2022 Issue 06

SUTD Honours And Research Programme (SHARP) Newsletter

Innovating Research with Design

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SINGAPORE UNIVERSITY OF TECHNOLOGY AND DESIGN

MATHEMATICAL MODELLING OF DYNAMICAL SYSTEMS FOR A BETTER WORLD!

By Ang Yee Sin, Assistant Professor, Science, Mathematics and Technology (SMT) and SHARP Instructor

In the upcoming Fall term, we will focus on the mathematical modelling of dynamical systems. Mathematical models provide powerful tools for us to understand the physical world around us. From simple phenomena, such as a durian falling on Newton's head, to complex systems such as climate dynamics and even human behaviors like elections and COVID-19 spread, mathematical modeling can be readily employed to describe these systems and predict their potential future behaviors. It is rather surprising that many complex systems with multiple entangling and interacting factors, e.g. infectious diseases versus vaccinations, supply and demand dynamics in commodity market, and traffic flow, can be quite accurately modelled using by just a few coupled differential equations. Mathematical modeling has become an indispensable tool in decision making, science and engineering.

Mathematical modeling in physical sciences

Mathematical modelling forms the core of many branches of science (especially physics, and other 'physics-based' chemistry and biology, such as physical chemistry and biophysics). Taking physics as an example, human (and likely many other animals) have the built-in intuition that if a durian is detached from its branch, it would fall towards the ground. However, the concrete quantitative descriptions of a durian's position $x(t)$ as a function of time, t, via the Newton's 2nd Law, $d^2x(t)/dt^2 = F_{grav}/m_{durian}$ (where F_{grav} is the gravity force and m_{durian} is the mass of durian), represents a major conceptual step forward in describing our physical world. The very core of physics can thus be viewed as the combination of physical insights, such as the coming up with the concepts of *force* and *inertia* in Newton's 1st Law, and the mathematical modeling of physical process using concrete mathematical expressions, such as $F = ma$ (or more precisely, $F = dp/dt$ where p is the object's momentum) in Newton's 2nd Law. An important daily task of physicists is thus the development of a workable 'mathematical modeling' of Nature that is in sufficiently good unison with our observations (or experiments). Of course, in SUTD, we go one step further – not just the discovery of 'new mathematical models' for science and engineering, but also how these new mathematical models can be harnessed to design a smarter, more sustainable, and better world.

Designing a 'useful' mathematical models

One of the greatest statisticians of the 20th century, George E.P. Box, once famously remarked that "Essentially all models are wrong, but some are useful". This remark very nicely captures the core spirit of mathematical modeling, and it also reveals the importance of *balance* between '*completeness*' and '*usefulness*' in mathematical modeling. For example, we could come up with a 'beautifully complete' mathematical model of free fall motion that includes all complex quantum mechanical effects, or even general relativity, but in a practical sense, such a model would not be significantly useful in the modeling of a falling durian. The "$F = ma$" is already sufficiently accurate to model how long it takes for the durian to hit Newton's head and at what impact velocity. Here, "$F = ma$" is a useful model that maximally capture the physics of a free-falling body with the minimal amount of mathematics.

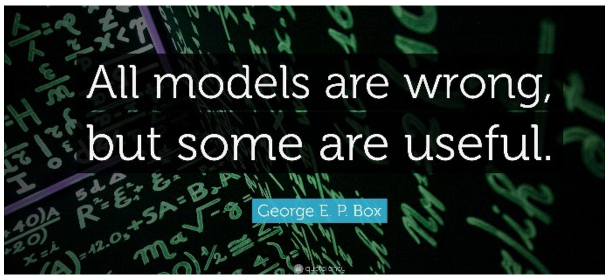
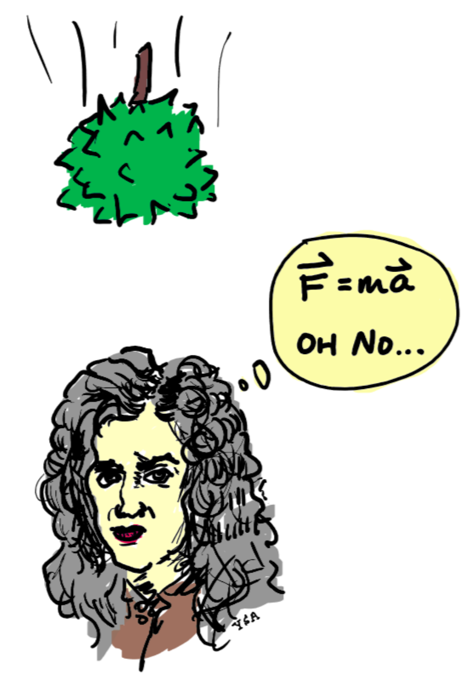


