, if the three candidate vehicle structure designs A, B, and C have bending stiffnesses of 3000, 3200, and 3400 N/mm respectively, Axiom 1 states that any individual is free to express the preference $\mathrm{C} \succ \mathrm{A} \succ \mathrm{B}$. A vehicle structures group, however, which proposed this order to management, would be criticized for "irrational" preferences *over bending stiffness*. The order that ranks $\mathrm{C} \succ \mathrm{A} \succ \mathrm{B}$ is transitive, and any transitive order must be considered rational in the social choice problem; it would be an acceptable final order of candidate designs. With respect to the particular evaluation criterion of bending stiffness, however, it is not rational; many such transitive orders would be considered irrational in an engineering context. No individual is given veto power in the social choice context; almost any attribute of an engineered design has a level so unacceptable as to veto the entire design.

Recall that Arrow proposed *single-peaked* preferences as a way to resolve the contradictions of the Impossibility Theorem: preferences are single-peaked

[2] This does not mean that there is one optimal solution that fits all situations; a design situation includes the attitudes and preferences of the people and corporate entities involved. Given the need for personal transportation, some auto manufacturers choose to make luxury cars, while others choose to make economy cars.

when all options are ordered on an external scale, and each individual has one preferred option and holds descending regard for alternatives to the two sides of that preferred option. Engineering variables are almost always ordered on an external scale, and preferences for engineering requirements are commonly single-peaked around an ideal target. Indeed, nearly all engineering requirements are of one of three forms: less is better, more is better, or closer to a particular target is better [3]. All three of these forms are single-peaked.

There are (rare) evaluation criteria that do not behave in a single-peaked manner. A design preference for availability of a particular material stock may be one criterion for a design, and it may change over time and take on any order. The preference for the frequency of the first acoustic mode is often to *avoid* a particular unpleasant range. However, designers can and do restrict criteria that are not globally single-peaked to regions of local single-peakedness. The vehicle structure designer seeking to avoid a particular range of frequencies of the first acoustic mode, for example, chooses to target either higher or lower frequencies, thus considering only a range over which the criterion is single-peaked. Thus, while the completely generic design decision problem should obey Arrow's axiom of unrestricted domain, designers strive to avoid the generic problem, and rather to cast each problem so that domains, rather than being unrestricted, are single-peaked along the obvious external scales provided by the design parameterization. Indeed, in terms of the decision problem, the parameterization of a design serves to restrict domains. For the multi-criteria decision problem, the axiom of unrestricted domain is replaced by an exhortation to the designer to verify that criteria are single-peaked, or restrict the problem until they are. It is understood that this may not always be possible, but when it is not, the designer realizes that the design problem is not completely well-conditioned. Such problems are difficult for formal methods and informal methods alike.

The axioms of the multi-criteria decision problem are thus not identical to those of the social choice problem. Axiom 1 is crucial to the social choice problem, while in the engineering design problem, it appears in a modified form that is known to overcome Arrow's Theorem. This difference in Axiom 1 allows multi-criteria methods to operate on some large classes of problems without violating Arrow's Theorem, even without the use of a numerical scale to compare preferences across attributes. However, the differences in axiomatic structure are minor compared to the difference in one fundamental assumption: the social choice problem does not admit interpersonal comparison, while the multi-criteria decision problem would be meaningless without inter-attribute comparisons. The next section will discuss how a numerical scale for the inter-attribute comparison of preference is used in two contexts: the aforementioned multi-criteria decision problem, and the related problem of decision-making under uncertainty.

There is a further, practical, difference between social choice and engineering design: designs have constraints. A maximum stress indicates the point at which a design breaks and fails; government regulations must be fulfilled or a design is not allowed on the market. However, it is usually far from obvious, *a priori*, which candidate designs violate constraints, and the engineer must be free to consider all potentially viable candidates. In decision theory, the decision maker chooses among a neatly defined set of viable alternatives; in engineering design, deciding which are the viable alternatives is a major task, often involving considerably more resources than the final decision. Furthermore, the early consideration of infeasible designs is often a crucial part of the refinement process.

A decision method that captures this special feature of engineering design would, in principle, violate Axiom 4. The positions of alternatives that fail on the basis of a single criterion could be seen as imposed (to be last) in any aggregated order. Constraints in engineering design, if translated into social choice terms, are a sort of veto that individual criteria may exercise over the entire decision, and thus violate the axiom of no imposed orders. However, when the design is acceptable on the basis of each individual criterion, there is no reason to abandon the axiom of no imposed orders. For the multi-criteria engineering decision problem, the axiom of no imposed orders is weakened:

Axiom 4.4a (limited imposed orders) *Axiom 4 holds, with the exception that some alternatives may be declared unacceptable, and thus last in any combined order, on the basis of an unacceptable ranking on a single criterion. All unacceptable alternatives are equally unacceptable.*

This modification of Axiom 4 is not a significant theoretical objection to the application of Arrow's Theorem in engineering design, as the same end can be achieved by simply removing alternatives, after analysis, from consideration. It does, however, provide a framework for decision methods for engineering design that are capable of considering alternatives that may turn out later to be infeasible.

4. Inter-attribute comparison of preference

Decision with multiple criteria differs empirically from social choice in an important way. In the former, there is always a well-defined aggregated order among alternatives, which is available to anyone with the time and resources to query a decision maker directly about all possible combinations; in the latter, Arrow's Theorem calls into question the very existence of a well-defined aggregated order. A direct specification of preference in many dimensions in the multi-criteria problem presents no more theoretical difficulties than a direct specification of preferences in one dimension; the practical implementation, however, can present great difficulty.

There is more than one way to assign a value function when a weak order among alternatives is given. In this section, two distinct approaches to the assignment of a value function are discussed: the *utilities* (or *von Neumann–Morgenstern utilities*) [14, 8] that are used for decision-making under uncertainty, and directly specified *preferences* such as those used in many multi-criteria decision systems. Both techniques use inter-attribute comparison to guarantee consistency (and avoid the pitfalls of the Impossibility Theorem), but the emphasis in each is different.

4.1 Utility

The specification of utility depends upon a weak order among alternatives, and on the mathematics of expectation. To determine a utility function, the so-called *lottery question* must have an answer: "Given that A is preferred to B, and B is preferred to C, at which probability p is there indifference between the two choices 'B with probability 1' and 'a lottery that yields A with probability p and C with probability $(1-p)$'?" (Note that the question need not have a *direct* answer; see [15], for instance, for a discussion of the elicitation of von Neumann–Morgenstern utilities with little or no probability information.) Von Neumann and Morgenstern [14] show that, given the assumption that utilities combine with the mathematics of expectation, the numerical utility scale is determined up to an affine transformation.

The assumption of the use of mathematical expectation arises because utility theory is intended to treat questions of decision-making under probabilistic uncertainty, such as those that are germane to gambling. This makes the specification of *relative* utilities with probabilities natural. The lottery method provides an elegant method to determine not only that A is preferred to B and B is preferred to C, but also how great the preference is for A over B relative to the preference for B over C.

However, the development of utility specifically excluded the notion of interpersonal comparison of utility as too difficult to address:

> We do not undertake to fix an absolute zero and an absolute unit of utility. [14, footnote, p. 25]

Utility theory is intended for use in decision-making under uncertainty or risk, rather than as a solution of the multi-criteria decision-making problem. Decision under uncertainty is often required in multi-criteria settings. [8] present conditions under which a decision maker's overall utility is a simple function of individual, independent utility functions. The overall utility function is determined by direct comparison of several alternatives in order to determine appropriate weighting factors for the individual utility scales determined with regard to the separate attributes. Arrow's Theorem presents no difficulty whatsoever to this sort of calculation; even if several people are involved in the decision, there is an idealized single decision maker.

