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MODELING IMPRECISION AND UNCERTAINTY IN PRELIMINARY ENGINEERING DESIGN

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Abstract—Each stage of the engineering design process, and particularly the preliminary phase, includes both imprecision and uncertainty. This paper presents a method by which the imprecision and uncertainty in the description of the design's components can be represented. Fuzzy set theory provides the foundation of the approach. *Extended hybrid numbers* are introduced to handle the two separate representations of imprecision and uncertainty. These representations include the designer's judgements. An application of the theory to machine design is presented, emphasizing the imprecision usually found in the preliminary design of machine elements. Results show the performance of each design alternative compared to its functional requirements, as well as the coupling betwen the design parameters and the resulting performance parameters.

NOMENCLATURE

d_p = pulley (sheave) diameter, belt configuration
d_s = shaft diameter
m = module
m_G = speed ratio
n_f = factor of safety, fatigue strength
n_{rpm} = speed, rpm
n_R = rated speed, bearing life
n_s = factor of safety, surface durability
w_F = face width
y_s = acceptable shaft deflection
C = bearing load rating
C_c = belt force conversion factor
C_p = belt force conversion factor
E = modulus of elasticity
F_p = peak belt force
H_B = Brinell hardness
K_a = surface finish factor, fatigue strength
K_b = belt bending force factor
K_c = belt centrifugal force factor
K_d = depth factor, V-belt
K_{df} = design factor, shaft strength
K_e = misc. effects factor
K_{elas} = elastic coefficient
K_f = surface finish and environment factor
K_k = service factor, shaft strength
K_l = load-distribution factor
K_0 = overload factor
K_{os} = oscillation factor
K_p = preloading factor
K_r = reliability factor, fatigue strength
K_s = size factor, fatigue strength
K_{sc} = stress concentration factor, shaft strength
K_{sf} = service factor
K_{sh} = application shock factor
K_t = temperature factor, fatigue strength
K_A = velocity factor
K_L = life factor
K_J = geometry factor
K_R = reliability factor, surface durability
K_T = temperature factor, surface durability
L_e = expected belt life
L_s = shaft length
L_D = bearing design life
L_R = rated bearing life
N = number of teeth
N_T = total belt passes for the belt sheaves
P = power
R = Reliability
S_f = failure stress
S_t = tensile strength
T_r = tension ratio
ϕ = pressure angle, spur gear
ϕ_t = transverse pressure angle, helical gear
ϕ_n = normal pressure angle, helical gear
ψ = helix angle

Note: superscripts s and h denote spur gear and helical gear configuration, respectively: Superscript r denotes "requirement."

1. INTRODUCTION

The engineering design process can be characterized as a collection of phases by which a description of a need is transformed into a physical artifact, proceeding from a highly imprecise preliminary stage to a final configuration in the form of a precise, physical design description. Many tools exist for the later design stages (e.g. solid modeling and mechanism analysis) by which designers may analyze precisely described configurations in order to verify configuration performance or to make final configuration choices. Because of the imprecision and uncertainty inherent in the preliminary design phase, computational tools developed for this phase must be able to manipulate both imprecise and uncertain representations of design alternatives, while directly incorporating the engineer's judgement. Many of the most important design decisions are made at the preliminary stage. The objective of the work reported here is to develop an approach to modeling imprecision *and* uncertainty in preliminary engineering design, leading to the development of computational tools that make more information available to the designer earlier in the design process than conventional computer-aided design techniques. The method for representing and manipulating imprecision has been introduced in a previous publication [1]. This paper introduces a technique for handling uncertainty in addition to imprecision, and demonstrates the method with a design example.

1.1. Terminology

In a previous publication [1], terminology relating to the design process was introduced. A summary of those definitions follows.

Design parameters (DPs). DPs are variables to be determined during the design process.

Performance parameters (PPs) PPs have specified target values.

Functional requirements (FRs). FRs are the specified target values for each of the PPs. Functional requirements are not restricted to the equality and inequality forms normally encountered. Qualitative statements, such as "maximize heat dissipation", or functional relationships relating certain imprecise design parameters to the functional requirement, e.g. "brake torque must be less than or equal to three-fourths of the force on one wheel multiplied by the wheel radius", may be specified.

Performance parameter expressions (PPEs). PPEs contain (usually in equation form) the relationship between the DPs and the PPs.

1.2. Imprecision and uncertainty in preliminary design

Geometry and other physical characteristics are commonly described approximately or imprecisely in the preliminary phase of design. The length of a beam, for example, may be represented as "about 5 m", or an irregular three-dimensional object may be represented by a sphere or cube for preliminary analysis. Imprecision is also common in the specification of material properties. The material used for a brake shoe lining may be described as "having a coefficient of friction of about 0.4". These imprecise descriptions are a natural consequence of the simplifications and approximations made in the preliminary phase of engineering design. The level of imprecision decreases as the design process proceeds, and the design becomes more detailed. The nominal value of the imprecisely represented DPs, and their imprecisions, are chosen by the designer, and as a result contain his or her judgement and experience.

Uncertainty is also present at all stages of the design process. The usual sources are tolerances in manufacturing processes resulting in uncertain dimensions and uncertainties in material properties. Uncertainties may also exist related to application. The coefficient of friction between a tire and the road can take on a wide range of values depending on road, tire and weather conditions. Sometimes these data will represent measured (objective) probability data, sometimes they will represent subjective possibility data. As will be seen later, we keep these two types of design uncertainty distinct.

This paper presents a method for representing and manipulating both imprecision and uncertainty. Calculations are performed with these variables according to the governing performance expressions of the system. Qualitative relations between the input and output performance parameters are determined such that the designer is able to rank the design parameters according to their impact on the performance results. The designer is also provided with the necessary information by which a design alternative may be rated according to its merit in relation to the design's functional requirements, and in relation to the other alternatives under consideration.

The method has led to the development of two specific computational tools: one for calculating performance parameters using the approximate input design parameters; and one for calculating a measure of the coupling between design parameters.

The general technique described above, when combined with its computational application, forms a *semi-automated* approach to design. The intent is not to remove the designer from the loop, nor to fully automate the design process, but instead to rely upon the designer's skill at idea generation and judgement such that he or she is provided with the means of evaluating alternatives in less time and with greater confidence.

2. BACKGROUND

There are many ways that approximate or imprecise parameters can be represented. Finite real intervals, probability distributions, sensitivity analysis, and others all address this area. We have chosen to use fuzzy sets for this purpose [1, 2] (except for measured probability density data which will be represented and manipulated using probabilistic techniques such as those described in Refs [3, 4]). Fuzzy sets provide distinct advantages in representation and manipulation of imprecise parameters for several reasons: computational efficiency; a more useful design interpretation than the methods mentioned above; and the ability to provide more information on the performance of a design at the preliminary stage. The publications referenced above discuss these points in detail.

A fuzzy set (as originally developed by Zadeh [5]) is a set with boundaries which are not sharply defined. Membership in the set can take on any value between zero (0) and one (1). Zero membership indicates that an entity has no membership in the set. A black object would have zero membership in the set of white objects. Membership of one (1) indicates perfect membership in the set. A light-beige, or off-white colored object would have partial membership. A cube has perfect membership in the set of rectangular parallelepipeds. Many real objects will have partial membership in these sets.

In the design method described here, membership functions are used to represent imprecision or approximation in design parameters. This is done by assigning membership values to particular parameter values based on the designer's preference, or desire. The more a designer desires to use a value, the greater its membership. In this way the knowledge and judgement of the designer are utilized during the semi-automated design process.

Zadeh [6] developed the extension principle to extend crisp mathematics to fuzzy sets. Kaufmann and Gupta [7] showed that Zadeh's analytical formulation is equivalent to an alpha-cut form. Finally Wong and Dong [8] developed a discrete version of the mathematics utilizing interval analysis at discrete alpha-cuts. We use Wong and Dong's FWA algorithm [8] in a computationally efficient implementation we developed [1, 2].

Essentially the fuzzy set representations can be operated on with any of the usual engineering operations: addition, multiplication, exponentiation, transcendentals, etc. The results of these calculations are themselves fuzzy sets, and can be interpreted in a similar way to the input parameters.

A distinct advantage of the use of fuzzy sets and their mathematics is the ability to not only find output sets from calculations on inputs, but to then find input values that correspond to a particular output. We refer to this as the *backward path* of our implementation of the FWA algorithm. This facility is absent with probability or interval analyses. Peak values of the inputs [membership of one (1)] correspond to the peak in the output set. Off-peak values of the output at a particular membership level are created by one or more off-peak inputs with the same membership. In this way, if the peak value of a performance calculation is not a usable result, an off-peak value can be selected, and the corresponding input values can easily be determined [1]. Our implementation takes advantage of this consequence of fuzzy mathematics.

Finally, the authors have developed a measure of the contribution of each input's fuzziness (imprecision) to the imprecisions of the output. This is called the γ-level measure, and is described in detail in Ref. [1], and will be discused in the example below.

3. COMBINING IMPRECISION AND UNCERTAINTY

In the introductory section, both imprecision and uncertainty (in the preliminary engineering design phase) were described. These two effects are usually present at the same time. For example, a dimension of a part may be only imprecisely known to the designer, *and* the manufacturing method will introduce an uncertainty (tolerance). Similarly for a coefficient used in the design process, such as the convection coefficient in heat-transfer: some contributions to this coefficient the designer can choose, such as geometry or surface finish; others, such as the conditions under which the device will operate, he or she has no control over. The first of these effects we represent as imprecision, determined by the designer's desires. The second of these we represent as uncertainty.

We have previously [1] developed a method for representing and manipulating imprecision (also briefly described in Section 2). To include uncertainty effects, an additional method must be employed. Several methods already exist for representing and manipulating uncertainty in engineering design [3, 4, 9–12]. We wish to combine the effects of uncertainty with imprecision. Since we have modeled imprecision with fuzzy sets, a logical way to include uncertainty effects is to also transform them into a fuzzy sets representation. For example, an uncertain parameter which is described by a probability density function can be normalized to have a peak of one (1), as shown by Kaufmann and Gupta [7, p. 79–82].