Image from Quotefancy

The design of a simple model that can be quickly computed 'at the back on an envelope' is much more challenging and more impactful, than developing an overly complex model that is 'expensive' to compute yet bring only minute gains in terms of model accuracy. A 'useful' mathematical model can thus be understood as a model capable of capturing maximal features of a set of given (or measured) real world data with the simplest possible mathematics. Coming up with a 'useful' model is especially challenging for mesoscopic or 'gray area' systems – simply put a system that is not too 'small' or not too 'large', or a system that sits at the boundary between two very different regimes. In this case, it is not immediately clear the laws of the 'big' or the laws of the 'small' should be applied in the mathematical models. Modelling such mesoscopic systems (for example, semiconductor devices and large ensemble of qubits) is thus a key research area that many scientists and engineers are working intensively now.



In a complex world, simplicity is king

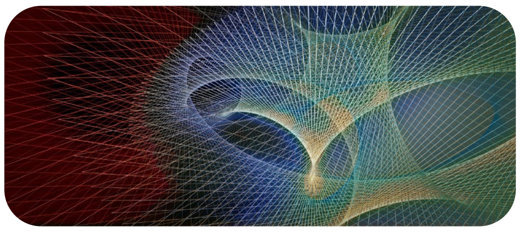
In the upcoming AY2022 Term 1 SHARP Honours Session, we will introduce the concept of 'mathematical modelling' through linear (first module led by me) and nonlinear as well as chaotic (second module led by Professor Ricky Ang) dynamical systems. Through the study of real-world examples and 'case problems', the power of simple mathematical models in describing our complex world will be introduced. We hope that this module can convince you that "in a complex world, simplicity is king".

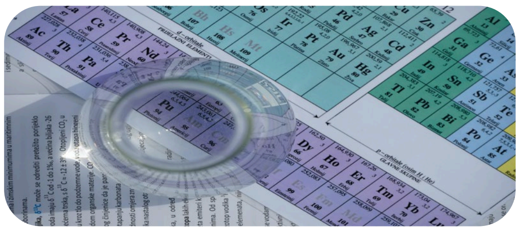
Suggested Reading: I. Mazin, Nature Phys. 18, 367 (2022) [Preprint URL: https://arxiv.org/abs/2204.08284].

* SHARP Honours Sessions are advanced classes offered on top of the regular Freshmore courses in Terms 1 to 3. It is specially designed to equip SHARP students with research methodology and to deepen their subject domain knowledge.

AY2021/2022 SHARP HONOURS SESSION

Week	Subjects	Faculty
SHARP Honours Session Term 1		
Week 2 to 4	Modelling of Dynamical Systems	Asst. Prof. Ang Yee Sin, SMT ^
Week 4 to 6	Nonlinear Systems and Chaos	Prof. Ricky Ang, SMT ^
Week 8 to 12	Group Projects	
SHARP Honours Session Term 2		
Week 1 to 4	Organic Chemistry	Dr. Tan Mei Xuan, SMT ^
Week 5 to 12	Experimental Projects	
SHARP Honours Session Term 3		
Week 1 to 3	Statistical Physics	Assoc. Prof. Dario Poletti, SMT ^
Week 4 to 6	Cryptography	Dr. Wong Wei Pin, SMT ^
Week 8 to 12	Group Projects	





^ Science, Mathematics and Technology (SMT) Cluster

RESEARCH PROJECT

AY2020 Term 3 Honours Session # ^

Project Title: **Modelling aircraft runway movements in Changi Airport**

Group Member: **Tran Nguyen Bao Long, Liu Renhang**

Project Supervisor: **Asst. Prof. Nuno Ribeiro**

Motivation: Changi Airport will undergo several stages of future development in the next years (i.e. Stage 1 - Changi West + Runway 1 + Runway 3; Stage 2 - Changi West + Runway 1 + Runway 2 + Runway 3; Stage 3 - Changi West + Changi East + Runway 1 + Runway 2 + Runway 3). The capacities of the runway system for each stage of development need to be recomputed. This is a critical task in airport management since it defines an upper limit on the amount of traffic that the airport can schedule per hour. In this project we aim to develop and apply analytical and queueing models to estimate runway capacity levels and corresponding predicted delays.