The success of the von Neumann–Morgenstern utility paradigm, and the ease of its application in terms of quantified risk, have led to a situation in which many decision problems are treated as problems of (economic) decision-making under uncertainty. The lottery question seems natural, and so it is assumed that the lottery question is the right way to impose a numerical scale on preferences. Nevertheless, engineering design may *not* be best classified as decision-making under risk and uncertainty. Utility theory is one paradigm for decision-making, appropriate for a particular set of problems, those where the "estimation of expectations for each option" is the most pertinent information. When design reaches the manufacturing stage, and probability distributions over manufacturing variations are usually the most relevant uncertainties, the design decision problem is indeed similar to the problem addressed by utility theory. Earlier in the design process, where uncertainty will be resolved by refinement of a design alternative, rather than by random selection from a perceived distribution among alternatives, a utility model is less appropriate. Direct preference specification, for uncertainty other than quantified risk, will be discussed in the next section.

From the point of view of classical utility theory, the design decision problem examined here is a case of decision-making under certainty. In the classical utility theory view, the construction of the utility function and the choice of a "best" solution with limited computation are uninteresting problems. The construction of the utility function is only support for the subsequent question of which course of action to select in the face of uncertain consequences.

4.2 Direct specification of preference

The engineering design decision problem that is the subject of this paper is not primarily a problem of decision-making under uncertainty. While methods such as those proposed by [8] could be applied to the problem, and the resulting utility function would indeed provide a basis for comparison of any two alternatives, the relative complexity of these methods is not justified in cases when no probabilistic uncertainty is to be incorporated in the problem. This is particularly true as the number of individual performance criteria increases (the number of coefficients to be determined is $2^n - 1$, where n is the number of criteria). Thus methods intended to solve the multi-criteria decision problem as stated at the beginning of this paper, without the incorporation of uncertainty, commonly use direct specification of preference (*e.g.*, QFD [6], AHP [13]). Direct preferences, unlike utilities, are expressed on an absolute scale. For example, a preference of $\mu = 1$ could indicate a completely acceptable value, and $\mu = 0$ a completely unacceptable value. Alternatively, each criterion may be judged on a discrete scale (such as 1–3–9). The individual numerical orderings associated with each criterion are then aggregated into an overall numerical ordering of all alternatives based on all criteria simultaneously. The functions used for aggregation of the orderings vary; a weighted arithmetic mean is a common choice.

It was made clear earlier that Arrow's Theorem does not apply to the engineering design decision problem, whose chief concern is to compare preferences based on multiple attributes. Nevertheless, Arrow's Theorem does bear indirectly on the legitimacy of these methods: if the comparison of preferences is effected by the arbitrary assignment of numbers to alternatives, then those preferences contain no more information than weak orders, as discussed in Section 2.5 and the Appendix. In that case no aggregation method is adequate. A method that delivers an overall preference as a function of individual preferences must explicitly, and justifiably, compare individual preference values.

The individual preference orders may be generated by, or associated with, different people or groups involved in a design. One of the motivations for the engineering design decision problem as described in this paper is the need to assess design alternatives for overall worthiness when the most readily accessible information comes from many independent performance measures, each of which is the responsibility of a different portion of a design team. If these groups or individuals must be treated as autonomous citizens, then there is no fair and rational way to combine their preferences. However, team design is not driven by the need to respect the sovereignty of the individuals involved, but rather by a desire to create superior products.

The team engineering decision problem thus has two aspects: the multiple criteria problem, to which Arrow's Theorem does not apply, and the problem of collaboration, to which Arrow's Theorem would apply if it were necessary to guarantee sovereignty of team members. The mere involvement of groups does not make it group decision making; in engineering, rather than social systems, attributes, not people, must be reconciled. There are certainly difficult and interesting issues in the management of groups in an engineering context, but Arrow's Theorem does not make such management impossible.

5. Conclusions

This paper establishes the legitimacy of constructing aggregated preferences in engineering design, even though such aggregation is not permissible in the social choice problem. It does so by a thorough examination of the context and results of Kenneth J. Arrow's General Possibility Theorem. The design decision problem has structural (axiomatic) peculiarities which make comparisons in the manner of social choice acceptable in engineering design; more fundamentally, however, the aggregation relies on a procedure for comparing the relative importance and interaction of individual preferences that violates an assumption of citizens' sovereignty in the social choice milieu.

Decision with multiple criteria and a single, idealized decision maker is one problem in decision theory; social choice, in which each idealized individual's sovereignty must be guaranteed, is another. Engineering design often falls somewhere in the middle, but even when many people are involved, the group nature of the decision is subordinate to the multi-criteria aspect, as the sovereignty of individuals is not necessary. Interesting parallels are evident between the ideas expressed in this paper and different management styles [5]. A top-down management style would correspond to forcing a single decision maker on the problem, whereas distributed control would correspond to the group decision environment described here; in either case, the decision problem is framed in terms of engineering requirements rather than individuals' desires.

The aggregation of preference in engineering design is often discussed as the assessment of a utility function, though utility is a particular form of preference assessment that is useful in decision under uncertainty and risk. The multi-criteria decision problem discussed in this paper does not include the incorporation of probabilistic uncertainty. The specification of a multi-attribute utility function for decision under uncertainty, and the use of direct specification of preferences for the resolution of the multi-criteria decision problem, both avoid the problems raised by Arrow's Theorem, but their emphases are different.

While Arrow's Theorem does not prove that the engineering design decision problem treated here cannot have a rational answer, it does offer a caution to methods that attempt to solve the multi-criteria decision problem: comparison of preferences must be made explicit. The arbitrary assignment of numbers to alternatives can lead to undesired conclusions. Similarly, the arbitrary assignment of an aggregation method can lead to undesired conclusions. A decision method must have an explicit procedure for assigning values to alternatives, and for combining those values into a single performance function, and the two must agree. If a decision method gives inadequate answers, it is not because Arrow's Theorem declares that comparison is impossible; it is because the particular implementation is flawed — the representation and comparison of preferences in the method are insufficiently rich.

Finally, the need in engineering design to consider and evaluate alternatives that may turn out to be infeasible is not well captured by the classic formulation of the decision problem, where only feasible alternatives are considered. This difficulty can be overcome by reformulating the decision problem whenever an infeasible candidate arises. Alternatively, a decision methodology can handle such constraints by a modification of the underlying decision axioms.

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References

[1] ARROW, K. J. *Social Choice and Individual Values*, 1st ed. J. Wiley, New York, NY, 1951.

[2] ARROW, K. J., AND RAYNAUD, H. *Social Choice and Multicriterion Decision-Making*. The MIT Press, Cambridge, MA, 1986.

[3] BYRNE, D. M., AND TAGUCHI, S. The Taguchi approach to parameter design. In *Quality Congress Transaction – Anaheim* (May 1986), ASQC, pp. 168–177.

[4] FRENCH, S. *Decision Theory: An Introduction to the Mathematics of Rationality*. Ellis Horwood, Chichester, 1988.

[5] GOOLD, M., AND CAMPBELL, A. Many best ways to make strategy. *Harvard Business Review 65*, 6 (November-December 1987), 70–76.

[6] HAUSER, J., AND CLAUSING, D. The House of Quality. *Harvard Business Review 66*, 3 (May/June 1988), 63–73.