With the uncertain parameter represented as a fuzzy set, and assuming we have an expression relating the uncertain parameters to the imprecise parameters, we may perform a fuzzy calculation which results in a fuzzy output. The fuzzy output in this case has both uncertain and imprecise contributions. Unfortunately, as pointed out by Kaufmann and Gupta [7, p. 82], information is lost by this process:

> "Basically, we have transformed a measurement of an objective (measured) value to a valuation of a subjective (fuzzy) value, which results in the loss of information. Although this procedure is mathematically correct, it decreases the amount of information that is available in the original data, and we should avoid it."

There is no way to determine which portion of the result is due to uncertainty and which is due to imprecision. Since these two effects are independent, and we wish to be able to determine their ramifications separately, a method for keeping uncertainty and imprecision distinct in design calculations must be developed.

Kaufmann and Gupta [7] have proposed the notion of *hybrid numbers*, as a method of representing uncertainty and fuzzy sets, without reducing the information content. This is somewhat similar to complex numbers, with a real component and an imaginary component. Here, instead, we have a fuzzy component, and an uncertain one. In our model of the engineering design process, we will use the fuzzy component of the hybrid numbers to represent imprecision (or approximation); the other component will represent stochastic uncertainty.

In engineering design, the random-variable frequency-based model of probability won't adequately represent all uncertainties. Some uncertainties are subjective, rather than measured. For example, the coefficient of friction of a brake shoe under a variety of possible operating conditions. These subjective uncertainties can be represented by *possibility*, introduced by Zadeh [13]. We have chosen to adopt this representation of uncertainty (rather than incorporating all uncertainty into a multi-logic probability formulation) for computational efficiency, and to adhere to the interpretation of these two distinct forms of uncertainty proposed by Kubic and Stein in Ref. [11].

Because we wish to use these two separate forms of uncertainty in the preliminary engineering design process, in addition to imprecision, we have augmented Kaufmann and Gupta's hybrid numbers to represent all three components. This new representation we call *extended hybrid numbers*.

The three distinct components that can comprise design parameters (imprecision, possibilistic uncertainty, and probabilistic uncertainty) can be operated on in a design calculation, and then recombined into an extended hybrid representation of the result. Imprecision is represented and calculated, and the result is interpreted as discussed in Section 2. The same fuzzy mathematics can be applied to the possibilistic uncertainties [13], but the interpretation of the inputs and outputs are different. The input interpretation corresponds to Kubic and Stein's and the output represents the performance over the range of possible values of the input parameters. The designer's judgement can be incorporated at this stage by determining over what range of possible values the design should function, or a specification may require the design to operate in some range of possible conditions. For example, in extreme cases the design must operate over all possible conditions, and the range of performance would extend over the entire possible output range.

Operation rules may be constructed from Cox's [14] formulation of the calculus for probability logic in order to carry out calculations with the probability component of the extended hybrid numbers. The method to perform these calculations are the subject of another publication of the authors' [15], and will not be discussed, nor presented in detail here. The usual likelihood interpretation of probability theory is applied to the normalized output results of computations with these operation rules.†

The calculation method for extended hybrid numbers may be summarized as follows:

- Given performance expressions for the design, relating the design parameters to the performance parameters.
- Calculate the imprecision using the nominal values for the possibilistic and probabilistic uncertainties.
- Repeat with the nominal values for the imprecision and probabilistic uncertainties.
- Repeat with the nominal values for the imprecision and possibilistic uncertainties, and normalize this result to get a probability function.
- Combine these resulting output functions.

The extended hybrid representation of the result can be shown on a single graph. All three output functions will have a common peak (at the nominal value for the output). The three curves represent the imprecision, possibility and probability, respectively, and can be compared to the design's functional requirements. For example, consider the output performance for fatigue strength of a spur gear system as shown in Fig. 3. Assuming that the desired performance (functional requirement) of the factor of safety is two (2.0), the corresponding membership value (α) of the imprecision curve can be directly read from the ordinate. In this case α is approximately equal to 0.8. This implies that at least one design parameter must decrease in membership to 0.8 in order to achieve a factor of safety equal to two (2.0) and satisfy the performance requirement.

The uncertainty, however, must also be accounted for in the design. In Fig. 3 the *possible* ranges of the fatigue strength factor of safety for the spur gear are indicated by the uncertainty curve. Because a minimum value (2.0) must be achieved for this parameter, the left portion (from the peak) of the uncertainty curve represents the possibilistic uncertainty which will be present. If a PP must meet a maximum constraint (instead of a minimum) then the right portion of the uncertainty curve would be used. The effect of uncertainty is to introduce a range of possible values for each output PP value. Since the uncertain range may include unsatisfactory performance values, the value of the PP must be chosen such that all *possible* values satisfy the required performance.

A particular range of uncertainty is determined by the degree of uncertainty the designer chooses to include. For example, the designer may wish this device to operate over no less than 50% of the possible operating conditions. In this case the range of possible outputs would be determined by the interval between the left and right portions of the uncertainty curve at a membership of 0.5. For this example PP, shown in Fig. 3, the nominal value is 1.61 for the spur gear configuration. The value of the left portion of the uncertainty curve at $\alpha = 0.5$ is 0.99 and 1.70 for the right. Thus the range of uncertainty in this instance is 0.71, however it is asymmetrically distributed, with the major portion (0.62) to the left of the PP value. If the nominal value for this PP is used (1.61), the possible range of performance would be from 0.99 to 1.70. Since the requirement on this PP is a minimum of 2.0, a value larger than the peak (nominal) output value must be used, such that the lower end of the possible range still meets the requirement. A satisfactory PP value can be obtained by adding the left portion of range to the required value. In this case the designer would have to use a value no less than $0.62 + 2.0 = 2.62$ to be sure that no less than 50% of the possible values of the output would meet or exceed the requirement.

A similar procedure can be carried out if a probabilistic component of uncertainty were also present on the output curve, where the chosen probabilistic uncertainty is subsequently added to the functional requirement to obtain an equivalent uncertainty performance specification in terms of both a possibilistic and probabilistic contribution.

4. A MACHINE DESIGN EXAMPLE

To demonstrate the approach described briefly above, an example preliminary design problem will be presented. The problem is to design a single speed power transmission for the "spin" cycle

†The design example shown in this paper will only make use of imprecision and possibilistic uncertainty, as no objective (measured) probabilistic design data is included.

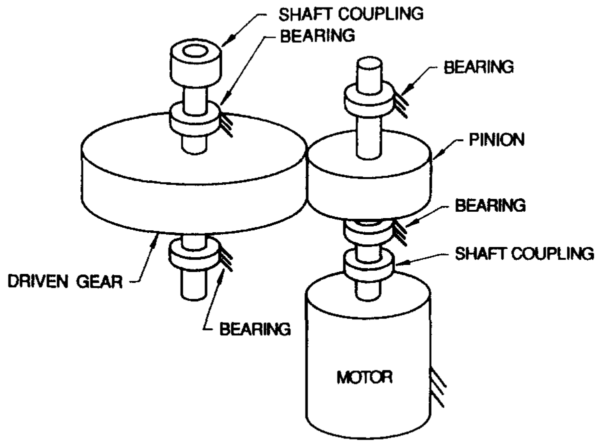

Fig. 1. Gear drive configuration.

of a conventional domestic clothes washing machine. The details of changing modes from agitation to spin are omitted for clarity. The drive motor is a 1.5 kW (2 h.p.) electric motor with a nominal no-load speed of 1750 rpm. The desired top speed of the drum is approx. 350 rpm. Both the motor shaft and the drive shaft on the washer drum are vertical. As a design goal we wish to minimize cost, as well as achieve satisfactory performance in terms of strength, durability and belt life. For simplicity in this example, cost will be assumed to be directly related to the diameter of the shafts. We wish to compare three different alternative drives: spur gears, helical gears and a V-belt. The configuration of the gear drives is shown in Fig. 1, and the V-belt is shown in Fig. 2.

4.1. *Performance expressions*

A variety of performance issues arise when designing a speed reduction system as described in the problem statement. For the spur and helical gear configurations, gear strength, surface durability, lubrication and reliability should be considered in order to rate the design's performance. Similarly, a successful *V*-belt design should perform satisfactorily with respect to belt life, efficiency, reliability, etc.

We will not discuss all performance aspects in this design example, Instead, a typical set of performance characteristics has been chosen for each of the three proposed configurations. For the spur gear and helical gear configurations, the choice of performance parameters include: factor of safety for fatigue strength n_f; factor of safety for surface durability n_s; and bearing load rating for the resultant gear forces C. In addition, two calculations relating to shaft diameter will be considered: shaft diameter for deflection $d_{s,\mathrm{def}}$; shaft diameter for strength (bending) $d_{s,\mathrm{str}}$. Using,

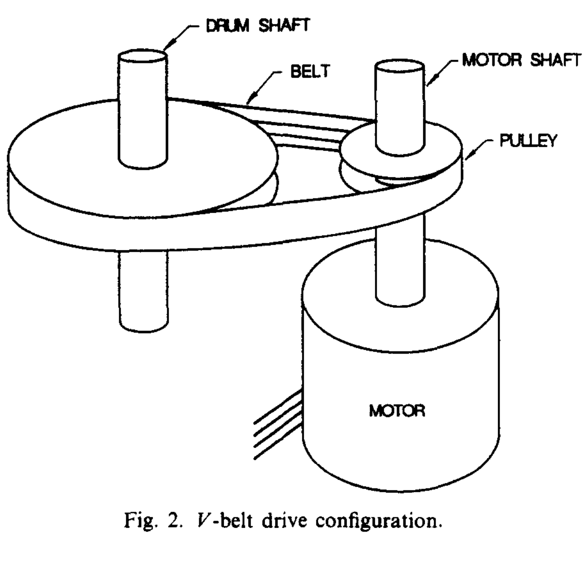

Fig. 2. *V*-belt drive configuration.

