Partial Third-Party Information Exchange with Network Coding

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Abstract—In this paper, we consider the problem of exchanging channel state information in a wireless network such that a subset of the clients can obtain the complete channel state information of all the links in the network. We first derive the minimum number of required transmissions for such partial third-party information exchange problem. We then design an optimal transmission scheme by determining the number of packets that each client should send, and designing a deterministic encoding strategy such that the subset of clients can acquire complete channel state information of the network with minimal number of transmissions. Numerical results show that network coding can efficiently reduce the number of transmissions, even with only pairwise encoding.

Index Terms—Network coding, cooperative data exchange, channel state information.

I. INTRODUCTION

TO increase throughput and network efficiency, it is always beneficial for clients in a wireless network to have a certain level of global knowledge of the network, for example link loss probability that is related to the network connectivity, or channel state information (CSI) that is related to the channel quality. Generally, such information, e.g. link loss probability and CSI, on a link (i,j) is regarded as a local and common information between two connected nodes i and j, and it is unknown to a *third-party* node, e.g., the node $k \neq i,j$. Thus, the problem of letting third-party nodes to know the information that is local to the other nodes is an important problem for network design [1], [2].

Recently, cooperative data exchange [3]–[6] with network coding [7], [8] has become a promising approach for efficient data communication. In cooperative data exchange, the clients in a network exchange the packets via a lossless common/broadcast channel. Inspired by cooperative data exchange, the works in [1] [2] proposed a network coding based *third-party information exchange* where the clients exchange their local CSI through a lossless common/broadcast channel such that each client finally gets the complete CSI of the whole network. Specifically, the work in [1] aims to minimize the total number of transmissions, while the work in [2] tries to minimize the total transmission cost required to complete the information exchange. However, both of these works assume that all the clients in the network need to get the complete CSI. In a practical system, there could be only a subset of the clients, e.g. only the one with information to transmit or receive, need to get the complete CSI at the same time in most cases.

Hence, different from the previous works [1] [2], we study a practical scenario where only a subset of the clients in the network need to acquire the complete CSI in this paper. To simplify the presentation, such third-party information exchange for only a subset of clients is denoted as *partial third-party information exchange*. We aim to propose a network coding based solution to minimize the total number of transmissions required during partial third-party information exchange. The main contributions of this paper can be summarized as follows:

- We derive the minimum number of transmissions for such partial third-party information exchange.
- We propose an optimal transmission scheme, which determines the number of packets that each client should send so as to achieve the minimal number of transmissions.
- We design a deterministic encoding strategy to make sure that with the proposed transmission scheme, the subset of clients that require the complete CSI can successfully decode/obtain the full information.
- Numerical results show that the proposed transmission scheme and encoding strategy can efficiently reduce the number of transmissions.

II. PROBLEM DESCRIPTION

Consider a network with N clients in $C = \{c_1, c_2, \cdots, c_N\}$. Let $x_{i,j}$ represent the link loss probability or CSI of the link between clients c_i and c_j, where each client c_i only knows the local information initially, i.e., client c_i only holds the CSI in $X_i = \{x_{i,j} | \forall j \in \{1, 2, \cdots, N\} \setminus \{i\}\}$. We assume that the links are symmetric, i.e., $x_{i,j} = x_{j,i}$ for $\forall i,j$, so for every two clients c_i and c_j, they hold one common CSI $x_{i,j}$. Thus, the set of all the CSI in the network is $X = \{x_{i,j} | 1 \leq i,j \leq N, i \neq j\}$, and the total number of the packets is $|X| = \frac{N(N-1)}{2}$.

Instead of letting all the clients get the complete CSI [1] [2], in this paper, we consider the case that only a subset of clients, $C' \subseteq C$, need to get the complete CSI in X. Without loss of generality, we assume that the first k clients in C want all the CSI in X, i.e. $C' = \{c_1, c_2, \cdots, c_k\}$, $k \leq N$. We also use $\overline{X_i}$ to denote the set of "wanted" packets by client $c_i \in C'$, i.e., $\overline{X_i} = X \setminus X_i \subseteq X$.

As in [1] [2], there is a lossless common/broadcast channel for clients to exchange information. Let y_i be the number of packets that client c_i should send. Then, the total number of transmissions sent by all the clients in C (notice that although only a subset of clients C' requires the full information, all clients in C should participate in sharing their information) can thus be written as

$$Y = \sum_{i=1}^{N} y_i \quad (1)$$

Recent works [1]–[6] show that network coding can efficiently save the number of transmissions for data exchange

problem. Thus, after determining the number of transmissions that each client should send, we design an encoding strategy based on network coding [7], where a linear encoded packet will be generated based on the packets that the sender initially has over a finite field.