[7] HAZELRIGG, G. A. The implication of arrow's impossibility theorem on approaches to optimal engineering design. *ASME Journal of Mechanical Design 118*, 2 (jun 1996), 161–164.

[8] KEENEY, R., AND RAIFFA, H. *Decisions with Multiple Objectives: Preferences and Value Tradeoffs*. Cambridge University Press, Cambridge, U.K., 1993.

[9] KRANTZ, D., LUCE, R., SUPPES, P., AND TVERSKY, A. *Foundations of Measurement*, vol. I. Academic Press, New York, NY, 1971.

[10] LUCE, R., AND RAIFFA, H. *Games and Decisions*. J. Wiley and Sons, New York, NY, 1957. Dover Edition, 1985.

[11] PAPALAMBROS, P., AND WILDE, D. *Principles of Optimal Design*. Cambridge University Press, New York, NY, 1988.

[12] PUGH, S. *Total Design*. Addison-Wesley, New York, NY, 1990.

[13] SAATY, T. L. *The Analytic Hierarchy Process: Planning, Priority Setting, Resource Allocation*. McGraw Hill, New York, NY, 1980.

[14] VON NEUMANN, J., AND MORGENSTERN, O. *Theory of Games and Economic Behavior*, 2nd ed. Princeton University Press, Princeton, NJ, 1953.

[15] WAKKER, P., AND DENEFFE, D. Eliciting von Neumann-Morgenstern utilities when probabilities are distorted or unknown. *Management Science 42*, 8 (Aug. 1996), 1131–1150.

A Appendix: Voting

In Section 2.5 the allotment of ten votes to each individual was proposed as a potential way around Arrow's Theorem, and was shown to be inadequate. That point is discussed in more detail here.

Any of the sets of numerical preferences attributed to individual voters in Section 2.5 can be recast as weak orders held by more voters. Combining all the weak orders of the fictitious voters using the majority method of decision-making will then yield the same social order as a weighted sum aggregation of the numerical rankings, with ties in the majority method corresponding to equal overall numerical rankings. To express 10 preference points requires 20 individual weak orders: for example, if alternative A received 1 of the 10 possible points, that is expressed by one weak order $\mathrm{A} \succ \mathrm{B} \succ \mathrm{C}$, and one $\mathrm{A} \succ \mathrm{C} \succ \mathrm{B}$. (Two orders are required so that B and C are indifferent.) Voter 2's numerical preferences, for instance, are equivalent to the following 20 weak orders:

1st	2nd	3rd	instances of this order
A	B	C	1
A	C	B	1
B	C	A	7
B	A	C	7
C	A	B	2
C	B	A	2

The entire weighted sum aggregation is equivalent to the majority method applied to 60 individual voters. However, these 60 individual orders do not obey Axiom 1, as they are actually 30 dependent pairs. This clearly demonstrates that the assignment of ten votes to each individual imposes restrictions that are not in the original problem, which by definition must obey all the axioms of Arrow's Theorem. Indeed, this numerical method can be seen as an attempted end run around Arrow's Theorem, by the means of eliminating Axiom 1. Thus, Arrow's Theorem still applies: the use of a numerical method does not by itself guarantee both fairness and consistency.

Chapter 10

USING INDIFFERENCE POINTS IN ENGINEERING DECISIONS

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Abstract Multi-criteria decision support methods are common in engineering design. These methods typically rely on the specification of importance weights to accomplish trade-offs among competing objectives. Such methods can have difficulties, however: they may not be able to select all possible Pareto optima, and the direct specification of importance weights can be arbitrary and *ad hoc*. The inability to reach all Pareto optima is shown to be surmountable by the consideration of *trade-off strategy* as an additional parameter of a decision. The use of *indifference points* to select a best solution, as an alternative to direct specification of importance weights, is presented, and a simple truss design example is used to illustrate the concepts.

Keywords: multicriteria analysis, engineering design, design decision-making, aggregation functions, trade-offs, strategies

1. Introduction

Multi-criteria decision making is an important part of design. There are many methods, both informal and formal, that support such design decision making, such as Pugh charts [5], QFD [3], and the Analytic Hierarchy Process, or AHP [6]. These design decision methods share several key features. All rely on the aggregation of preferences to choose among designs, and most methods allow for the assignment of importance to individual attributes through the use of weights. These importance weights are meant to allow for meaningful comparison of many options when two or more attributes must be traded-off against each other. Among decision methods, weighted-sum aggregation of preferences is common, as is direct specification of importance weights.

Multi-criteria decision methods are related to multi-criteria optimization and the calculation of the Pareto frontier. Decision methods can be used to avoid unnecessary computation by optimizing directly to the most desirable configuration without calculating other Pareto points. It is implicit in the use of any decision method that the selection of its parameters, usually weights, enables the selection of the most desirable points.

Decision methods are important for decision support, and are crucial for semi-automated design, yet their underlying decision representations have rarely been examined or justified. Even if standard decision methods worked all the time, a formal investigation of the underlying mathematics of decision would still be warranted. It shall be demonstrated below that weighted-sum methods have serious drawbacks; in fact, any method that relies exclusively on importance weights to define a decision runs the risk of missing "optimal" options. A complete model of a decision requires an additional parameter to specify the level of compensation between criteria [7]. Also, the direct specification of importance weights is an *ad hoc* process, and the answers it produces may not always be reliable. Several relevant results are presented here:

- In addition to importance weights, the level of compensation between attributes is a parameter that defines a decision.
- For decision-making that conforms to the axioms of rational design [4], a parameterized family of functions (with compensation parameter s) was shown to span a complete range of degrees of compensation [8].
- The compensation parameter s increases with the level of compensation, which is demonstrated formally in two different ways. The compensation parameter, together with weights, defines a decision.
- By the use of these functions, a weight/strategy pair to select any Pareto optimal point can always be found.
- The ability to choose any Pareto point is *not* present when degree of compensation is pre-selected (as with the use of a weighted sum).

The concept of *indifference points* as a structured alternative to *ad hoc* specification of parameters is presented below, and its application illustrated by an example. The notion of level of compensation is a less intuitive concept than importance weighting, and a structured method is even more essential when both compensation and importance weights are considered. The relation of these results to multi-criteria optimization will be discussed below.

2. Example: simple truss structure

Consider the structure shown in Figure 10.1. This is a pin-jointed two-member bracket to support a load of one kilogram (1000 g) at a distance of

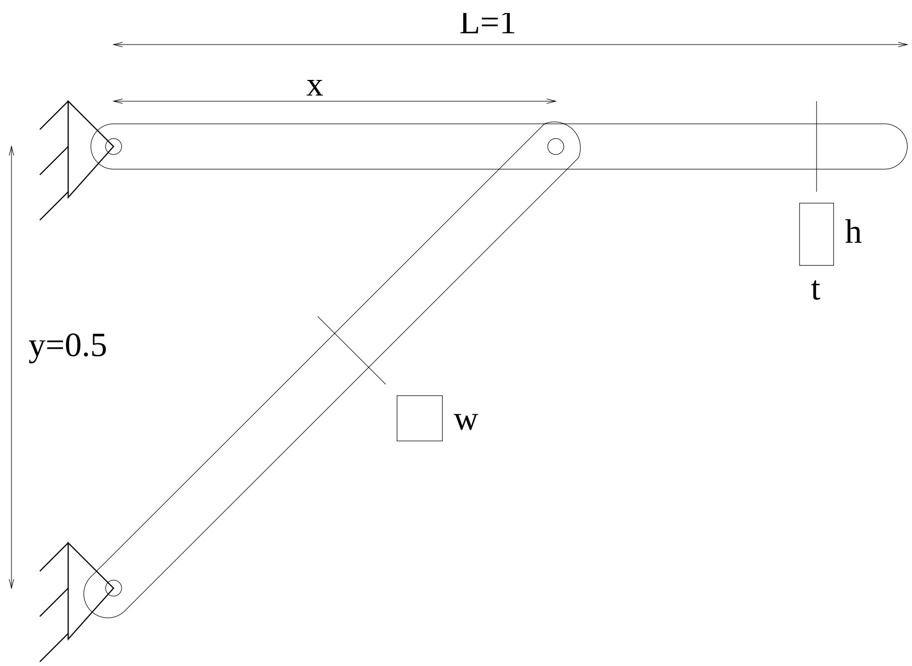

Figure 10.1 Example: a simple truss structure.