in part, the mathematical formulations found in Refs [16–19], the corresponding performance expressions for these PPs may be specified as:†

$$n_{\mathrm{f,spur}} = \frac{0.5\ \pi\ K_a K_s K_r K_t K_e K_J K_A^s S_t N m^2 n_{\mathrm{rpm}} w_F}{60(10)^3 K_o K_l P\left[K_A^s + \left(\frac{\pi N m n_{\mathrm{rpm}} B}{10^3}\right)\right]}, \tag{1}$$

$$n_{\mathrm{f,helical}} = \frac{0.5\pi K_a K_s K_r K_t K_e K_J \sqrt{K_A^h}\, S_t N m^2 n_{\mathrm{rpm}} w_F}{60(10)^3 K_o K_l P \cos^2\psi \sqrt{\left[K_A^h + \left(\sqrt{\frac{\pi N m n_{\mathrm{rpm}} B}{10^3}}\right)\right]}}, \tag{2}$$

$$n_{\mathrm{s,spur}} = \frac{\left(\frac{K_L}{K_T K_R K_{\mathrm{elas}}}(2.76 H_B - 70)(10)^6\right)^2 (\pi K_A^s w_F N^2 m^2 n_{\mathrm{rpm}} \cos\phi \sin\phi m_G)}{2(60)(10)^3 P K_o K_l\left[K_A^s + \left(\frac{\pi N m n_{\mathrm{rpm}} B}{10^3}\right)\right](m_G + 1)}, \tag{3}$$

$$n_{\mathrm{s,helical}} = \frac{\left(\frac{K_L}{K_T K_R K_{\mathrm{elas}}}(2.76 H_B - 70)(10)^6\right)^2 \cdot (A_1 \cdot A_2)}{2(60)(10)^3 P \cos^2\psi \cos\phi_n K_o K_l \sqrt{\left[K_A^h + \sqrt{\left(\frac{\pi N m n_{\mathrm{rpm}} B}{10^3}\right)}\right]}(m_G + 1)}, \tag{4}$$

where

$$A_1 = ((0.95)\sqrt{K_A^h}\, w_F N^2 m n_{\mathrm{rpm}} \cos\phi_t \sin\phi_t m_G),$$

$$A_2 = \left[\sqrt{\left(\frac{Nm}{2\cos\psi} + m\right)^2 - \left(\frac{Nm\cos\phi_t}{2\cos\psi}\right)^2} + \sqrt{\left(\frac{m_G N m}{2\cos\psi} + m\right)^2 - \left(\frac{m_G N m \cos\phi_t}{2\cos\psi}\right)^2} - \left(\frac{3Nm\sin\phi_t}{\cos\psi}\right)\right],$$

$$d_{\mathrm{s,str}}^{s} = \left(\frac{32(60)(10)^3 P L_s K_f K_k K_{sc} K_{df}\sqrt{1 + \tan^2\phi}}{4(0.55)\pi^2 N m n_{\mathrm{rpm}} S_f}\right)^{1/3}, \tag{5}$$

$$d_{\mathrm{s,def}}^{s} = \left(\frac{4(60)(10)^3 P L_s^3 \sqrt{1 + \tan^2\phi}}{3\pi^2 N m n_{\mathrm{rpm}} E y_s}\right)^{1/4}, \tag{6}$$

$$d_{\mathrm{s,str}}^{h} = \left(\frac{32(60)(10)^3 P \cos\psi K_f K_k K_{sc} K_{df}\sqrt{\left[\frac{\tan\phi_t L_s}{4} + \frac{\tan\psi N m}{4\cos\psi}\right]^2 + \frac{L_s^2}{16}}}{(0.55)\pi^2 N m n_{\mathrm{rpm}} S_f}\right)^{1/3}, \tag{7}$$

$$d_{\mathrm{s,def}}^{h} = \left(\frac{4(60)(10)^3 P \cos\psi L_s^3 \sqrt{1 + \tan^2\phi_t}}{3\pi^2 N m n_{\mathrm{rpm}} E y_s}\right)^{1/4}, \tag{8}$$

$$C_{\mathrm{spur}} = \left(\frac{60(10)^3 K_{sh} K_p K_{os} P \tan\phi}{\pi N m n_{\mathrm{rpm}}}\right)\left[\left(\frac{L_D}{L_R}\right)\left(\frac{n_{\mathrm{rpm}}}{n_R}\right)\left(\frac{1}{6.84}\right)\right]^{1/3}\left(\frac{1}{\left[\ln\left(\frac{1}{R}\right)\right]^{1/3.51}}\right), \tag{9}$$

†The performance parameters will not be derived in this article. The equations shown for factor of safety, rated belt load, and belt life may be formulated from the material in the cited references. Likewise, the shaft diameter equations come directly from beam bending theory, and may be reproduced by considering maximal moments from the moment and loading diagrams.

$$C_{\text{helical}} = \left(\frac{60(10)^3 P}{\pi N m n_{\text{rpm}}}\right) \cdot (0.5 K_{\text{sh}} K_{\text{p}} K_{\text{os}} \cos\psi \tan\phi_{\text{t}} + 1.4 \sin\psi)$$

$$\times \left[\left(\frac{L_{\text{D}}}{L_{\text{R}}}\right)\left(\frac{n_{\text{rpm}}}{n_{\text{R}}}\right)\left(\frac{1}{6.84}\right)\right]^{1/3} \left(\frac{1}{\left[\ln\left(\frac{1}{R}\right)\right]^{1/3.51}}\right). \tag{10}$$

From these performance expressions, the design parameters for the spur and helical gear configurations are as follows: module m; speed n_{rpm}; face width w_{F}; acceptable shaft deflection y_{s}; Brinell hardness H_{B}; surface finish factor K_{a}; design factor K_{df}; miscellaneous effects factor K_{e}; surface finish and environment factor K_{f}; service factor K_{k}; load-distribution K_{l}; overload factor K_{o}; elastic coefficient K_{elas}; preloading factor K_{p}; oscillation factor K_{os}; shock factor K_{sh}; reliability factor K_{r}; size factor K_{s}; temperature factor K_{t}; velocity factor K_{A}; life factor K_{L}; geometry factor K_{J}; reliability factor K_{R}; temperature factor K_{T}; shaft length L_{s}; failure stress S_{f} and tensile strength S_{t}. The speed ratio m_{G} and the input power P are constants as specified in the problem statement. The pressure and helix angles, ϕ and ψ, respectively, as well as the number of teeth N, rated bearing life L_{R}; rated bearing speed n_{R}, and modulus of elasticity E are also considered as "constants" for this design. Although these terms could be represented as imprecise parameters, their contribution is viewed as either negligible (modulus of elasticity) or prescribed by requirements on the design problem (e.g. the number of teeth is set at the minimum value as prescribed in design tables).

The final configuration to be considered is the V-belt system shown in Fig. 2. The same performance parameters exist for the V-belt alternative as found for the gear systems, with one exception. Instead of safety factors for fatigue strength and surface durability, belt life is used as a measure of performance, where belt life is directly related to the *peak force* encountered per belt-pass. Denoting expected belt life by L_{e} and peak force by F_{p}, the governing expressions, relating the V-belt design parameters to the performance parameters, may be written as:

$$F_{\text{p}} = \frac{60(10)^3 P S_{\text{f}} T_{\text{r}}}{(T_{\text{r}} - 1)\pi d_{\text{p}} n_{\text{rpm}}} + \frac{K_{\text{b}} C_{\text{b}}}{d_{\text{p}}} + \frac{K_{\text{c}} C_{\text{b}} (\pi d_{\text{p}} n_{\text{rpm}} C_{\text{c}})^2}{10^6}, \tag{11}$$

$$L_{\text{e}} = \frac{N_{\text{T}} \left\{ [3.57(d_{\text{p}}(m_{\text{G}} + 1) - C_{\text{d}} m_{\text{G}})] + \dfrac{[d_{\text{p}}(m_{\text{G}} - 1) - C_{\text{d}} m_{\text{G}}]^2}{4(d_{\text{p}}(m_{\text{G}} + 1) - C_{\text{d}} m_{\text{G}})} \right\}}{60\pi d_{\text{p}} n_{\text{rpm}}}, \tag{12}$$

$$d^{\text{b}}_{\text{s,str}} = \left(\frac{32(60)(10)^3 K_{\text{sf}} P L_{\text{s}} K_{\text{f}} K_{\text{k}} K_{\text{sc}} K_{\text{df}} \sqrt{\left\{\sqrt{3}\dfrac{T_{\text{r}} + 1}{T_{\text{r}} - 1}\right\}^2 + 1}}{8(0.55)\pi^2 d_{\text{p}} n_{\text{rpm}} S_{\text{f}}} \right)^{1/3}, \tag{13}$$

$$d^{\text{b}}_{\text{s,def}} = \left(\frac{4(60)(10)^3 K_{\text{sf}} P L_{\text{s}}^3 \sqrt{T_{\text{r}}^2 + T_{\text{r}} + 1}}{3\pi^2 (T_{\text{r}} - 1) d_{\text{p}} n_{\text{rpm}} E y_{\text{s}}} \right)^{1/4}, \tag{14}$$

$$C_{\text{belt}} = \left(\frac{60(10)^3 K_{\text{sf}} K_{\text{sh}} K_{\text{p}} K_{\text{os}} P}{\pi d_{\text{p}} n_{\text{rpm}}} \right) \left[\left(\frac{L_{\text{D}}}{L_{\text{R}}}\right)\left(\frac{n_{\text{rpm}}}{n_R}\right)\left(\frac{1}{6.84}\right)\right]^{1/3} \left(\frac{1}{\left[\ln\left(\frac{1}{R}\right)\right]^{1/3.51}}\right). \tag{15}$$

The additional design parameters for the V-belt configuration are: pulley diameter d_{p}; belt bending force factor K_{b}; belt centrifugal force factor K_{c}; service factor K_{sf}; total belt passes N_{T}; and tension ratio T_{r}.

4.2. *Performance specifications*

In this design example we are assuming that preliminary performance criteria (functional requirements) have been specified for: the fatigue strength factor of safety; surface durability safety factor; shaft diameter; belt life and bearing load rating. For the first two of these, the designer will usually use a "rule-of-thumb" to determine the factor of safety PPs. The result represents a minimum safety factor that must be achieved by the design. In other words, n_f and n_s must be greater than or equal to n_f^r and n_s^r, i.e.

$$n_f \geqslant n_f^r = 2.0,$$

$$n_s > n_s^r = 1.0.$$

In a similar manner, a minimum limit may be specified for the expected belt life performance parameter. Assuming under nominal conditions, that a washing machine will be used an average of 10h per week, the resultant minimum expected life of the belt is:

$$L_e \geqslant L_e^r = 16.0\,\text{kh}.$$

A single functional requirement or specification is usually not given for the two remaining output parameters (d_s and C) in the preliminary design phase. For this example, we have chosen to *minimize shaft diameter* to minimize material and manufacturing costs. This specification is denoted by d_s^r. A similar specification, C^r, will be used for the bearing load rating.