In the following sections, we will derive the minimum number of transmissions required for partial third-party information exchange in Section III. Then, in Section IV, we propose an optimal transmission scheme, which can achieve the minimum number of transmissions. Based on the proposed transmission scheme, in Section V, we design a deterministic encoding strategy to make sure that each client in C' can successfully decode/obtain the complete CSI. We compare the performance in Section VI.

III. THE MINIMUM NUMBER OF TRANSMISSIONS

In this section, we theoretically derive the minimum number of transmissions for the partial third-party information exchange problem.

We use Y_{min} to denote the minimum number of transmissions required for the partial third-party information exchange problem. We have the following lemma.

Lemma 1 *The minimum number of transmissions required for the partial third-party information exchange problem is lower bounded as*

$$Y_{min} \geq \frac{(N-1)(N-2)}{2} + \lceil \frac{k-2}{2} \rceil \qquad (2)$$

Proof: Since the number of packets required by each client $c_i \in C'$ is $|\overline{X_i}| = \frac{(N-1)(N-2)}{2}$, the number of packets received by client c_i should be at least $\frac{(N-1)(N-2)}{2}$, otherwise, c_i cannot get the complete information. In other words,

$$\sum_{i' \in \{1,\cdots,N\} \setminus \{i\}} y_{i'} \geq \frac{(N-1)(N-2)}{2} \qquad (3)$$

By considering all the clients in C', we have

$$\sum_{i=1}^{k} \sum_{i' \in \{1,\cdots,N\} \setminus \{i\}} y_{i'} \geq \frac{(N-1)(N-2)k}{2}$$

$$k \sum_{i'=1}^{N} y_{i'} - \sum_{i'=1}^{k} y_{i'} \geq \frac{(N-1)(N-2)k}{2} \qquad (4)$$

That is

$$\sum_{i'=1}^{N} y_{i'} \geq \frac{(N-1)(N-2)}{2} + \frac{\sum_{i'=1}^{k} y_{i'}}{k} \qquad (5)$$

According to [2], for each client $c_i \in C'$, the packets received from other $k-1$ clients in $C' \setminus \{c_i\}$ should satisfy

$$\sum_{c_{i'} \in C' \setminus \{c_i\}} y_{i'} \geq \binom{k-1}{2} \qquad (6)$$

By considering all the clients in C', we have

$$\sum_{i=1}^{k} \sum_{i' \in \{1,\cdots,k\} \setminus \{i\}} y_{i'} \geq \frac{k(k-1)(k-2)}{2}$$

$$(k-1) \sum_{i'=1}^{k} y_{i'} \geq \frac{k(k-1)(k-2)}{2} \qquad (7)$$

That is

$$\sum_{i'=1}^{k} y_{i'} \geq \lceil \frac{k(k-2)}{2} \rceil \qquad (8)$$

According to Eq. (5) and Eq. (8), we can obtain that

$$\sum_{i'=1}^{N} y_{i'} \geq \frac{(N-1)(N-2)}{2} + \lceil \frac{k-2}{2} \rceil \qquad (9)$$

We thus proved Lemma 1.   ∎

We can also get the following Theorem.

Theorem 1 *The optimal number of transmissions required for partial third-party information exchange problem that minimizes the number of transmissions is*

$$Y_{min}^{opt} = \frac{(N-1)(N-2)}{2} + \lceil \frac{k-2}{2} \rceil \qquad (10)$$

Proof: According to Lemma 1, we know that the lower bound of the minimum number of transmissions required is $\frac{(N-1)(N-2)}{2} + \lceil \frac{k-2}{2} \rceil$. Thus, we get the optimal minimum number of transmissions, as denoted in Eq. (10).   ∎

In the following sections, we will show that the above lower bound can be achieved with an optimal transmission scheme and a deterministic encoding strategy.

IV. AN OPTIMAL TRANSMISSION SCHEME

In this section, we first propose a feasible transmission scheme. We then prove that the proposed transmission scheme can achieve the minimum number of transmissions required for partial third-party information exchange problem.

A. A Feasible Transmission Scheme

We first describe the transmission scheme, which determines the number of packets that each client should send.