1 meter from a wall (L $=$ 1). The positions of the wall mounts are fixed, with the lower support one half meter below the upper support ($y = 0.5$). Both members are made of aluminum (6061-T6). The designer controls four design variables:

$x \in [0.1 \text{ m}, 0.9 \text{ m}]$	distance from wall to pin
$t \in [5 \text{ mm}, 20 \text{ mm}]$	thickness of bending member
$h \in [5 \text{ mm}, 20 \text{ mm}]$	height of bending member
$w \in [5 \text{ mm}, 20 \text{ mm}]$	width of (square) compression member

For this example, the performance measures to consider are the mass (M) of the structure, and the safety factor (S). The example is simple enough that both can be expressed analytically, but let us start by treating the performance calculation as a black box. The details of the calculations are presented in the Appendix. For purposes of this paper, the design problem is to minimize the mass of the structure while maximizing the factor of safety.

Further suppose that no additional advantage is gained from factors of safety above ten. Also, designs with safety factors below one should not be considered. Using optimization or other means, it can be determined that the minimum mass achievable with a factor of safety of one is 123 grams, while the

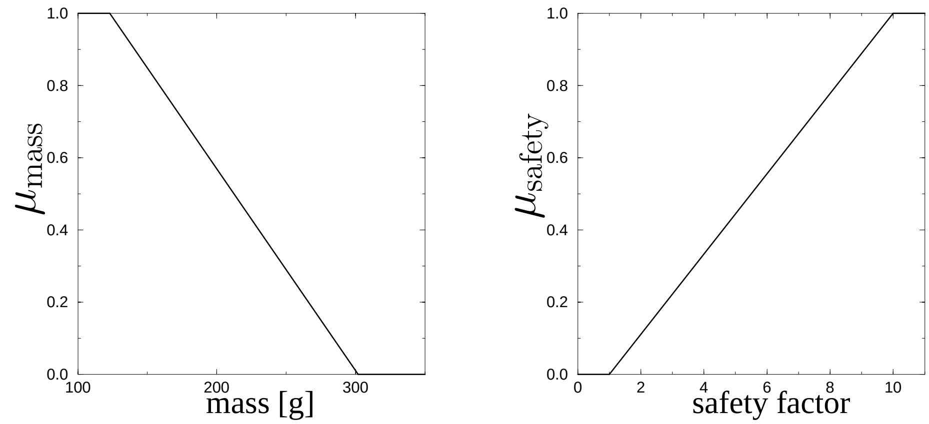

Figure 10.2 Preferences for mass and safety factor

minimum mass achievable with a factor of safety of ten is 302 grams. The best designs will be trade-offs between the safety factor and the mass.

Assuming that both 123 g (the lowest possible mass for the acceptable range of safety factors) and 302 g (the mass corresponding to the highest safety factor) are acceptable, it is common to normalize the performance measures. Here we follow the approach of the Method of Imprecision, or M_oI [9, 7], and specify preferences on the performance measures, which incidentally normalizes the performances to the interval $[0, 1]$. The results presented apply to any normalization scheme, or to no normalization at all. Let the preferences for mass and safety be as follows:

$$\begin{aligned} \mu_{\text{mass}}(\text{M}) &= \frac{302 - \text{M}}{179} \\ \mu_{\text{safety}}(\text{S}) &= \frac{\text{S} - 1}{9} \end{aligned}$$

so that $\mu_{\text{mass}}(123) = 1$, $\mu_{\text{mass}}(302) = 0$, $\mu_{\text{safety}}(1) = 0$, and $\mu_{\text{safety}}(10) = 1$, as shown in Figure 10.2. Note that these simple linear preferences are chosen for convenience; all the results presented here hold for more complicated preferences as well.

Weighted sum

As was discussed above, a common way to select a best design is to assign importance weights to the two criteria, and then use a weighted sum to aggregate preferences; the best designs will have the highest overall preference. Let the importance weights assigned to mass (ω_1) and safety (ω_2) be normalized