These five performance specifications make up the preliminary functional requirement set for this transmission example. When combined with the imprecise output and γ-level results (shown later), this set provides a means of rating each design alternative according to individual performance, and in comparison to other alternatives.

4.3. Specifying input design parameters

As discussed earlier, each input parameter represents a triplet of information: imprecision modeled with fuzzy set theory; uncertainty modeled with possibility and uncertainty modeled with probability theory. Triangular and linear (fuzzy) functions have been used to represent the imprecision and possibilistic uncertainty in the input parameters. Probability density functions may be used to represent the last component of information; however, for simplicity, only possibilistic uncertainty will be modeled in this design example. Table 1 lists the data for constructing the imprecise and possibilistic components of the input parameters, where the three data values for each DP represent the following left-extreme value for membership equal to zero, peak value for membership of unity, and right-extreme value with zero membership. Table 2 lists the values used for the design "constants." The interpretation and use of these data conforms to the explanations in Sections 2 and 3.

4.4. Output performance parameters

Using the performance expressions [equations (1)–(15)] along with the design parameter data given in the previous section, the FWA procedure was applied to obtain the imprecise and possibilistic performance outputs. For designs where probability data is present, it would be combined to produce an extended hybrid representation for the performance parameters of the design. Figures 3–12 show the performance outputs for the spur gear, helical gear and V-belt alternatives.

4.4.1. Spur gear output performance parameters. The spur gear output results contained in Figs 3–7 will be compared to the performance criteria: n_f^r, n_s^r, d_s^r, and C^r. Figure 3 shows the imprecise and uncertainty output for the fatigue strength factor of safety [equation (1)]. The imprecision curve is denoted by $\tilde{n}_f^s$. Checking the output at the peak of $\tilde{n}_f^s$ (membership of unity, $\alpha = 1$), we find that $\tilde{n}^s_{f,(\text{at}\,\alpha=1)}$ equals 1.61. This output safety factor does not satisfy the performance specified for the design, $n_f^r \geqslant 2.0$. To reach this required value the input parameters must deviate from their peak values. To achieve the minimum desired performance at least one design parameter must decrease in membership of approx. 0.2, i.e. $\tilde{n}^s_{f,(\text{at}\,\alpha=0.8)} = 2.15$. Only imprecision has been considered to arrive at this conclusion. If we also take into consideration the uncertainty effects shown in the figure, the membership of one or more input parameters must decrease by between 0.3 and 0.4 [$\tilde{n}^s_{f,(\text{at}\,\alpha=0.7)} = 2.47$ and $\tilde{n}^s_{f,(\text{at}\,\alpha=1)} = 2.83$] to satisfy the performance specification including both the functional requirement and uncertainty involved. These results demonstrate that the fatigue strength factor of safety may be satisfied by the spur gear configuration, but only with a large change in membership, and therefore also a large change in the choice of DP values. They

Table 1. Machine design example: fuzzy design parameter data

	Imprecision (i)/uncertainty (u)					
DPs (units)	$\alpha_i = 0$	$\alpha_i = 1$	$\alpha_i = 0$	$\alpha_u = 0$	$\alpha_u = 1$	$\alpha_u = 0$
K_a	0.63	0.70	0.85	0.69	0.70	0.71
K_s				0.8	1.0	1.0
K_r	0.702	0.814	0.897	0.780	0.814	0.830
S_t (MPa)	350.0	1065.0	1550.0			
m (mm)	1.2	1.4	1.6			
n_{rpm} (rpm)	1500.0	1750.0	2000.0			
w_F (mm)	6.0	9.5	13.0			
K_t				0.95	1.0	1.0
K_e				1.31	1.33	1.35
K_A^s (m/s)				800.0	1200.0	1300.0
K_A^h (m/s)				60.0	78.0	100.0
K_J				0.34	0.35	0.36
K_o				1.0	1.0	1.4
K_I				1.3	1.3	1.5
K_L	1.0	1.0	1.4	1.0	1.0	1.1
K_R	0.8	0.8	1.2	0.8	0.8	0.9
K_{elas} (k$\sqrt{\text{Pa}}$)				187.0	191.0	191.0
H_B	100.0	310.0	460.0			
K_T				0.95	1.0	1.0
K_f	1.0	1.1	1.3	1.05	1.1	1.15
L_S (mm)	125.0	150.0	205.0			
S_f (MPa)	175.0	350.0	1050			
K_K				1.0	1.2	1.3
K_{sc}				1.4	1.6	1.8
K_{df}				1.2	1.5	1.7
y_s (mm)	1.0	2.0	3.0			
L_D (kh)	12.0	16.0	16.0			
R	0.95	0.99	0.99			
K_{sh}				1.4	1.5	1.6
K_p				1.0	1.0	1.1
K_{os}				0.95	1.0	1.05
T_r	3.3	3.6	3.8			
d_p (mm)	75.0	85.0	125.0			
K_c	0.1	0.6	2.0	0.4	0.6	0.9
K_b				125.0	175.0	225.0
N_T (Mpasses)	0.5	123.5	1400.0	75.0	123.5	175
K_{sf}				1.0	1.2	1.3

also show that care must be taken when adjusting PPs which are coupled, because adjusting one DP may affect more than one PP.

A similar situation occurs in the case of the output surface durability safety factor for the spur gear, as shown in Fig. 4. Once again, the performance criterion n_s^r is not satisfied at the peak (nominal value), $\tilde{n}^s_{s,(\text{at } \alpha=1)} = 0.79$. Considering output values to the right of the peak and taking into account the represented uncertainty, satisfactory performance will be achieved at an output between $\tilde{n}^s_{s,(\text{at } \alpha=0.8)} = 1.26$ and $\tilde{n}^s_{s,(\text{at } \alpha=0.7)} = 1.63$. Of course, this means that a corresponding decrease in membership of the design parameters is required; however, the decrease in not as drastic when

Table 2. Machine design example: "constant" data

"Constant" (units)	Value
N	18
P(kW)	1.5
B	3.2809
C_b	4.4482
C_c	0.00328
ϕ (deg)	20.0
ψ (deg)	15.0
ϕ_t (deg)	20.65
ϕ_n (deg)	20.0
E (GPa)	207.0
L_R (kh)	3.0
n_r (rpm)	500.0
m_G	5

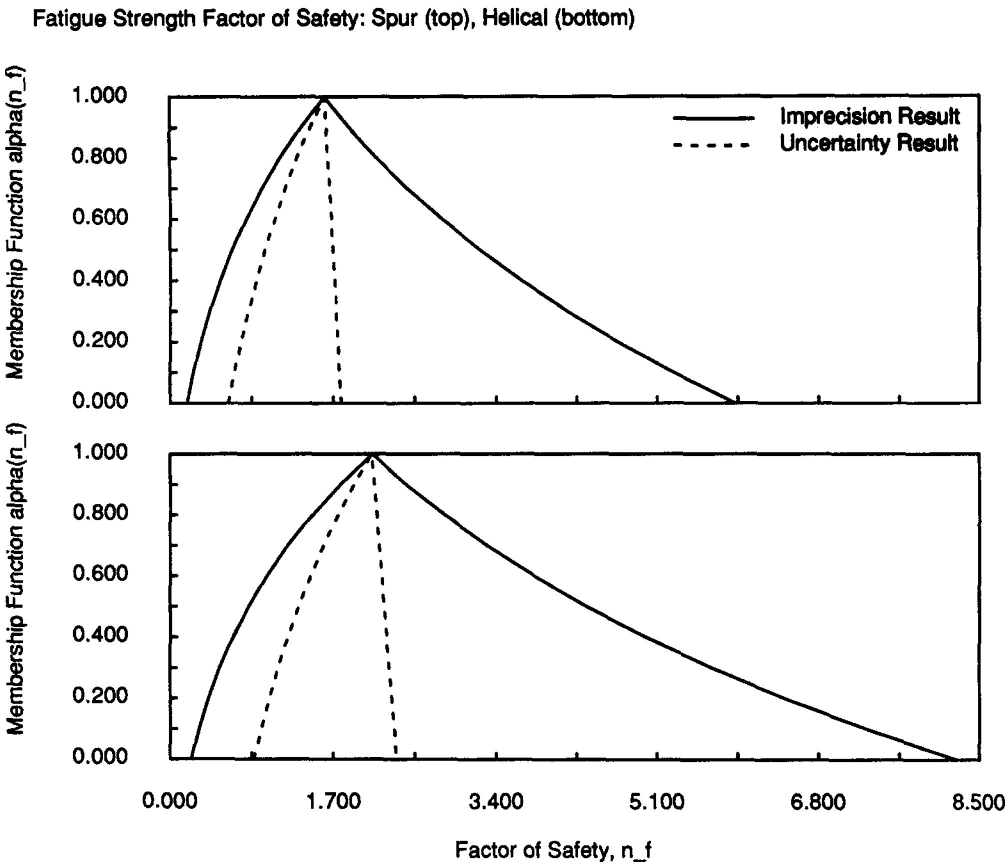

Fig. 3. Spur and helical gear: fatigue strength factor of safety n_f.

