A SET BASED CONCURRENT ENGINEERING METHOD FOR PARAMETER DESIGN

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ABSTRACT

A method for parameter design is initially proposed; it is based on the sequential application of
the Labeled Interval Calculus to search for the feasible design space, and the Method of Imprecision
to choose a design solution, in a Sequential Filters approach. Further improvement to these methods
and approach is then developed, by the unification of their tools of parametric representation and
mathematical manipulation. The result is the Labeled Fuzzy Sets method, with the capability to
represent and manipulate simultaneously possibility, necessity, and imprecision uncertainty in a
Concurrent Filters approach.

INTRODUCTION

A sequential engineering approach for parameter design consists of the filtering of the
parametric design space to first eliminate unfeasible design alternatives and then select a preferred
solution [Eschenbach and Tesar. 1969]. Two set-based parameter design methods capable to perform
these sequential filters functions are the Labeled Interval Calculus (LIC) [Ward and Scering, 1993]
and the Method of Imprecision (MI) [Wood and Antonsson, 1991]. A schematic diagram of a set-based
sequential engineering approach based on these methods is shown in Figure 1 [Hernández-
Luna, 1994]. First, the LIC is applied to determine feasible design space by filtering unfeasible design
options using necessary requirements, followed by the MI to select a desirable design solution based on
the manipulation of preference. An important criterion for selection of these methods is their
compatibility with respect to their parametric representation and mathematical manipulation,
allowing the feasible design space found using the LIC to be directly inputted into the MI, facilitating
the flow of data.

In the conceptual design stage of Figure 1, alternative solutions are conceptualized, and their
corresponding design space is defined. Not all

Figure 1. A Set-Based Sequential Engineering Approach
The developed representation form, referred here as the labeled fuzzy sets, has the unified capability to represent possibility, preference, and necessity. The labeled fuzzy sets are based on the unification of the representation tool of the LIC, the labeled interval language, and the MI, the fuzzy sets for preference. An additional objective of this paper is the unification of the mathematics of the LIC and the MI. The resultant mathematical tool is referred here as the Labeled $\alpha$-cut Algorithm. The integration of the Labeled Fuzzy Sets and the Labeled $\alpha$-cut Algorithm in a unified set-based concurrent engineering approach form the basis of a parameter design method presented in this paper, the Labeled Fuzzy Sets Method.

1.0 Background

The idea of unifying the LIC and the MI has been proposed since the development of these two methods. This can be confirmed by referring to the doctoral thesis of Allan Ward, who developed the LIC [Ward, 1989], and Kris Wood, who developed the MI [Wood, 1989]. Extracts from their doctoral thesis referring to this subject are quoted below:


Ward and Seering [Ward and Seering, 1989] have developed a theory of quantitative inference for artifact sets as applied to a mechanical design compiler. . . Because Ward and Seering's approach relies on interval operations, the mathematics may be extended to include the uncertainty method such that further subjective and objective data of the designer may be included. [K. L. Wood, A Method for the Representation of Uncertainties in Preliminary Engineering Design, California Institute of Technology, 1989].

An effort to incorporate the LIC's representation of necessity within the MI's fuzzy sets for preference is reported in [Otto, 1992; Otto and Antonsson, 1992]. Necessity is represented within fuzzy sets using the degree of satisfaction $\alpha$, which is assigned a value in the interval $[0,1]$. A value of $\alpha = 0$ means that only necessary crisp values must be satisfied. A value of $\alpha = 1$ means all necessary and possible values must be satisfied. Therefore, $\alpha$ determines which fraction of all possible values requires that they be satisfied. For example, a robotic actuator is required to provide every torque up to 600 lb-ft. According to the definition of the degree of satisfaction $\alpha$, using a value of $\alpha = 0.9$ ensures that 90% of the possible torque range is satisfied. The resultant necessary parameter is represented in the fuzzy set of Figure 2.

The necessity interval that must be satisfied is $[20,560]$, out of the possible interval $[0,600]$. It is assumed in this paper that the elimination of values from the $\alpha$-cuts is done symmetrically, that is, $5\% = (1-\alpha)/2$ of the values are removed from each limit of the interval. No formal procedure about this issue is provided in [Otto, 1992; Otto and Antonsson, 1992].

![Figure 2. Fuzzy Set for Actuator Torque](image)
The representation of necessity using the degree of satisfaction (α) must have the following capabilities to be considered for the integration of the LIC's representation of necessity labels with the MI's fuzzy sets for preference:

1. It must represent necessity in state variables and parameters.
2. It must be compatible with the LIC's mathematical operations.

Necessary parameters are defined as "those which every possible value needs to be satisfied... parameters in which a single value in the allowable range can be used are not modeled as necessity parameters." [Otto and Antonsson, 1992]. According to this definition, the design quantities defined in the LIC as state variables (variables that change during performance) can be assigned necessity, but LIC's parameters (variables fixed at manufacturing) cannot. Therefore, the definition of necessity does not satisfy the first requirement.