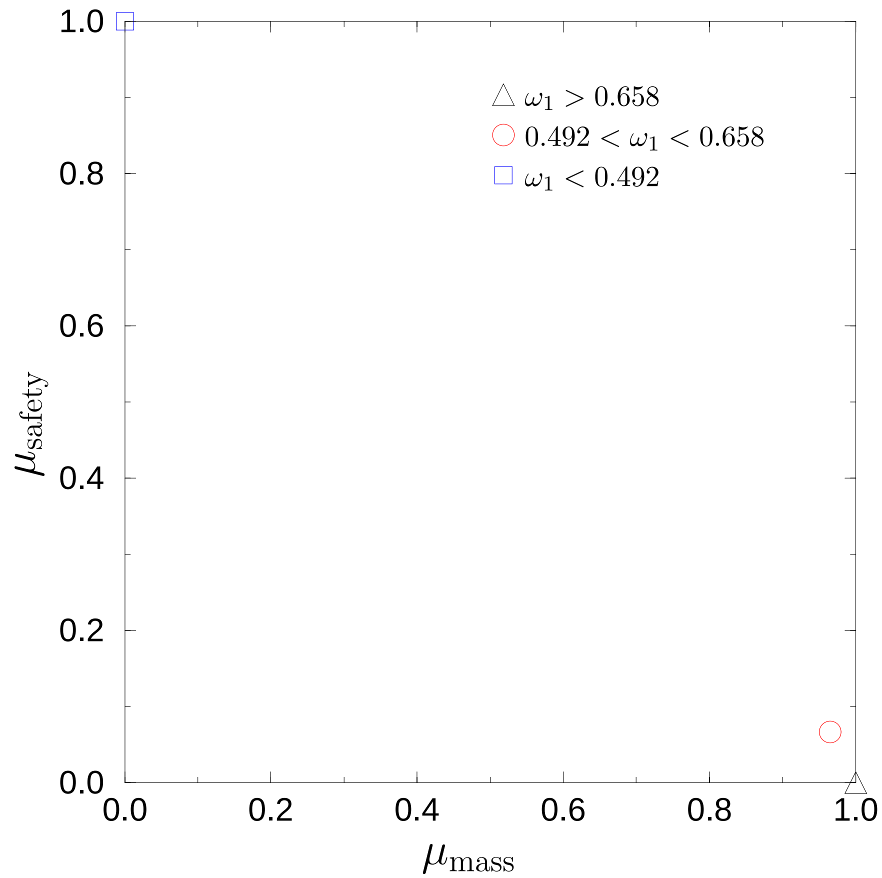

Figure 10.3 Three "best" points found using weighted sum exploration

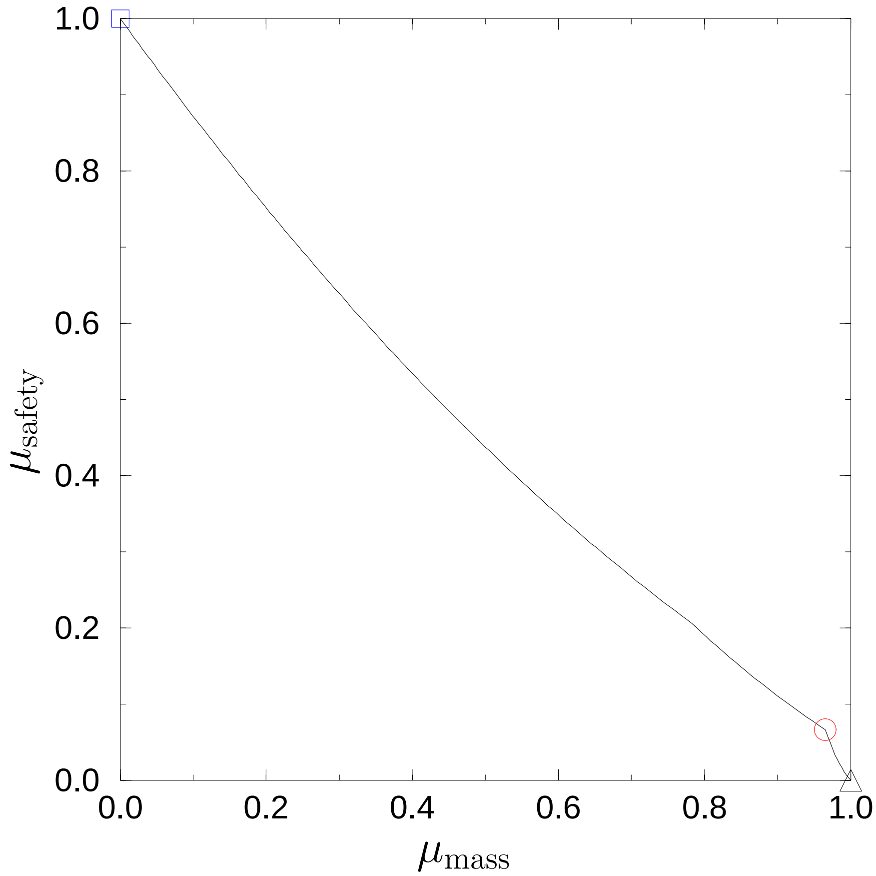

Figure 10.4 Pareto frontier with three "best" points.

so that their sum is one. Employing this approach, there are only three possible "best" points, summarized in the table below:

ω_1	mass [g]	safety factor	x	t	h	w
$\omega_1 > 0.658$	123	1	0.71	5	5	5
$0.492 < \omega_1 < 0.658$	129	1.6	0.82	5	5	5
$\omega_1 < 0.492$	302	10	0.9	5	9.34	8.16

According to this weighted sum aggregation, all other possible points are worse when both mass and safety factor are considered. These points are shown on the graph in Figure 10.3.

The three "best" points shown in Figure 10.3 do not represent the entire range of reasonable trade-offs between the two performance measures mass and safety. To make the notion of a "reasonable" decision more precise, we use the idea of Pareto optimality:

Definition 1 *The alternative* A dominates *the alternative* B *if* A *performs no worse than* B *on all attributes, and better than* B *on at least one attribute. In this case, regardless of the weights or the strategy, it is always better to choose* A *over* B*. A feasible solution is* undominated *(or* Pareto optimal*) if there is no other feasible solution which dominates it.*

Figure 10.4 shows a plot of all the feasible Pareto optimal points, normalized with respect to preference, with the three points from Figure 10.3 retained. (The calculation of the Pareto frontier is detailed in the Appendix). Examining the entire Pareto frontier, we see that it is made up of two concave sections. The weighted sum approach fails to identify any Pareto points on the concave sections of the frontier, though it is quite possible that one of those points represents the most desirable compromise. Some difficulties of the weighted sum have been discussed previously in an optimization context by [2] and [1], among others.

Clearly, a weighted sum approach to multi-criteria decision making is problematic if it cannot identify all possible best solutions. In the example presented here, the performance calculations are all analytic expressions which are easily evaluated, and the Pareto frontier is thus easily discovered. In such simple cases, a designer can choose to informally explore regions of the performance space that the formal decision model does not identify. When the design is more complicated, perhaps because evaluation is more costly (say, each point is a finite element calculation rather than an analytic expression), or because there are more than two competing objectives, informal exploration becomes much more difficult, and designers may rely more on automated techniques such as optimization. In these more complicated design situations, it is particularly important that design decision methods provide reliable guidance. If the preference aggregation is valid, it is not necessary to compute the entire Pareto frontier.

In the example presented above, it is clear that the choice of a point on the Pareto frontier depends on the trade-off between safety and mass. As is suggested by the problems exhibited by the weighted sum, a formal model of a design decision is more complex than a simple matter of choosing importance weights.

3. Compensation strategies: how to consider <u>all</u> designs

In the preceding section it was seen that a weighted sum cannot always identify all Pareto points for a design. This is one instance of a more general result about the aggregation of preference. All existing support methods for multi-criteria decision making ultimately rely on the aggregation of disparate preferences with *aggregation functions*. [4] presented axioms that an aggregation function must obey in order to be appropriate for rational design decision making. [8] showed that the operators that satisfy these axioms are a restricted set of weighted means, and that, in particular, there is a family of aggregation operators $\mathcal{P}_s$ that spans an entire range of possible operators between $\min$ and $\max$, given by:

$$\mathcal{P}_s(\alpha_1, \alpha_2; \omega_1, \omega_2) = \left(\frac{\omega_1 {\alpha_1}^s + \omega_2 {\alpha_2}^s}{\omega_1 + \omega_2} \right)^{\frac{1}{s}}$$

Here, the values α_1, α_2 are individual preference values to be aggregated. The values α_i are the result of applying preferences μ_i to performances x_i: $\alpha_i = \mu_i(x_i)$, or more generally, $\alpha = \mu(\vec{x})$. The parameter s can be interpreted as a measure of the *level of compensation*, or *trade-off*, and is sometimes referred to as the *trade-off strategy*. Higher values of s indicate a greater willingness to allow high preference for one criterion to compensate for lower values of another. The parameters ω_1 and ω_2 are importance weights, both assumed to be positive without loss of generality, and as they may be normalized, the ratio $\omega = \frac{\omega_2}{\omega_1}$ is sufficient to characterize the relative importance of two attributes. The definition is for two attributes, but can be extended to more than two. It is readily shown [8] that

$$\begin{array}{lclcl}
\mathcal{P}_{-\infty} & = & \lim_{s \to -\infty} \mathcal{P}_s & = & \min \\
\mathcal{P}_0 & = & \lim_{s \to 0} \mathcal{P}_s & = & \text{geometric mean } (\alpha_1^{\omega_1} \alpha_2^{\omega_2})^{\frac{1}{\omega_1 + \omega_2}} \\
\mathcal{P}_1 & = & \lim_{s \to 1} \mathcal{P}_s & = & \text{arithmetic mean } \frac{\omega_1 \alpha_1 + \omega_2 \alpha_2}{\omega_1 + \omega_2} \\
\mathcal{P}_{\infty} & = & \lim_{s \to +\infty} \mathcal{P}_s & = & \max
\end{array}$$

Thus the common weighted sum is one instance of this family of design-appropriate aggregation functions, with the compensation parameter s equal to 1.