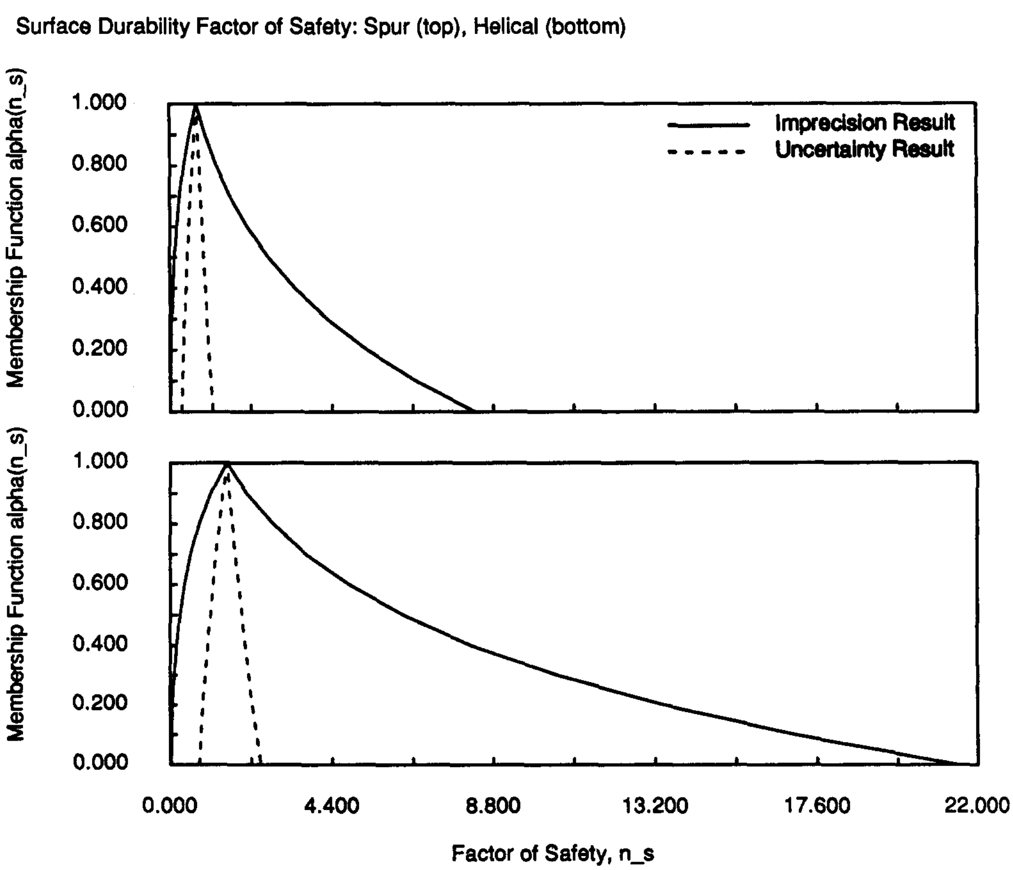

Fig. 4. Spur and helical gear: surface durability factor of safety n_s.

compared to $\tilde{n}_f^s$ due to the nearness of the peak output to the functional requirement, and due to the greater imprecision of the right portion of the curve ($\tilde{n}_s^s$ changes faster for a given change in membership than $\tilde{n}_f^s$). Even though $\tilde{n}_s^s$ is closer to meeting its FR than $\tilde{n}_f^s$, it may still be influenced by DP changes made to adjust other coupled PPs.

The remaining performance output performance results for the shaft diameter calculations and rated bearing load are shown in Figs 5–7. These will be discussed later, when the design alternatives are compared.

4.4.2. Helical gear output performance parameters. The imprecise and uncertainty output results for the helical gear configuration may be found in Figs 3–7. For the case of the fatigue strength factor of safety, Fig. 3 shows the output set as determined from equation (2). The peak of the imprecision curve corresponds to: $\tilde{n}^h_{f,(\text{at } \alpha=1)} = 2.1$. When compared with the requirement $n_f^r \geqslant 2$, the factor of safety is satisfactory. Considering the additional uncertainty effect, the peak output does not meet n_f^r. To satisfy the required performance, an output membership value between 0.8 and 0.9 must be used ($\tilde{n}^h_{f,(\text{at } \alpha=0.9)} = 2.46$ and $\tilde{n}^h_{f,(\text{at } \alpha=0.8)} = 2.85$). Combining this result with the fact that the output curve for $\tilde{n}_f^h$ has imprecision on the order of the difference between n_f^r and the output peak, special care must be taken when adjusting other PPs which are coupled to n_f^h. Despite these potential difficulties the performance is nearly satisfied with the nominal (peak) value for the helical gear fatigue strength safety factor.

Using Fig. 4, the helical gear configuration performs satisfactorily for the surface durability factor of safety, $\tilde{n}^h_{s,(\text{at } \alpha=1)} = 1.53$. Even with regard to uncertainty considerations the nominal output meets the functional requirement n_s^r. The only concern involved with $\tilde{n}_s^h$ is the coupling with other performance parameters. Because of the relatively small imprecision on the left-hand side of $\tilde{n}_s^h$, small changes of a design parameter's membership to the left will have only a small influence on performance (this may be verified by the *backward path* FWA implementation as described in Section 2). Thus, even though there exist combinations of input parameters which do not satisfy

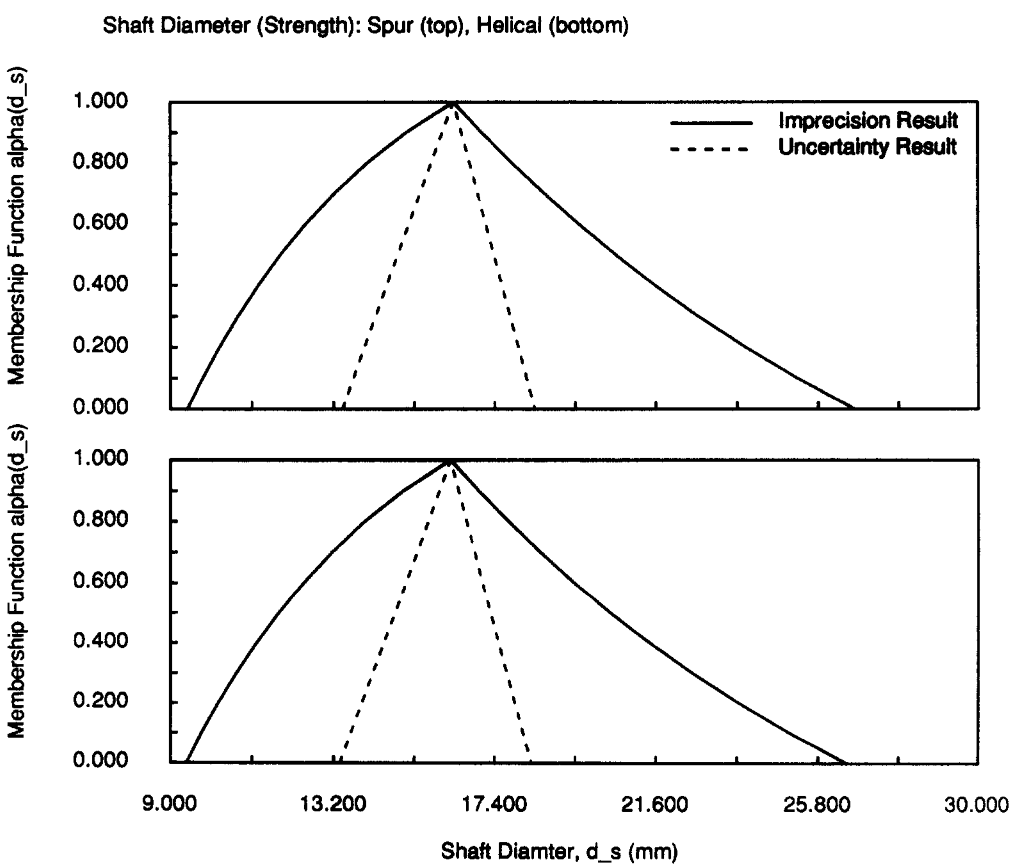

Fig. 5. Spur and helical gear: shaft diameter (strength) d_s.

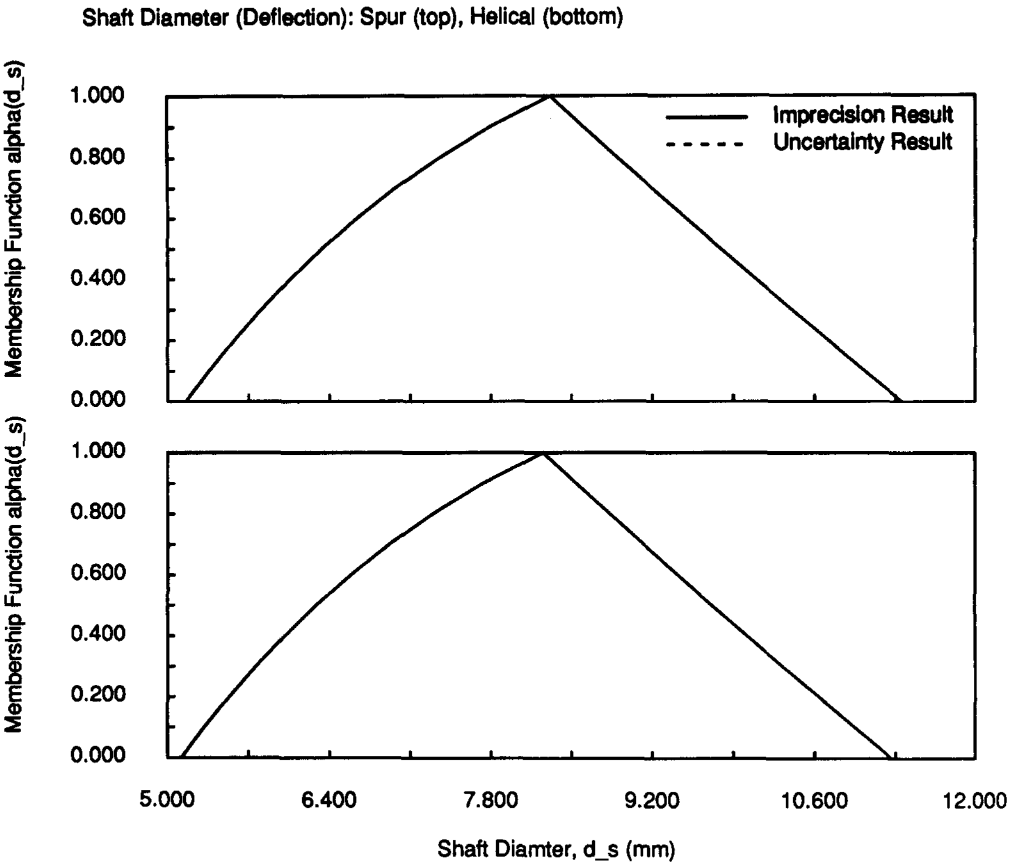

Fig. 6. Spur and helical gear: shaft diameter (deflection) d_s.