Definition 1 The transmission scheme: *the number of packets sent by each client is*

$$y_i = \begin{cases} \lceil \frac{k}{2} - 1 \rceil, & \text{if } 1 \leq i < k \\ \frac{k}{2} - 1, & \text{if } i=k \text{ and } k \text{ is even} \\ 0, & \text{if } i=k \text{ and } k \text{ is odd} \\ N+k-i-1, & \text{if } k+1 \leq i \leq N \end{cases} \qquad (11)$$

We can get the following lemma.

Lemma 2 *The transmission scheme defined in Definition 1 is a feasible solution for the partial third-party information exchange problem.*

Proof: We prove the above lemma by only considering the case when k is even. The case when k is odd can be proved in a similar way.

According to [2], there exists a feasible code design to make sure the client $c_i \in C'$ can successfully decode/obtain the complete information, if and only if the total number of packets received from any other l clients in $C \setminus \{c_i\}$ is at least

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TABLE I
THE TRANSMISSION SCHEME AND CODE DESIGN FOR $N = 6, k = 3$

	Initial Information	y_i	Code design
c_1	$x_{1,2}, x_{1,3}, x_{1,4}, x_{1,5}, x_{1,6}$	1	$x_{1,2} \oplus x_{1,3}$
c_2	$x_{1,2}, x_{2,3}, x_{2,4}, x_{2,5}, x_{2,6}$	1	$x_{2,3} \oplus x_{2,1}$
c_3	$x_{1,3}, x_{2,3}, x_{3,4}, x_{3,5}, x_{3,6}$	0	
c_4	$x_{1,4}, x_{2,4}, x_{3,4}, x_{4,5}, x_{4,6}$	4	$x_{1,4} \oplus x_{2,4},\ x_{1,4} \oplus x_{3,4}$ $x_{4,5},\ x_{4,6}$
c_5	$x_{1,5}, x_{2,5}, x_{3,5}, x_{4,5}, x_{5,6}$	3	$x_{1,5} \oplus x_{2,5},\ x_{1,5} \oplus x_{3,5}$ $x_{5,6}$
c_6	$x_{1,6}, x_{2,6}, x_{3,6}, x_{4,6}, x_{5,6}$	2	$x_{1,6} \oplus x_{2,6},\ x_{1,6} \oplus x_{3,6}$

$\binom{l}{2}$. In other words, the feasible solution of the partial third-party information exchange requires that for any $c_i \in C'$,

$$\sum_{t=1}^{l} y_{i_t} \geq \binom{l}{2}, \text{ for } \forall \{c_{i_1}, c_{i_2}, \cdots, c_{i_l}\} \subseteq C \backslash \{c_i\} \quad (12)$$

According to Eq. (11), for $\forall \{c_{i_1}, c_{i_2}, \cdots, c_{i_l}\} \subseteq C \backslash \{c_i\}$, when $l < k$, we have

$$\sum_{t=1}^{l} y_{i_t} \geq (\frac{k}{2} - 1)l \geq \frac{l(l-1)}{2} = \binom{l}{2} \quad (13)$$

When $l \geq k$, we have

$$\sum_{t=1}^{l} y_{i_t} \geq \sum_{j \in C' \backslash \{i\}} y_j + \sum_{j=N-l+k}^{N} y_j$$
$$= \frac{l(l-1)}{2} = \binom{l}{2} \quad (14)$$

We thus prove that the transmission scheme in Definition 1 is a feasible transmission scheme, i.e., there exists a feasible code design to make sure with the above transmission scheme, the clients in C' can obtain their "wanted" packets. ■

B. Performance Analysis

We now prove that the proposed transmission scheme can achieve the minimum number of transmissions for the partial third-party information exchange problem as specified in Theorem 1. To avoid confusion, we use Y_p to denote the number of transmissions required by the proposed transmission scheme. According to Definition 1, we can have the following lemma.

Lemma 3 *The number of transmissions required with the transmission scheme defined in Definition 1 is*

$$Y_p = \frac{(N-1)(N-2)}{2} + \lceil \frac{k-2}{2} \rceil \quad (15)$$

Proof: We analyze the number of transmissions required with the proposed transmission scheme by considering two cases: 1) k is even and 2) k is odd.