Several results about this family of aggregation functions can be proven [7], including:

- For any Pareto optimal point in a given set, there is always a choice of a weight ratio ω and a trade-off strategy s that selects that point as the most preferred.

- For any fixed strategy s, there are Pareto sets in which some Pareto points can *never* be selected by any choice of weights ω. This is what occurs in the truss example above, where $s = 1$ cannot select all Pareto points. This is related to the well-known result that non-convex portions of a Pareto surface are unreachable by weighted-sum minimization.

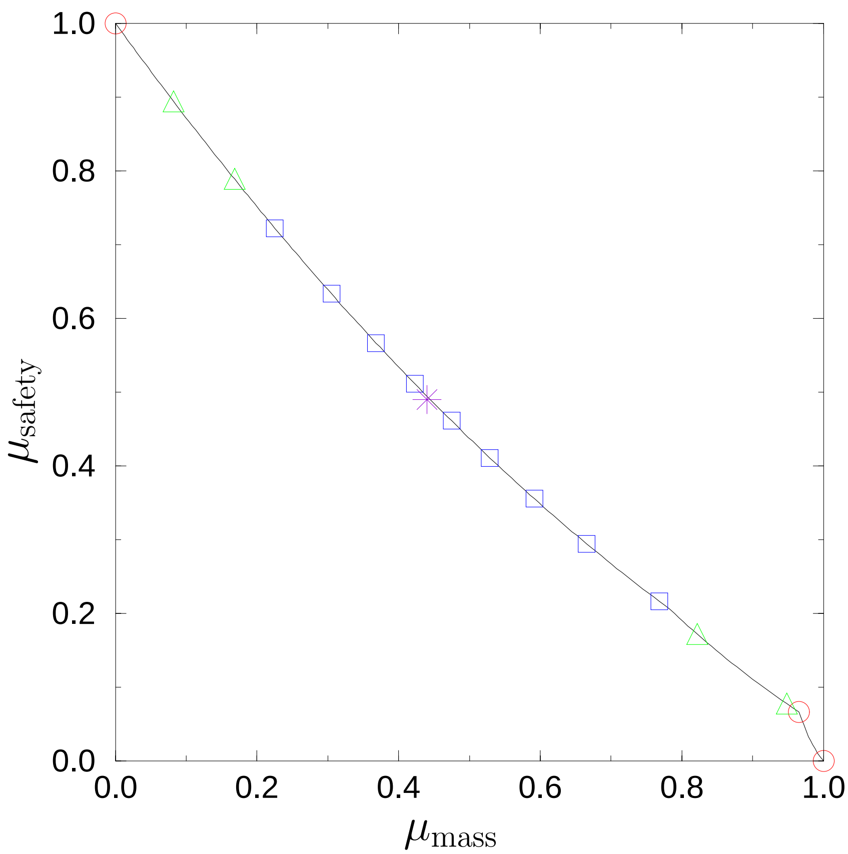

Figure 10.5 Normalized Pareto frontier

The second result does not say that for every strategy s, every Pareto set has some Pareto points that are unreachable. In the truss example, for instance, $s = -1$ can select every Pareto point if the correct weights are chosen. This can be seen in Figure 10.5. Here, the three "optimal" points found earlier by the weighted sum method are circled. Setting $s = -1$ and varying the weights allows for a more varied range of "best" designs:

ω_1	mass [g]	safety	x	h	w
0.1	262	7.5	0.9	8.09	7.59
0.2	248	6.7	0.9	7.65	7.38
0.3	236	6.1	0.9	7.30	7.21
0.4	226	5.6	0.9	6.99	7.06
0.5	217	5.15	0.9	6.70	6.91
0.6	207	4.7	0.9	6.41	6.76
0.7	196	4.2	0.9	6.05	6.57
0.8	183	3.65	0.9	5,64	6.34
0.9	164	2.95	0.9	5.07	6.01

These points are shown as squares on the graph in Figure 10.5.

By allowing the weight assigned to one attribute to be much larger than the weight assigned to the other, points much closer to the extremes of the Pareto frontier can be reached with $s = -1$:

ω_1	mass [g]	safety	x	h	w
0.01	288	9.05	0.9	8.89	7.96
0.05	272	8.1	0.9	8.41	7.74
0.95	155	2.55	0.89	5	5.77
0.99	132	1.7	0.83	5	5.10

These points are shown as triangles on the graph in Figure 10.5.

4. Using indifference points to determine strategies and weights

As was mentioned above, the direct specification of importance weights is an *ad hoc* process. If a designer says that safety is twice as important as mass, how are we to know that aggregation with any strategy will choose the best alternatives? The difficulty is only compounded by the consideration of trade-off strategies. In this section a technique is presented for determining the correct parameter pair of compensation strategy s and weight ratio $\frac{\omega_2}{\omega_1}$ for a particular decision. Rather than direct specification, the technique relies on the use of *indifference points* to establish the appropriate parameters.

Two points are considered *indifferent* if they have the same preference; it is not necessary that the numerical preferences be known. When a single individual has complete decision-making authority, strategies and weights can be considered simultaneously, and their values can be calculated from indifference points. The procedure is as follows:

1. Determine preferences $\alpha_1 = \mu_1(x_i)$ and $\alpha_2 = \mu_2(x_i)$ such that

$$\mathcal{P}_s(\alpha_1, 1; \omega_1, \omega_2) = \mathcal{P}_s(1, \alpha_2; \omega_1, \omega_2) = 0.5$$

(The values for s, ω_1, and ω_2 are to be determined.) In other words, at which value α_1 is there indifference between a design with a preference equal to α_1 for the first performance attribute and a preference values of 1 on the second attribute, and a design that achieves preferences of 0.5 on both attributes (and thus, by idempotency [4], has a combined preference of 0.5)? A similar question is asked for α_2. Sometimes it is easier to ask for values of x_i and calculate α_i; sometimes it is easier to seek α_i directly. Either approach to determining the indifference points is acceptable.

To see how the selection of indifference points might work on the truss example, start with the preference values from Section 2.:

μ	mass [g]	safety
0	302	1
0.5	214	5.5
1	126	10

The reference design is thus the one that yields a mass of 214 g and a safety factor of 5.5. It is important to note that the reference design does not need to be physically realizable. In the truss example, there is no feasible design with preferences $(0.5, 0.5)$. To determine the value of α_1, ask "If we start from the reference design and increase the safety factor to 10, how much can the mass increase so that the new design has the same overall preference as the reference design?" The answer could be, say, that the mass can increase to 260 g; since $\mu_1(260) = 0.29$, $\alpha_1 = 0.29$, indicating that there is indifference between the two points with performances $(x_1, x_2) = (260, 10)$ (preferences $(\alpha_1, \alpha_2) = (0.29, 1)$) and $(214, 5.5)$ (preferences $(0.5, 0.5)$).

The corresponding question is asked for α_2: "Comparing the reference design to one where the mass is 126 g, what is the safety factor that achieves indifference with the reference design (214,5.5)?" If the answer is that a safety factor of 3 ($\mu_2(3) = 0.22$) together with a mass of 214 g is indifferent to a safety factor of 5.5 together with a mass of 126 g, then $\alpha_2 = 0.22$.