With respect to the second requirement, let's analyze the fuzzy preference function of Figure 2. The maximum preference (μ = 1) is assigned to the α-cut corresponding to the labeled interval [every $T_a$ 20 560]. A subset of actuators C1 capable of satisfying this requirement is defined using a LIC elimination pattern [Ward and Seering, 1993] as

$$\{C1 \subseteq C / C1 \cap \{every \ T_a \ 20 \ 560 \ \mu = 1\} = C1\}$$

The minimum preference (μ = 0) is assigned to the α-cut corresponding to the labeled interval [every $T_a$ 0 600]. Also, a subset of actuators C2 capable of satisfying this requirement is defined using a LIC elimination pattern, too.

$$\{C2 \subseteq C / C2 \cap \{every \ T_a \ 0 \ 600 \ \mu = 0\} = C2\}$$

C1 and C2 are subsets of the set C of all possible actuators from a catalog.

On the basis of the MI's concept of fuzzy sets for preference, an actuator member of the subset of actuators C1 with the capability [every $T_a$ 20 560] has the maximum preference to be chosen from a catalog, while the set of actuator C2 with the capability [every $T_a$ 0 600] is assigned the minimum preference, and should be removed from the feasible design space. However, the operations of the LIC give us a different result. Using the "intersection" operation of the LIC [Ward and Seering, 1993], we obtain:

$$\cap ([\text{every } T_a 20 \ 560], [\text{every } T_a 0 \ 600]) \rightarrow [\text{every } T_a 20 \ 560]$$

or

$$C1 \cap C2 \rightarrow C1$$

This is true only if C1 is a subset of C2.

$$C1 \cap C2 \rightarrow C1 \subseteq C2$$

This means that any actuator member of the subset C2 capable of satisfying [every $T_a$ 0 600] can satisfy the [every $T_a$ 20 560] as well, and should not be removed from the feasible design space. The contrary is not necessarily true: an actuator member of the subset C1 capable of satisfying [every $T_a$ 20 560] does not necessarily satisfy [every $T_a$ 0 600]. Therefore, the subset of actuators C2 should have a higher preference than the set of actuators C1. This is not the case of Figure 2. Therefore, the representation of necessity in fuzzy sets of preference using the degree of satisfaction α has "a similar intention," but fails to satisfy the two requirements for compatibility with the labeled interval language of the LIC. The formal relation and unification of the LIC and MI remain to be developed, as mentioned by Otto in his doctoral thesis quoted below.

There has been other research work in the mechanical design domain with possibility and necessity Ward and Seering [Ward and Seering, 1989] have developed the "Labeled Interval

Another effort to unify the LIC and the MI methods is based on the simultaneous execution of the LIC's DOMAIN operation and the MI's Level Interval Algorithm, directly to functional requirements and design parameters represented using fuzzy sets of preference [Horng, Ward, and Wood. 1992]. The example used to motivate and illustrate this unification effort is an intermediate calculation performed during the design process of a conventional fan. The fan design parameters manipulated are the fan diameter (d) and speed (n). The performance parameter to calculate, by propagating the fuzzy sets for preference, is the fan static pressure \( P_s \). The performance parameter expression relating design and performance parameter is:

\[
P_s = k_{1s} d^2 \left( \frac{n}{1000} \right)^2 \rho
\]

where \( k_{1s} = 0.90 \), and \( \rho = 12.16 \text{ kg/m}^3 \). The fuzzy sets for \( d \) and \( n \) are represented as triangular fuzzy sets shown in Figures 3 and 4. Dividing these two fuzzy sets by \( \alpha \)-cuts and performing the LIC's DOMAIN operation for each \( \alpha \)-cut using the Label Interval Algorithm, results in the propagated fuzzy set of Figure 5.

An interpretation of the propagated preference function for \( P_s \) shown in Figure 5 is not provided in [Horng, Ward, and Wood. 1992]. instead, the question "What is the physical meaning of applying the constraint labels to preference functions?" is left to be answered by "future investigation."
pagated preference function correctly defines the feasible design space, as if it would be done applying the LIC to the intervals corresponding to the $\alpha$-cut = 0. However, the $\alpha$-cut values corresponding to preference between 0 and 1 are beyond the feasible support interval. For example, the value of fan pressure corresponding to the maximum preference ($\alpha=1$) is beyond the feasible support interval and therefore unfeasible, which makes no sense at all. Therefore, the propagation of the fuzzy sets of preference applying the LIC’s DOMAin operation is correct only at the support, that is, for the $\alpha$-cut = 0. The propagation of preference for all the rest of preference ($\alpha$-cut > 0) is incorrect.