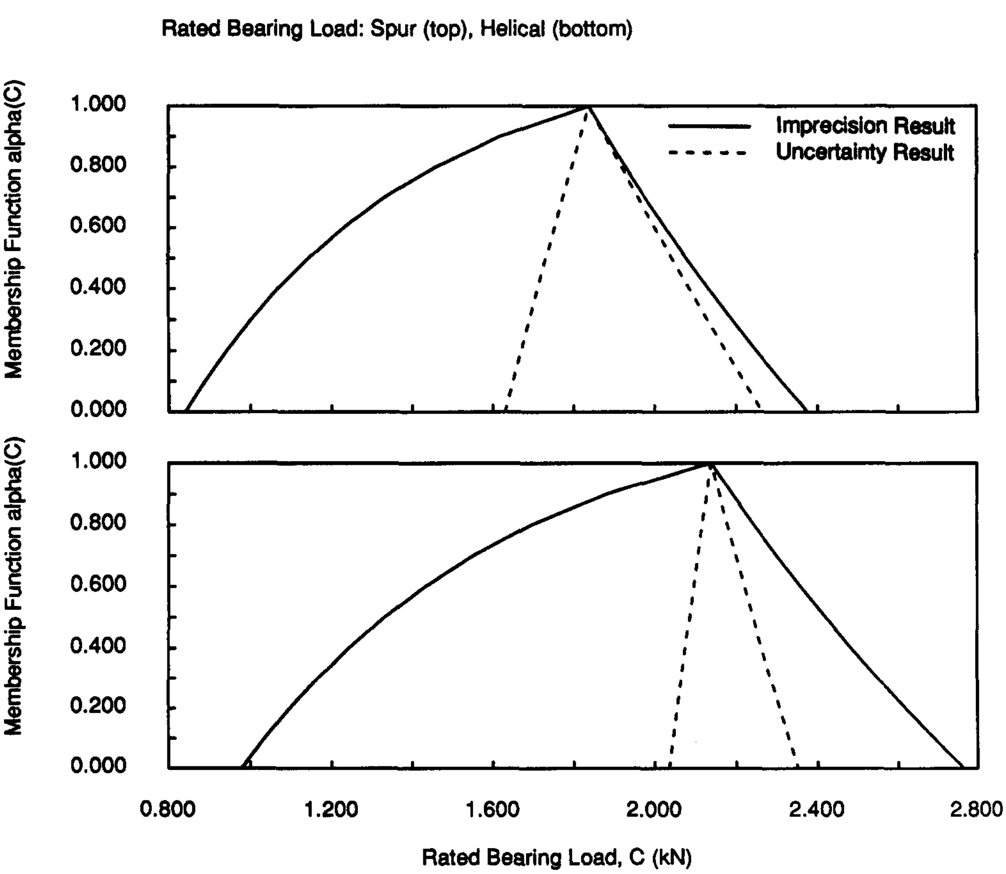

Fig. 7. Spur and helical gear: rated bearing load C.

the performance requirements, n_s^h is a performance parameter which is closer to its FR than n_f^h or n_s^s, and is not influenced as strongly by other coupled PPs as shown by the small left-hand side imprecision.

4.4.3. V-belt output performance parameters. Figures 8–12 list the output performance curves for the *V*-belt configuration. Equation (11) represents the performance parameter for belt peak force. No performance is specified for peak force in this design. Instead, the performance focus is on the expected life of the belt, as represented in PPE form in equation (12). Because the N_T design parameter implicitly depends on F_p which in turn depends on other design parameters in the problem, a method is required for determining L_e given the imprecise output for belt peak force. The approach used in this design is to calculate the imprecise and uncertain peak force performance for both pulleys, as shown in Fig. 8. Using design tables and these results, the input parameter N_T may be constructed (an approximation used for N_T may be found in Table 1) and used in equation (12). The result is the output for expected life as given in Fig. 9.

Analyzing Fig. 9, the peak output value is: $\tilde{L}_{e,(\text{at } \alpha=1)} = 8.3$ kh. Comparing this result with $L_e^r \geqslant 16.0$ kh, the nominal design does not satisfy the specified performance, by a factor of two. However, the imprecision of the output is on the order of the difference between L_e^r and $\tilde{L}_{e,(\text{at } \alpha=1)}$, implying that only a small change in input parameter membership is required. In the context of the uncertainty shown in the figure, satisfactory performance is obtained between $\alpha_{\tilde{L}_e} = 0.8$ and $\alpha_{\tilde{L}_e} = 0.9$, i.e. $\tilde{L}_{e,(\text{at } \alpha=0.8)} = 26.0$ kh and $\tilde{L}_{e,(\text{at } \alpha=0.9)} = 17.0$ kh.

4.5. Applying the γ-level measure

The γ-level measure may be applied to the performance parameter outputs using the procedure described in Ref. [1]. The results provide the designer with qualitative information concerning the importance and coupling of the input parameters for this transmission design. Tables 3–5 list normalized γ-level measures for the three alternatives under consideration.

When a design parameter has the greatest qualitative importance for a given performance parameter, the numerical measure produces a normalized value of one (1). As the measure

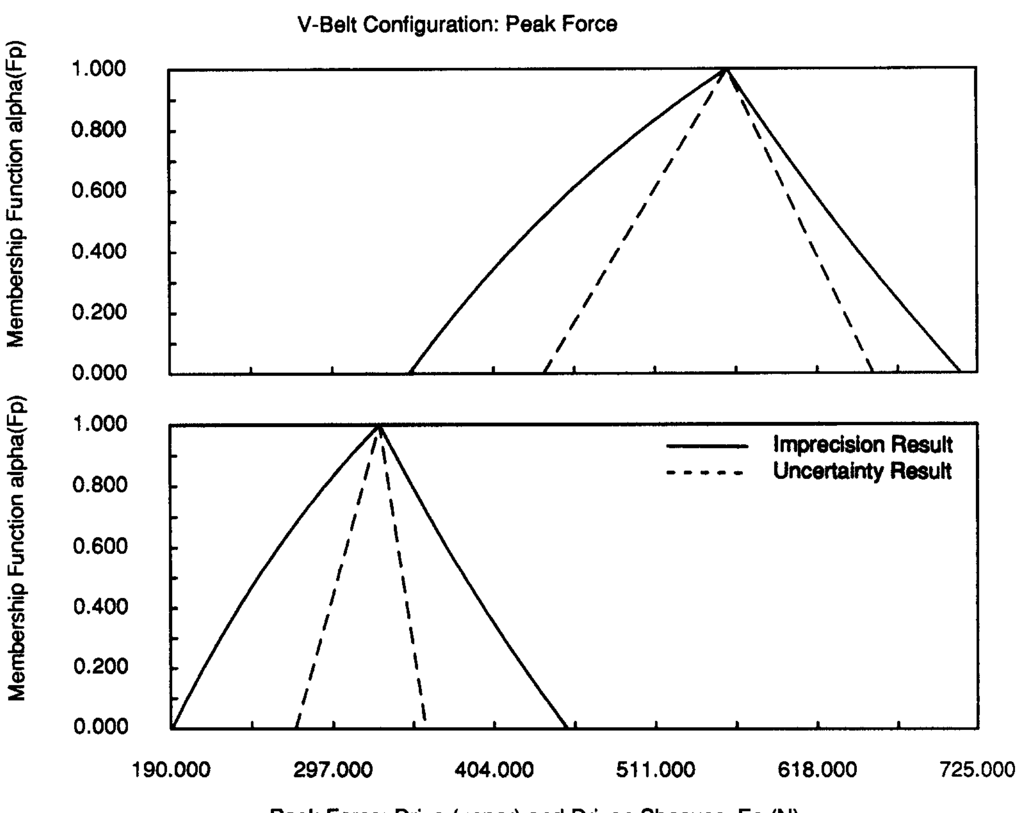

Fig. 8. *V*-Belt: peak force F_p.

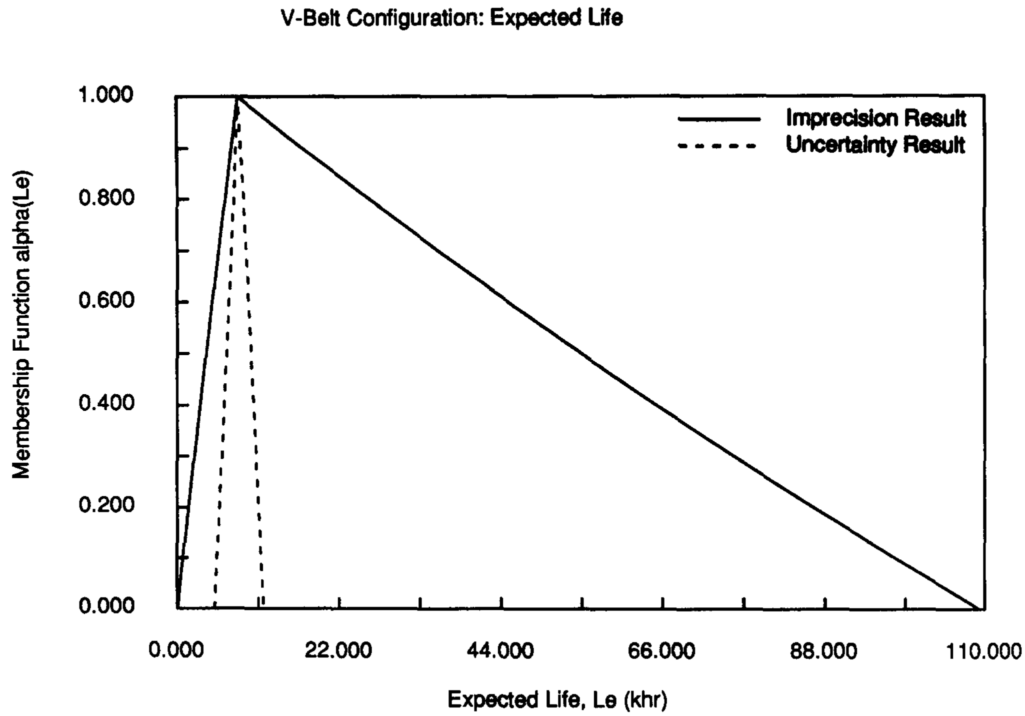

Fig. 9. V-belt: expected belt life L_e.

decreases in value, the corresponding input has little effect in determining the performance parameter, meaning that even a large change in the design parameter (decrease of membership) produces a small change in output. The output is loosely analogous to sensitivity, but applies to imprecise parameters, and represents the entire range of the parameters, not a single operating point. Moreover, this sensitivity is weighted by the designer's desires, as identified in the input parameters fuzzy set membership functions. The details of this weighting are discussed in Ref. [1].