If the person providing the indifference points is comfortable thinking in terms of preferences between 0 and 1, then it is not necessary to refer to performance values. Instead the question can be asked directly: "If we start with a reference design where preference for both mass and safety is 0.5, and we increase the preference for mass to 1, how low a preference for safety achieves indifference with the reference design?" This latter form of questioning is particularly useful when several attributes are combined hierarchically and thus groups of attributes must be compared. It is always possible, even in hierarchical aggregation, to specify particular values of all performance attributes in order to specify indifference points.

2. Let $\omega = \frac{\omega_2}{\omega_1}$.
3. If $\alpha_1 = \alpha_2$, then $\omega = 1$:
   (a) If $\alpha_1 = 0.5$, then $s = -\infty$.
   (b) If $\alpha_1 = 0.25$, then $s = 0$.
   (c) If $\alpha_1 > 0.25$, then $s \in (-\infty, 0)$. Solve ${\alpha_1}^s + 1 = 2(0.5)^s$ numerically.
   (d) If $\alpha_1 < 0.25$, then $s \in (0, \infty)$. Solve ${\alpha_1}^s + 1 = 2(0.5)^s$ numerically.
4. If $\alpha_1 \neq \alpha_2$, then $\omega \neq 1$. Note that if $s = 0$:

$$ {\alpha_1}^m = 0.5 = {\alpha_2}^{1-m} \Rightarrow {\alpha_2}^{1-\log_{\alpha_1} 0.5} = 0.5 $$

   Thus:

   (a) If ${\alpha_2}^{1-\log_{\alpha_1} 0.5} = 0.5$, then $s = 0$, and $b = \frac{1-\log_{\alpha_1} 0.5}{\log_{\alpha_1} 0.5}$

   (b) If ${\alpha_2}^{1-\log_{\alpha_1} 0.5} > 0.5$, then $s < 0$.
   If ${\alpha_2}^{1-\log_{\alpha_1} 0.5} < 0.5$, then $s > 0$.
   Solve numerically for s from

$$ \left(\frac{1 + \omega {\alpha_2}^s}{1 + b}\right)^{\frac{1}{s}} = \left(\frac{{\alpha_1}^s + \omega}{1 + \omega}\right)^{\frac{1}{s}} = 0.5 $$

   which reduces to

$$ \left({\alpha_1}^s - 0.5^s\right)\left({\alpha_2}^s - 0.5^s\right) = \left(1 - 0.5^s\right)^2 $$

   Once this is solved numerically for s, then ω can also be determined.

Applying steps 2–4 to the indifference values $\alpha_1 = 0.29$ and $\alpha_2 = 0.22$ determined above, the parameters that determine the aggregation are found to be:

$$ s^* = -0.5 \quad \omega^* = 1.23, \text{ or } (\omega_1, \omega_2) = (0.45, 0.55) $$

The best point on the Pareto frontier for that trade-off is:

mass [g]	safety	α_1	α_2	x	t	h	w
223	5.45	0.44	0.49	0.9	5	6.90	7.01

Note that this best point depends only on the preferences for safety and mass, and on the answers given above to the two indifference questions. Also note that this point, which is shown as a star on the graph in Figure 10.5, is intuitively appealing as an overall optimum, while the points provided by the weighted sum method are not.

It should be noted that if either α_1 or α_2 is close to 0, then the (s, ω) pair is quite sensitive to small differences in α_1 and α_2. In these cases, it might be preferable to elicit other indifference points to determine s and ω. In the procedure described above, points that are equivalent to $(0.5, 0.5)$ are chosen; the procedure can easily be modified to consider indifference to some other reference point. Indeed, if the procedure is applied more than once with different reference points, the redundant information serves as a check on the accuracy of the specification.

5. Conclusion

Weighted sum aggregation with importance weights is common to many methods for engineering design decision making. Two important difficulties with these methods are the inability of weighted-sum methods to select all Pareto points, and the arbitrary nature of direct assignment of importance weights.

It is shown here that a complete model of an engineering decision depends not only on the importance weights, but also on the level of compensation, or trade-off strategy. A weighted sum, or any other pre-determined aggregation procedure, is overly and inappropriately constraining. The appropriate strategy (or degree of compensation among attributes) is situation-dependent, and a "one-size-fits-all" decision method that dictates an aggregation method can lead to incorrect results. An easily computed family of preference aggregation functions is completely determined by two parameters that represent the trade-off strategy (degree of compensation) and the importance weighting.

The choice of strategy and importance weights, or even the choice of importance weights alone, can be *ad hoc* and arbitrary if accomplished by direct specification. A simple procedure is presented, along with straightforward calculations, to establish the proper importance weights and degree of compensation to reach rational engineering decisions.

The results in this paper regarding decision methods are related to previously known results about multi-criteria optimization [2, 1]. The preference aggregation operators presented here could be used to explore a non-convex Pareto frontier. From an optimization point of view, once an aggregated preference function (or utility function) is determined, locating the optimum is straightforward. If some point lies on a non-convex region of a Pareto frontier, and a utility function is constructed using a weighted sum, then that point is an inferior point. From a design decision point of view, however, it is appropriate to question the choice of a weighted sum to aggregate the preferences. The method presented in this paper, unlike a weighted sum, has the great advantage that it does not *a priori* exclude any Pareto points from consideration. Thus, if the aggregated preference is optimized, the selected "optimum" is actually what the designer desires, and is not artificially constrained by the geometry of the design and performance spaces.

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References

[1] CHEN, W., WIECEK, M., AND ZHANG, J. Quality utility - a compromise programming approach to robust design. *ASME Journal of Mechanical Design 121*, 2 (1999), 179–187.

[2] DAS, I., AND DENNIS, J. A closer look at drawbacks of minimizing weighted sums of objectives for pareto set generation in multicriteria optimization problems. *Structural Optimization 14*, 1 (1997), 63–69.

[3] HAUSER, J., AND CLAUSING, D. The House of Quality. *Harvard Business Review 66*, 3 (May/June 1988), 63–73.

[4] OTTO, K. N., AND ANTONSSON, E. K. Trade-Off Strategies in Engineering Design. *Research in Engineering Design 3*, 2 (1991), 87–104.

[5] PUGH, S. *Total Design*. Addison-Wesley, New York, NY, 1990.

[6] SAATY, T. L. *The Analytic Hierarchy Process: Planning, Priority Setting, Resource Allocation*. McGraw Hill, New York, NY, 1980.

[7] SCOTT, M. J. *Formalizing Negotiation in Engineering Design*. PhD thesis, California Institute of Technology, Pasadena, CA, June 1999.

[8] SCOTT, M. J., AND ANTONSSON, E. K. Aggregation Functions for Engineering Design Trade-offs. *Fuzzy Sets and Systems 99*, 3 (1998), 253–264.

[9] WOOD, K. L., AND ANTONSSON, E. K. Computations with Imprecise Parameters in Engineering Design: Background and Theory. *ASME Journal of Mechanisms, Transmissions, and Automation in Design 111*, 4 (Dec. 1989), 616–625.

A Appendix: Details of the Truss Example

Here are the details for the example of the bracket shown in Figure 10.1.