The main weakness of this effort of unification is in the representation of the fan speed $n$ with the fuzzy set of Figure 4. Previous to the assignment of preference to fan speed $n$, its feasible design space was determined in the reference as $[R \ n \ every \ 300 \ 500]$ RPM, applying the LIC [Horng, Ward, and Wood, 1992]. This labeled interval specification means that the fan is required to be capable of running at every speed within the interval [300 500] RPM. Looking at the fuzzy preference function of fan speed $n$ in Figure 4, this feasible interval is assigned as the support of the function, which seems to be correct because it is the minimum acceptable requirement. However, the most preferred interval is assigned to a fan speed of 400 RPM (a crisp value). Therefore, the most preferred value is out of the feasible interval. Using the elimination patterns of the LIC [Ward and Seering, 1993], the fans corresponding to the most preferred interval would be eliminated, for not satisfying the minimal requirement of feasibility.

$[R \ every \ n \ 400] \subseteq [N \ every \ n \ 300 \ 500] \rightarrow$ eliminate

This means that the sets of fans with the required capacity to satisfy 400 RPM is not a subset of the feasible fans with the capacity of satisfying every value in an interval of at least [300 500].

An alternative representation of Figure 5 is shown in Figure 6, based on Zadeh’s extension principle [Zadeh, 1965]. Notice that the support does not correspond with the feasible interval propagated by the LIC’s DOMAIN operation for $\alpha$-cut = 0. Therefore, the most preferred value is at the same time unfeasible in the fuzzy set of the fan velocity $n$.

![Figure 6 $P_s$ after Extension Principle](image)

The basic problem with this unifying effort is again in the wrong representation of the LIC’s every necessity label in a fuzzy set for preference. The unified representation of the LIC’s necessity labels and MI’s fuzzy sets for preference is the objective of the next section.

2.0 A Unified Representation of Necessity Labels in Fuzzy Sets of Preference

This section describes the development of a unified form of representation, the labeled fuzzy sets, based on the unification of the labeled interval language and the fuzzy sets for preference. The labeled fuzzy sets have the capability to represent simultaneously parametric possibility, necessity, and preference.

2.1 The Labeled Fuzzy Sets

Representation of necessity in fuzzy sets
paper as the labeled fuzzy sets. The representation of the necessity label every in a labeled fuzzy set is shown in the following example.

Example

The interval label every is an operating-region necessity label used exclusively for state variables, indicating that a variable can assume every value within the interval [Ward and Seering, 1993]. From our interpretation of the representation of necessity for actuator torque in Figure 2, it was concluded that the preference of the $\alpha$-cut corresponding to the labeled interval $[Ta \ every \ 0 \ 600]$ should have a higher preference than the $\alpha$-cut corresponding to the labeled interval $[Ta \ every \ 20 \ 580]$. In terms of degree of preference, this concept is expressed mathematically as

$$1 \geq \mu([Ta \ every \ 0 \ 600]) \geq \mu([Ta \ every \ 20 \ 580]) \geq 0$$

This assignment of preference means that actuators capable of satisfying at least the labeled interval $[Ta \ every \ 0 \ 600]$ are preferred to actuators capable of satisfying at least the labeled interval $[Ta \ every \ 20 \ 580]$. In addition, the values that a parameter set with an every label can take are not limited by the boundaries of the interval. For instance, the necessity requirement for an actuator represented as $[T_a \ every \ 0 \ 600]$ requires the satisfaction of at least every value within this interval. Therefore, actuators with capabilities larger than the interval $[T_a \ every \ 0 \ 800]$ are also feasible to satisfy the necessity requirement.

The representation of these two concepts for possibility, necessity, and preference in a labeled fuzzy set is shown in Figure 7. The most preferred value ($\alpha = 1$) is assigned to the labeled interval $[Ta \ every \ 0 \ 600]$, while the minimum preference ($\alpha=0$) is assigned to the interval $[Ta \ every \ 100 \ 400]$.

Therefore, a feasible actuator must have at least the capacity to satisfy the necessity requirements $[Ta \ every \ 100 \ 400]$. In addition, it is understood that an actuator capable of satisfying the labeled $\alpha$-cut $[Ta \ every \ 0 \ 600 \ \mu=1]$ (representation extended to include preference) can satisfy the labeled $\alpha$-cut $[Ta \ every \ 100 \ 400 \ \mu=0]$, as well. The contrary is not necessarily true. Notice that the labeled interval specifications are referred to as labeled $\alpha$-cuts to incorporate preference.

The fuzzy set is represented using a dashed line to emphasize that the values of the variable parameter are not constrained by the boundary lines of the labeled fuzzy set. The next step is to verify if this labeled fuzzy set form of representation has the following capabilities required to unify the MI and LIC representation forms:

1. It should represent necessity for both state variables and parameters.
2. It must be compatible with the LIC's mathematical operations.