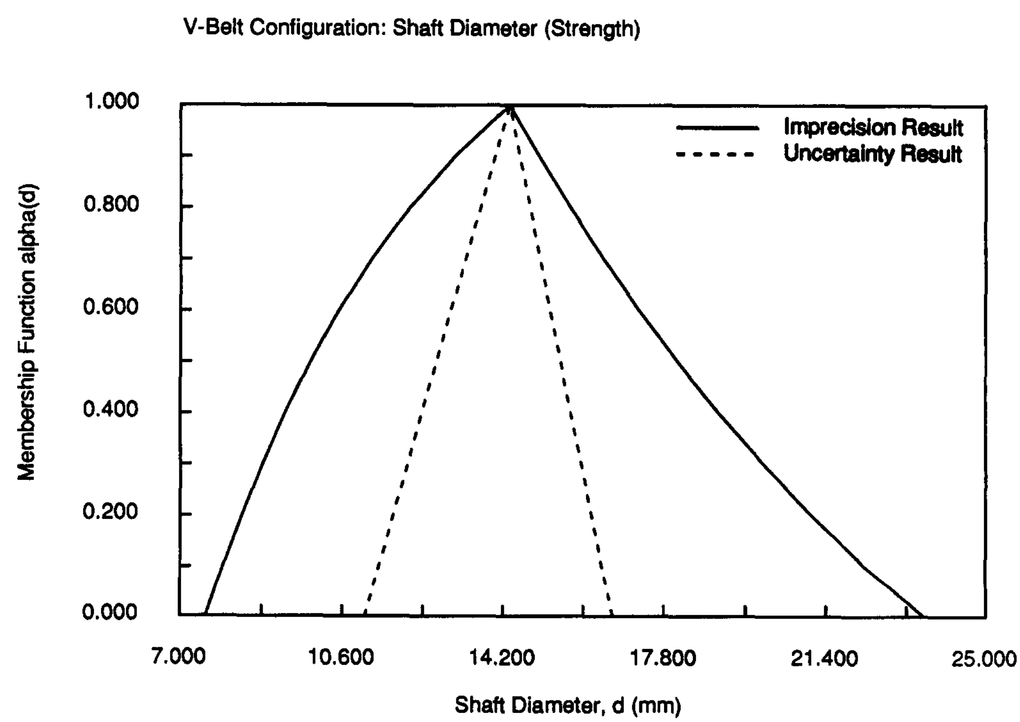

Fig. 10. V-belt: shaft diameter (strength) d_s.

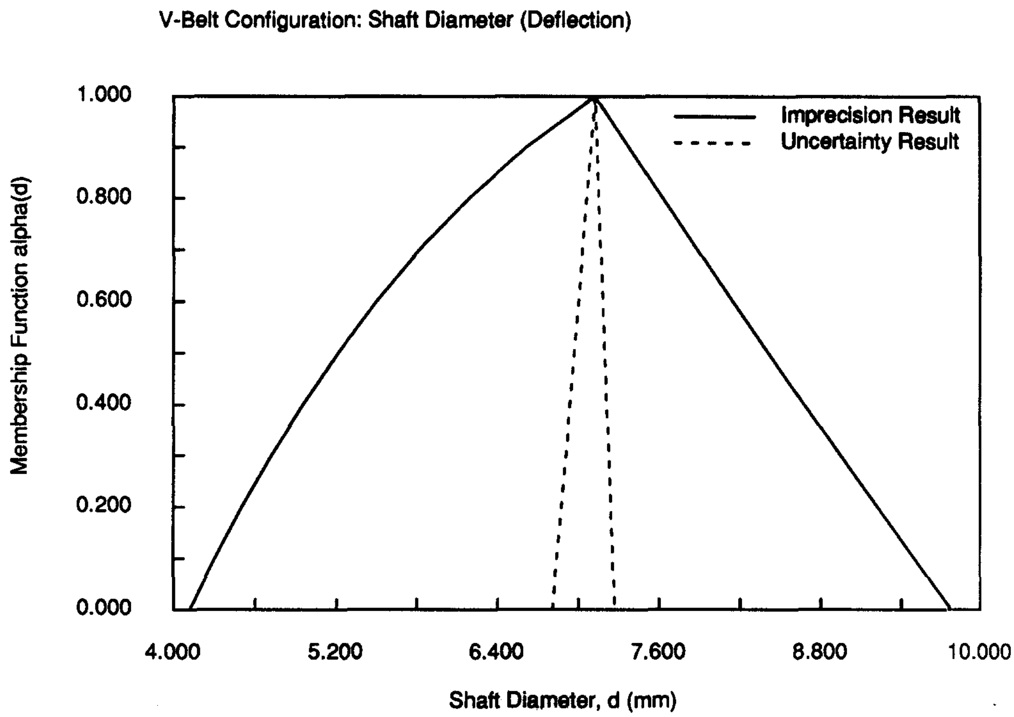

Fig. 11. V-belt: shaft diameter (deflection) d_s.

The γ-level measure may be used to determine importance of inputs simply by comparing the normalized measure of one input relative to the others. This information suggests that parameters with small γ-level measures may be fixed, as changes in those parameters will have only a small affect on performance. Coupling information is also obtained. If a design parameter has a high measure with respect to one PP but a very small measure with respect to another, the performance parameters in question may be viewed as uncoupled with respect to the design parameter. Further,

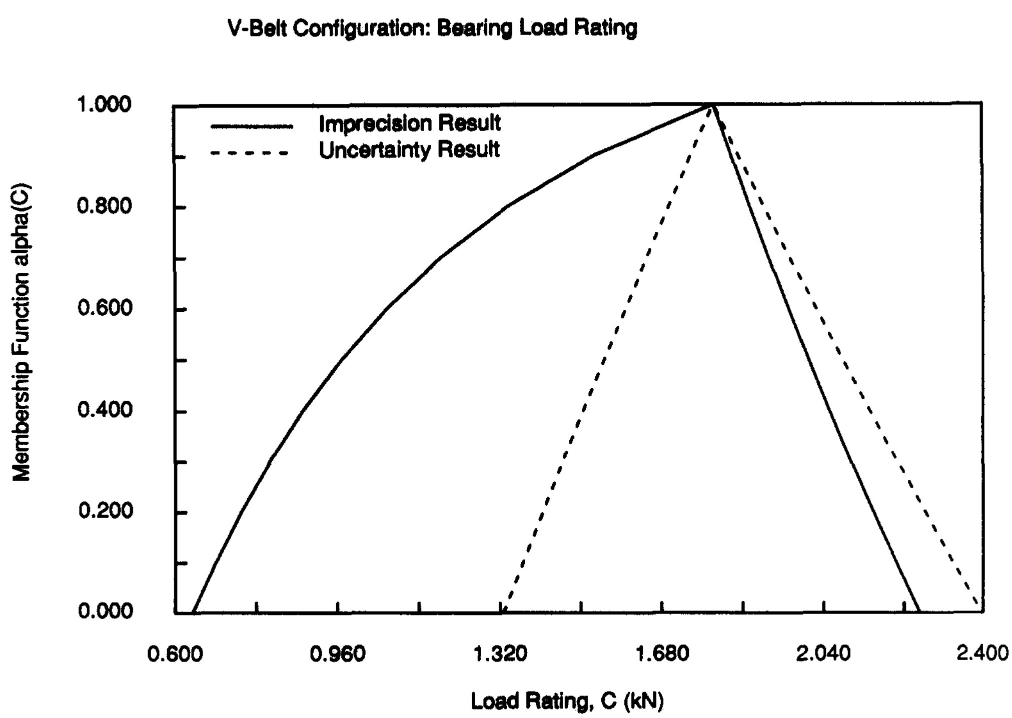

Fig. 12. V-belt: rated bearing load C.

Table 3. γ-level measure results for the spur gear (imprecision)

DPs	Performance parameters				
	n_f	n_s	d_s (str)	d_s (def)	C
m	0.44	0.21	0.16	0.18	0.55
n_{rpm}	0.18	0.09	0.16	0.18	0.37
w_F	0.65	0.31	0.0	0.0	0.0
S_t	1.0	0.0	0.0	0.0	0.0
S_f	0.0	0.0	1.0	0.0	0.0
K_a	0.28	0.0	0.0	0.0	0.0
K_r	0.21	0.0	0.0	0.0	0.0
K_{elas}	0.0	0.13	0.0	0.0	0.0
K_L	0.0	0.38	0.0	0.0	0.0
K_R	0.0	0.29	0.0	0.0	0.0
H_B	0.0	1.0	0.0	0.0	0.0
K_f	0.0	0.0	0.15	0.0	0.0
L_s	0.0	0.0	0.28	1.0	0.0
y_s	0.0	0.0	0.0	0.77	0.0
R	0.0	0.0	0.0	0.0	1.0
L_D	0.0	0.0	0.0	0.0	0.17

in terms of possibilistic uncertainty, the γ-level measure can also provide an indication of which parameters contribute the greatest to the uncertainty of the problem.