Case 1 (k is even): According to Eq. (11), the total number of transmissions sent by all the clients can be expressed as

$$\sum_{i=1}^{k} (\frac{k}{2} - 1) + \sum_{i=k+1}^{N} (N + k - i - 1)$$
$$= \frac{(N-1)(N-2)}{2} + \frac{k-2}{2} \quad (16)$$

Case 2 (k is odd): According to Eq. (11), the total number of transmissions required for this case is

$$\sum_{i=1}^{k-1} \lceil \frac{k}{2} - 1 \rceil + \sum_{i=k+1}^{N} (N + k - i - 1)$$
$$= \frac{(k-1)^2}{2} + \frac{(N-k)(N+k-3)}{2}$$
$$= \frac{(N-1)(N-2)}{2} + \lceil \frac{k-2}{2} \rceil \quad (17)$$

Thus, the number of transmissions required is

$$Y_p = \frac{(N-1)(N-2)}{2} + \lceil \frac{k-2}{2} \rceil$$

■

We then get the following Theorem.

Theorem 2 *The proposed transmission scheme achieves the optimal solution of the partial third-party information exchange problem.*

Proof: As the proposed feasible transmission scheme achieves the minimum number of transmissions in Eq.(10), the derived bound is thus reachable and the proposed transmission scheme achieves the optimal solution. ■

V. A DETERMINISTIC NETWORK CODE DESIGN

The above transmission scheme only gives the number of transmissions to be sent by each client. In this section, we will design a deterministic encoding strategy to decide the encoded packets that each client should send, so as to make sure that with the above transmission scheme, each client in C' can successfully decode/obtain all its "wanted" packets.

Definition 2 *Each client encodes the packets according to the following rules, where the number of packets sent by each client is determined by Definition 1.*

(1) For client c_i, where $1 \leq i \leq k$, the j-th packet sent by c_i is

$$x_{i,i\%k+1} \oplus x_{i,(i+j)\%k+1} \quad (18)$$

(2) For client c_i, where $k < i \leq N$,
- *if $j < k$, the j-th packet sent by c_i is*

$$x_{1,i} \oplus x_{1+j,i} \quad (19)$$

- *if $j \geq k$, the j-th packet sent by c_i is*

$$x_{i,i+j-k+1} \quad (20)$$

Note that the proposed code design is a simple pairwise coding (i.e. by encoding at most two packets only), which can be implemented easily with XOR operation.

Consider an example with 6 clients, where the first 3 clients want to get the complete CSI. According to the above definitions, the transmission scheme and the encoded packets sent by each client are shown in Table I.

We can also prove the following lemma.

Lemma 4 *With the code design in Definition 2 and the transmission scheme in Definition 1, every client in C' can successfully decode and obtain the complete CSI in X.*

TABLE II
THE MINIMUM NUMBER OF TRANSMISSIONS WITH AND WITHOUT
NETWORK CODING

	with network coding (NC)					without NC	
	$k=2$	$k=4$	$k=7$	$k=10$	$k=15$	$k=2$	$3 \leq k \leq N$
$N=4$	3	4	NA	NA	NA	5	6
$N=7$	15	16	18	NA	NA	20	21
$N=12$	55	56	58	59	NA	65	66
$N=15$	91	92	94	95	98	104	105

Proof: We first prove that every packet $x_{i,j} \in X$ is encoded in at least one transmitted packet.

- When $i, j \leq k$, packet $x_{i,j}$ must be encoded in at least one transmitted packet from client c_i or c_j, similar to data exchange among the k clients in C' [1], [2].
- When $i \leq k$ and $j > k$ (or $i > k$ and $j \leq k$), packet $x_{i,j}$ must be encoded in at least one packet sent by client c_j (or client c_i), according to Eq. (19).
- When $i, j \geq k$, packet $x_{i,j}$ must be sent by client $c_{min\{i,j\}}$, according to Eq. (20).

Thus, every packet in X will be encoded in at least one transmitted packet.

We now check the decoding process of the clients in C'. We first consider client c_1 as follows:

- For the packets sent by client c_i, where $1 < i \leq k$, c_1 must be able to decode them, as this process is similar to the data exchange among the k clients in C' [1], [2].
- According to Eq. (19), c_1 also can decode the first $k - 1$ packets sent by c_i, where $i > k$, as packet $x_{1,i}$ is participated in each of these packets and $x_{1,i}$ is initially available at c_1. In addition, as the other $N - i$ packets sent by c_i are original ones, c_1 can obtain them directly. In other words, c_1 can decode all the packets sent by any client c_i, where $i > k$.

As all the packets in X are participated in the packets sent by the clients and c_1 can decode all the packets sent by the clients, c_1 can thus decode/obtain all the CSI in X.