Innovation and Outcome: Based on Queueing Theory, when the utilization rate of an airport is over 80% (Figure 1), there would be an exponential increase in delay time which directly affects costs and the perceived level of services. Therefore, even with a third runway, Changi Airport flight volume is limited by its practical capacity envelope. (Figure 2)

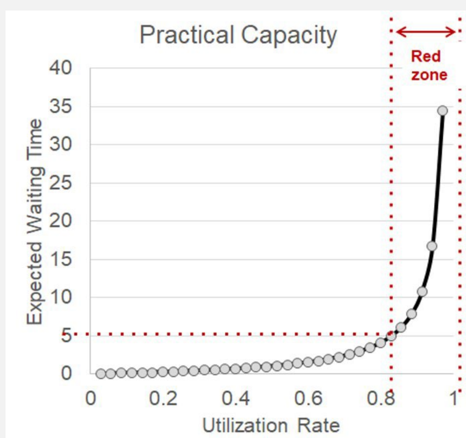


Figure 1: Practical Capacity plot based on Queuing Theory

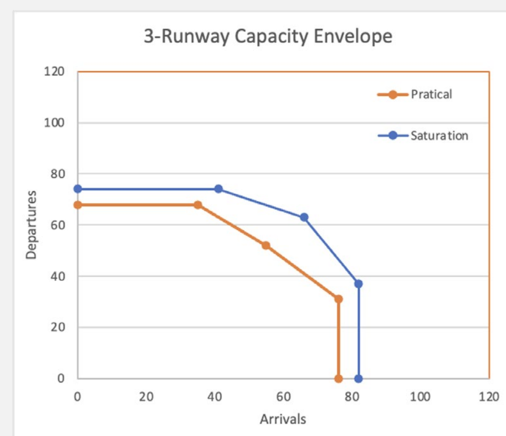


Figure 2: Capacity Envelope of 3 runway theoretical (saturation) vs practical

To further understand what affects taxi time (delay time during ground transition), we researched measurable features (such as operating distance to be covered, types of aircraft, terminal used for departures/arrivals,...) and use them to predict taxi time with a wide range of models from simple Linear Regression to advanced Machine Learning (such as XGBoost, Decision Trees, Random Forest,..).

SUMMARY OUTPUT								
Regression Statistics								
Multiple R	0.68867092							
R Square	0.47426763							
Adjusted R Square	0.47426228							
Standard Error	118.775922							
Observations	98219							
ANOVA								
	df	*SS*	*MS*	*F*				
Regression	1	1249977658	1249977658	88602.3886				
Residual	98217	1385617900	14107.7196					
Total	98218	2635595558						
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	-95.1953972	1.70834908	-55.7236214	0	-98.5437411	-91.8470532	-98.5437411	-91.8470532
X Variable 1	0.09593766	0.0003223	297.661534	0	0.09530594	0.09656937	0.09530594	0.09656937

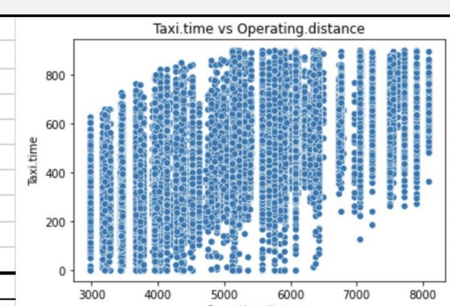


Figure 3: Simple Linear Regression model verifying the significance of operating distance in predicting delay time

Significance and Outlook: Operating Distance is a crucial feature which could explain 47.4% of the delay time data. However, many other features we tried did not show any significant correlation. Predicting taxi time to estimate delay time continues to be a challenging problem and it remains critical to find other features that could affect taxi time to better manage and coordinate flights in Changi Airport.

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^ This research project is related to the topic, **Mathematical Modelling of Physical and Complex Systems**, that was taught in AY2020 Term 3 Honours Session. In this topic, students will learn about the important concepts of mathematical models and the numerical methods of simulating physical and complex systems.

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