4.5.1. Spur gear γ-level measure. Table 3 categorizes the γ-level results for the spur gear configuration. Two important input parameters for a spur gear reduction unit are the speed n_{rpm} and the module m. Analyzing the γ-level measures shows that n_{rpm} contributes very little to any of the performance parameters. This implies that the speed of the motor may be fixed with respect to the spur gear design without affecting performance. While the module design parameter also contributes very little to the shaft diameter results, the γ-level measure shows that m may not be fixed relative to the factors of safety, n_f and n_s, and the bearing load rating, C. As a result, we may conclude that n_f, n_s and C are coupled with respect to m, but uncoupled with respect to d_s.

4.5.2. Helical gear γ-level measure. The γ-level measures for the helical gear alternative are shown in Table 4. Similar results occur for the speed n_{rpm} and the module m as found for the spur gear. Additional information which may be inferred from the γ-level results concerns the shaft diameter performance specification, d_s^r. The relevant design parameters for shaft diameter [strength calculation, equation (7)] include: S_f, L_s, n_{rpm}, m and f. From Table 4, the material property, S_f, obviously dominates the output performance, with secondary effects from L_s. This tells us that when attempting to satisfy d_s^r, minimization efforts should be first (and foremost) placed on choosing a material property just to the right of the $d_{s,str}$ peak value (Fig. 5), with subsequent effort placed in changing L_s.

Table 4. γ-level measure results for the Helical gear (imprecision)

DPs	Performance parameters				
	n_f	n_s	d_s (str)	d_s (*def*)	C
m	0.49	0.23	0.15	0.18	0.55
n_{rpm}	0.24	0.11	0.16	0.18	0.37
w_F	0.65	0.31	0.0	0.0	0.0
S_t	1.0	0.0	0.0	0.0	0.0
S_f	0.0	0.0	1.0	0.0	0.0
K_a	0.28	0.0	0.0	0.0	0.0
K_r	0.21	0.0	0.0	0.0	0.0
K_{elas}	0.0	0.13	0.0	0.0	0.0
K_L	0.0	0.49	0.0	0.0	0.0
K_R	0.0	0.29	0.0	0.0	0.0
H_B	0.0	1.0	0.0	0.0	0.0
K_f	0.0	0.0	0.15	0.0	0.0
L_s	0.0	0.0	0.28	1.0	0.0
y_s	0.0	0.0	0.0	0.77	0.0
R	0.0	0.0	0.0	0.0	1.0
L_D	0.0	0.0	0.0	0.0	0.17

Table 5. γ-level measure results for V-belt (imprecision)

DPs	Performance parameters				
	F_p (sm pull)	L_e	d_s (str)	d_s (def)	C
d_p	1.0	0.001	0.29	0.33	0.95
n_{rpm}	0.33	0.03	0.16	0.18	0.37
T_r	0.07	0.0	0.04	0.05	0.0
S_f	0.0	0.0	1.0	0.0	0.0
K_c	0.07	0.0	0.0	0.0	0.0
K_f	0.0	0.0	0.15	0.0	0.0
L_s	0.0	0.0	0.28	1.0	0.0
y_s	0.0	0.0	0.0	0.78	0.0
N	0.0	1.0	0.0	0.0	0.0
R	0.0	0.0	0.0	0.0	1.0
L_D	0.0	0.0	0.0	0.0	0.17

4.5.3. V-belt γ-level measure. Table 5 lists the γ-level measures for the V-belt configuration. A preliminary analysis of the table shows that the pulley diameter and speed contribute very little to belt life performance L_e, with N_t dominating the imprecision. However, because N_T implicitly depends on F_p, the peak force design parameters will bias the imprecision of expected belt life to the greatest extent. Thus, considering the γ-level results for F_p, we find that the most important parameters are indeed pulley diameter (a γ-level measure of 1.0) and speed (0.334).

Another interesting result with regard to the peak force is the contribution of the centrifugal force factor, K_c. When the equation for belt peak force was derived, all force contributions were considered, including tension force, bending force and centrifugal force. Of course, for the given belt tensions and bending, the centrifugal component will contribute very little. The γ-level measure verifies this result.

Finally, as with the helical gear, the failure stress design parameter plays the most significant role in terms of imprecision of the shaft diameter output. In order to meet the performance criteria on shaft diameter, efforts should once again be focused on S_f, with secondary considerations of shaft length L_s and pulley diameter d_p as these parameters have the largest γ-level measures.

4.6. Discussion

This example has demonstrated the evaluation of the imprecise and uncertain output performance with respect to individual functional requirements. γ-level measure results have also provided information concerning the relative importance of certain design parameters of the problem. Major differences of the alternative configurations may now be determined. When comparing the spur gear and helical gear alternatives, both configurations do not satisfy the nominal fatigue strength performance requirement n_f^r, especially when uncertainty is included. Even though both the spur and helical gears only require a small deviation in input membership from the most desired, the helical configuration slightly out-performs the spur gear configuration. Figure 3 illustrates this higher performance due to the closer proximity of the peak of n_f^h to n_f^r, and due to the slightly higher imprecision of $\tilde{n}_f^h$ to the right of the peak.

More drastic differences occur when comparing n_s^h and n_s^s. The helical system satisfies the performance specification without change of input parameters, whereas a significant change, by comparison, is required for the spur alternative. The order of the imprecision is also significantly different. Figure 4 shows that a given change in membership to the right of the peak will produce a change in output of more than a factor of two for n_s^h compared to n_s^s. We may conclude from this that the helical gear alternative is less sensitive to variance in design parameters, especially in terms of coupling with any other performance parameters.

The performance criteria for the shaft diameter and rated bearing load may be compared directly from Figures 5–7. Both shaft diameter calculations [equations (5)–(8)] were carried out in order to determine the minimum requirements for satisfying both strength and deflection considerations. Because the strength calculation produces higher shaft diameters, Fig. 5 will be used as a basis for comparison. Considering Fig. 5, the output performance curves are essentially identical for both the spur and helical configurations. This implies that the requirement d_s^r will not be satisfied to any greater extent by choosing one gear system over the other. The γ-level measure suggests that efforts should be placed in varying the material property S_f and subsequently L_s in order to decrease diameter.

Figure 7 shows the performance outputs for the rated bearing load. In this case, the spur gear out-performs the helical system with respect to the nominal output of the imprecision gear. Notice however that greater uncertainty exists for the spur gear output. Considering the greatest uncertainty to the right of the peak for both curves, the spur gear rated bearing load is less than the helical system, but not significantly less.

At this point, both gear configurations may be compared directly with the V-belt drive. However, the direct comparison of the gear systems showed that the *only*advantage of the spur gear in terms of the functional requirements is the smaller rated bearing load. Because the difference in bearing load was not significant, and because the helical system out-performed the spur gear alternative in terms of fatigue strength and surface durability, only the helical gear configuration will be compared with the V-belt.

Using Fig. 9 and the results discussed earlier, the nominal V-belt alternative does not satisfy the expected life requirement nominally, but requires a change in design parameter membership. Although the expected life of the helical gear is not determined by a performance expression, the fatigue strength and surface durability implicitly depend upon life through the factors involved in the equations. Comparing L_e to n_f^h and n_s^h in this sense, we find that the helical gear requires similar changes of design parameters to achieve the specified performance.

Similar results occur for the shaft diameter and rated bearing load. Even though the V-belt alternative has lower nominal values at the peak of the imprecision curve for both cases, the addition of uncertainty considerations shows little difference between the V-belt and helical system. Thus, very little advantage exists for choosing one configuration over the other to minimize either shaft diameter or rated bearing load. Because of these results, a cost function might be devised for the next design stage, in which material cost or volume might be used as a measure.

5. CONCLUSIONS

This power transmission design example demonstrates an application of our semi-automated approach to representing and manipulating both imprecision and uncertainty in preliminary design. The designer is able to represent preliminary descriptions of design alternatives, even when they are very imprecisely described. In the design example shown above, none of the three alternatives was precisely described, yet conclusions could be drawn regarding the performance of each in this application, subject to the requirements specified here. For example, we found that the helical gear drive is less sensitive to changes in design parameters, and appears to meet the functional requirements more easily than the spur gear configuration. Nearly the same performance was obtained for the helical gear and V-belt alternatives, when compared to the functional requirements used here.

Further, the introduction of uncertain data (to complement the imprecise data in the design process) contributes additional information on the performance of each design alternative. The uncertain data is comprised of two distinct components: a probabilistic (objectively measured) part, and a possibilistic (subjective) portion. These three data are combined by use of extended hybrid numbers, which provide a consistent representation for input design parameters, and evaluation of results. In this example, the possibilistic data played an important role in indicating how the imprecise performance results might change over the possible range of input parameters.

The γ-level measure also helps in the process of determining the relative importance of design parameters. Input parameters are seen to have a profound effect on performance parameters in some cases, and small in others, indicated by large or small (respectively) γ-level measures. This is an indication of the coupling between the DPs and the PPs. In some cases, parameters with small γ-level measures can be fixed at a value of the designer's choosing, and removed from further design consideration, thus reducing the number of parameters that need to be considered. In the design example used here the γ-level measure showed that the motor speed, in the case of the gears, affects the performance of the transmission very little.

Imprecision and uncertainty data both play an important role in engineering design. The approach described and demonstrated here comprises of a method for representing and manipulating both aspects simultaneously. We believe that this method will provide more information to the designer at the preliminary stage than is available using conventional design tools.

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MODELISATION DES IMPRECISIONS ET DES INCERTITUDES DES ETUDES TECHNIQUES PRELIMINAIRES

Résumé—Chaque étape du procédè de conception technologique, et en particulier la phase préliminaire, comprend à la fois des imprécisions et des incertitudes. Cet article présente une méthode par laquelle il est possible de représenter ces imperfections et incertitudes dans la description de chaque composant conçu. La théorie des ensembles flous (fuzzy sets) est à la base de cette approche. Les nombres hybrides étendus (Extended Hybrid Numbers) sont introduits pour pouvoir étudier séparément les deux représentations de l'imprécision et de l'incertitude. Ces représentations incluent le jugement de l'ingénieur. Une application de cette théorie à la conception d'une machine est présentée, en soulignant les imprécisions les plus souvent retrouvées pendant l'etude préliminaire de chaque pièce de la machine. Les résultats montrent la performance de chacune des machines conçues possible par rapport à leurs besoins fonctionnels, aussi bien que le lien entre les paramètres de ces machines et les paramètres de leurs performances respectives.