We then check the decoding process of client c_i, where $1 < i \leq k$. Similar to the decoding process of c_1, c_i must be also able to decode all the packets sent by $c_{i'}$ where $1 \leq i' \neq i \leq k$. In addition, according to Eq. (19), the set of the first $k - 1$ packets sent by client $c_{i'}$, where $i' > k$, is $\{x_{1,i'} \oplus x_{2,i'}, x_{1,i'} \oplus x_{3,i'}, \cdots, x_{1,i'} \oplus x_{k,i'}\}$. In other words, packet $x_{i,i'}$ must be encoded in one packet sent by $c_{i'}$ where $i \leq k, i' > k$. Thus, client c_i can decode all the first $k - 1$ packets sent by $c_{i'}$ where $i' > k$. Finally, as the last $N - i$ packets sent by $c_{i'}$ are original packets, c_i thus can get them directly. That is, c_i successfully decodes/obtains all the packets sent by the other clients. As all the packets are participated in the packets sent by the clients, c_i gets the complete CSI in X.

To summarize, with the proposed transmission scheme and encoding strategy, all the clients in C' can successfully obtain all the CSI in X, which thus proved Lemma 4. ∎

Still considering the example in Table I, we can easily verify that after receiving all the packets sent from other clients, c_1, c_2 and c_3 can successfully decode their "wanted" CSI in $\overline{X_1}, \overline{X_2}$ and $\overline{X_3}$ respectively.

Note that the proposed transmission scheme and the encoding strategy can be implemented in a distributed manner. They only need to know the sequence of the clients and the indices set of the clients in C'.

VI. PERFORMANCE COMPARISON

We now compare the minimum number of transmissions required with and without network coding for various values of $k \leq N$ and $N = 4, 7, 12, 15$. As shown in Table II, we can see that the number of transmissions with network coding is much less than that without network coding. Without network coding, the number of transmissions required for $k \geq 3$ and $k = 2$ are $|X| = \frac{N(N-1)}{2}$ and $|X| - 1$ respectively (because when $k \geq 3$, each packet in X is required by at least one client in C'; while when $k = 2$, the two clients in C' must share one common packet). It can also be verified easily that our result in Theorem 2 includes [1] as a special case, where [1] considers to minimize the total number of transmissions only when $k = N$. However, the encoding scheme proposed in this paper is totally different from [1], [2] due to different problem setting.

VII. CONCLUSION

In this paper, we aim to design a network coded transmission scheme to minimize the total number of transmissions required for partial third-party information exchange problem. We first derive the minimum number of required transmissions for the partial third-party information exchange. Then, we design an optimal transmission scheme to determine the number of packets that each client should send so as to achieve the optimal minimal number of transmissions. Finally, a simple deterministic encoding strategy, based only on XOR operation, is designed to make sure that with the proposed optimal transmission scheme, all the clients that require the complete information can eventually decode/obtain their "wanted" packets.

REFERENCES

[1] D. J. Love, B. M. Hochwald, and K. Krishnamurthy, "Exchanging third-party information in a network," in *2007 UCSD Information Theory and Applications Workshop.*

[2] X. Wang, W. Song, C. Yuen, and J. L. (Tiffany), "Exchanging third-party information with minimum transmission cost," *2012 IEEE GLOBECOM.*

[3] S. El Rouayheb, A. Sprintson, and P. Sadeghi, "On coding for cooperative data exchange," *2010 IEEE ITW.*

[4] A. Sprintson, P. Sadeghi, G. Booker, and S. El Rouayheb, "A randomized algorithm and performance bounds for coded cooperative data exchange," *2010 IEEE ISIT.*

[5] S. Tajbakhsh, P. Sadeghi, and R. Shams, "A generalized model for cost and fairness analysis in coded cooperative data exchange," *2011 IEEE NetCod.*

[6] N. Milosavljevic, S. Pawar, S. El Rouayheb, M. Gastpar, and K. Ramchandran, "Deterministic algorithm for the cooperative data exchange problem," *2011 IEEE ISIT.*

[7] R. Ahlswede, N. Cai, S. Li, and R. Yeung, "Network information flow," *IEEE Trans. Inf. Theory*, vol. 46, pp. 1204–1216, 2000.

[8] S. Katti, H. Rahul, W. Hu, D. Katabi, M. Medard, and J. Crowcroft, "XORs in the air: practical wireless network coding," in *2006 ACM SIGCOMM.*