Material properties and performance measures

The material is aluminum (6061-T6), which has the following relevant material properties:

Young's modulus (E)	$69 \cdot 10^9$ Pa
Density (ρ)	$2660\ \mathrm{kg/m^3}$
Yield stress (σ)	$275 \cdot 10^6$ Pa

There are four design variables:

$x \in [0.1\ \mathrm{m}, 0.9\ \mathrm{m}]$	distance from wall to pin
$t \in [5\ \mathrm{mm}, 20\ \mathrm{mm}]$	thickness of bending member
$h \in [5\ \mathrm{mm}, 20\ \mathrm{mm}]$	height of bending member
$w \in [5\ \mathrm{mm}, 20\ \mathrm{mm}]$	width of (square) compression member

The first performance measure is total mass (in kilograms, here, so the load P below is equal to 1):

$$\mathrm{M} = \rho\left(htL + w^2\sqrt{x^2+y^2}\right) = 2660\left(ht + w^2\sqrt{x^2+0.25}\right)$$

The safety factor S has two components, the safety factor for the bending member S_b , and the safety factor for the compression member S_c. Since the yield stress in the bending member is σ, and the maximum stress in the bending member is $\frac{12P(L-x)}{th^2}$, the factor of safety in bending is the ratio:

$$\mathrm{S}_b = \frac{\sigma t h^2}{12P(L-x)} = \frac{\sigma t h^2}{120(1-x)}$$

Similarly, using the Euler buckling load, the safety factor in the compression member is:

$$\mathrm{S}_c = \frac{\pi^2 \mathrm{E} x y w^4}{12PL(x^2+y^2)^{1.5}} = \frac{\pi^2 \mathrm{E} x y w^4}{120(x^2+0.25)^{1.5}}$$

The safety factor for the entire design is defined to be the minimum of the two:

$$\mathrm{S} = \min\left(\frac{\sigma t h^2}{120(1-x)}, \frac{\pi^2 \mathrm{E} x y w^4}{120(x^2+0.25)^{1.5}}\right)$$

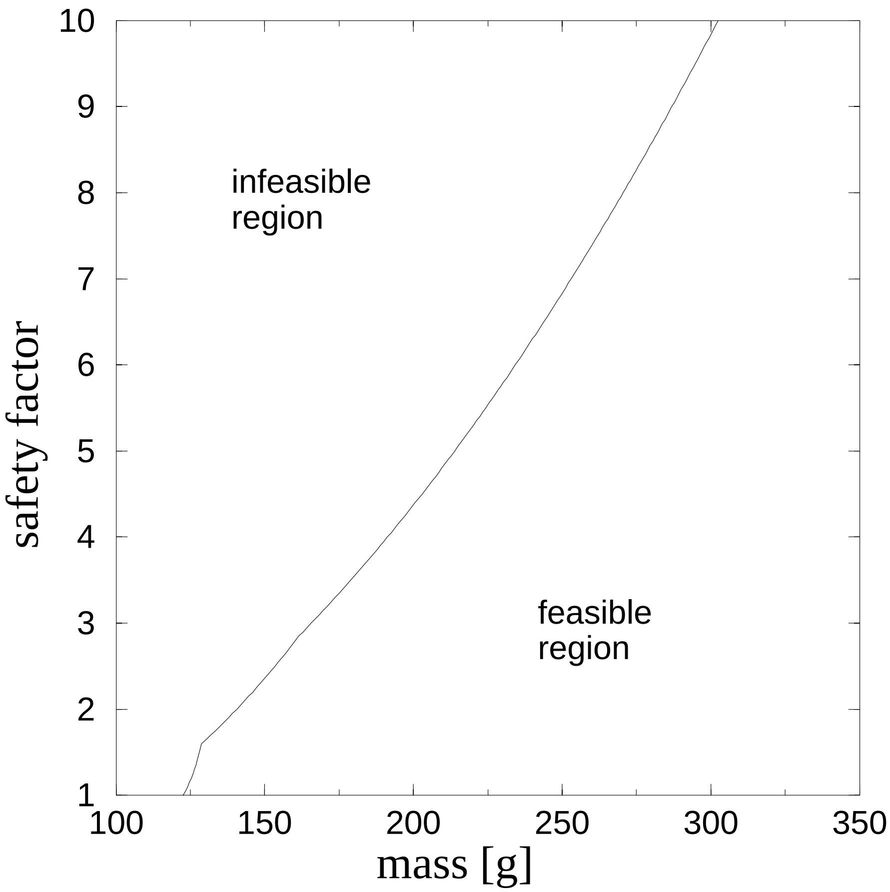

Figure A.1 Pareto frontier of best performances

Calculation of the Pareto frontier

The design problem is to minimize the mass while maximizing the factor of safety; both are analytic expressions. First, note that mass is linear in both t and h, while the factor of safety in bending is linear in t but quadratic in h. Thus, as long as no other design variables reach the maximum acceptable dimensions, it will always be preferable to increase h rather than t. Setting $t = t_{\min} = 5\ \mathrm{mm}$ reduces the problem to the three design variables x, h, and w. Both h and w can be expressed as functions of x and a safety factor, and thus finding the minimum possible mass for a given safety factor requires solving a rational equation in x. These solutions yield a Pareto frontier of designs, which is shown in Figure A.1.

The values of x, h, and w which generate these optimal designs are included in Figure A.2. It can be seen from Figure A.2 that at each Pareto point, at least one domain constraint is active: in particular, for low mass, h takes its minimum acceptable value of 5 mm, while for higher mass, x takes its maximum acceptable value of 0.9 m. Nevertheless, along most of the Pareto frontier two design variables are changing as the frontier is traversed.

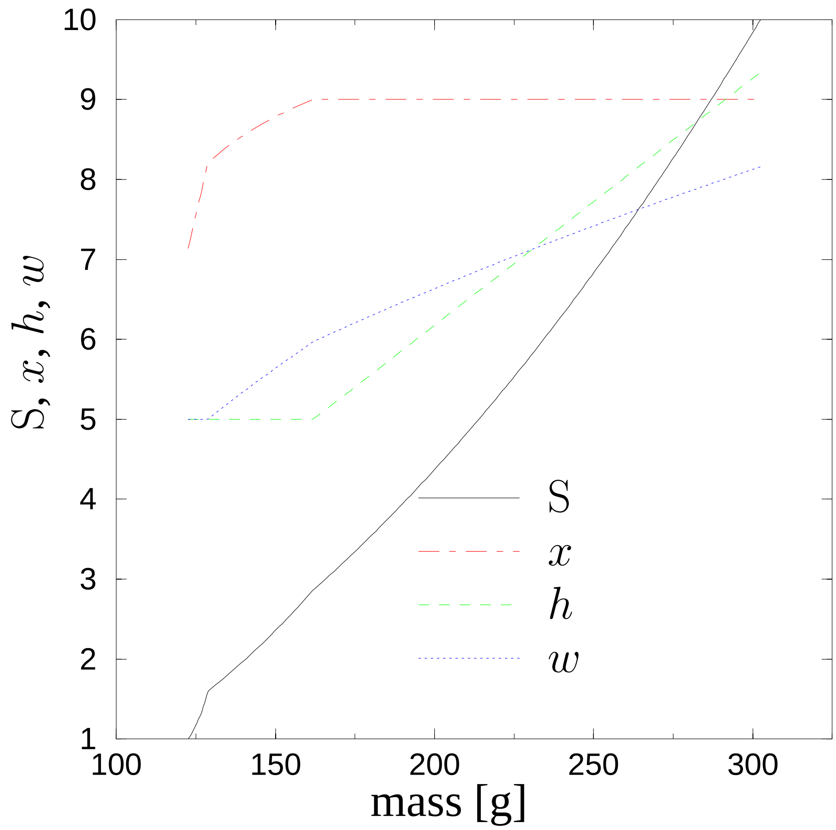

Figure A.2 Pareto frontier with design variable values