First, the representation of parametric necessity in labeled fuzzy sets is similar to the representation using the fuzzy sets for preference of the MI. In addition, the labeled fuzzy sets represent necessity in state variables, as shown in Figure 7.
With regard to the second requirement, feasible design space is based on the propagation of the interval corresponding to the minimum preference: that is, the \( \alpha \)-cut = 0. This agrees with the operations of the LIC. However, for complete compatibility, the operations of the LIC should be valid for all the other degrees of preference larger than zero and up to 1 (0 \( \leq \alpha \leq 1 \)). A procedure to propagate label fuzzy sets by their mathematical manipulation using the operations of the LIC is required. This is the objective of the next section.

3.0 Manipulation of Labeled Fuzzy Sets

This section presents a mathematical tool to manipulate Labeled Fuzzy Sets, the Labeled \( \alpha \)-cut Algorithm (L\( \alpha \)A), based on the unification of the mathematical operations of the LIC [Ward and Scerri, 1993] and the Level Interval Algorithm of the MI [Wood, Otto, and Antonsson, 1992]. The L\( \alpha \)A allows the manipulation of parametric possibility, necessity, and preference represented in Labeled Fuzzy Sets. Development of this tool enhances the compatibility and unification of the LIC and the MI.

3.1 The Labeled \( \alpha \)-cut Algorithm (L\( \alpha \)A)

The L\( \alpha \)A performs the labeled interval operations of the LIC for each prescribed number of \( \alpha \)-cuts that a labeled fuzzy set is discretized. A description of the L\( \alpha \)A is presented below.

Algorithm 1. The L\( \alpha \)A takes an implicit performance parameter expression in three parameters and a pair of labeled \( \alpha \)-cut intervals in two of the parameters, and returns the compatible labeled \( \alpha \)-cut interval in the third parameter. The following algorithm steps lead to the execution of this process.

1. For each one of the two parameters to be propagated, discretize their labeled preference function into a number of \( \alpha \) values, \( \alpha \ldots \alpha_m \), where \( M \) is the number of steps in the discretization.

2. Determine the labeled \( \alpha \)-cut for each of the two parameters to be propagated for each \( \alpha \)-cut, \( \alpha_j \), \( j=1 \ldots M \).

3. Identify the LIC inference pattern and operation (RANGE, DOMAIN, SUFPT) that correspond to the two labeled \( \alpha \)-cut to be propagated.

4. For the labeled \( \alpha \)-cut of the two parameters to be propagated, apply the corresponding identified LIC inference pattern and operation.

Application of the L\( \alpha \)A for propagation of labeled fuzzy sets considering the RANGE, DOMAIN and SUFPT operations of the LIC is demonstrated in the following examples.

**Example: RANGE operation.**

The actuator shown schematically in Figure 8 is required to have an output torque capability up to 600 N-m only, to avoid damage to a certain specified load. As a safety measure, to avoid any possibility of failure, a torque capability from 100 to 400 N-m only is preferred. The transmission to use on the actuator is required to have a ratio between 30 and 100 only. A transmission ratio of 60 is preferred. The labeled preference functions for these specifications are shown in Figures 9 and 10.

![Figure 8 Actuator](image)

Notice that the labeled fuzzy sets of Figures 7 and 8 are defined using the same possibility intervals, but with different necessity interval
label, which results in the opposite interval preference. The necessity label in Figure 7 is every, represented as a dashed line, while the necessity label in Figure 9 is only represented as a solid line like in a conventional fuzzy set for preference.

The labeled fuzzy sets of $T_a$ and $r$ are discretized into labeled $\alpha$-cut intervals. The LIC's RANGE operation is identified for the propagation of the labeled $\alpha$-cuts.

$$(R \text{ only } [s] \alpha) \land (A \text{ only } [p] \alpha) \rightarrow (R \text{ only } [s] \alpha \text{ RANGE})$$

The propagation of the labeled fuzzy functions for $T_a$ and $r$, using the RANGE operation, yields the labeled fuzzy function for $T_m$ shown in Figure 11.

![Figure 9 Actuator Torque](image)

![Figure 10 Transmission Ratio](image)

The feasible interval for the motor torque represented as a labeled $\alpha$-cut is:

$[T_m \text{ only } 0 \ 20 \ \mu=0]$  

Motors with a capacity higher than this interval are not feasible and should not be selected. The most preferred interval for the motor torque represented as a labeled $\alpha$-cut is:

$[T_m \text{ only } 1.67 \ 6.67 \ \mu=1]$  

Motors with a capability within this interval are preferred. The next example shows the application of the Labeled $\alpha$-cut Algorithm to a DOMAIN operation.

**Example**

Assume that the actuator of the previous example is desired to have the capability of providing every torque from 0 up to 600 N-m, with a minimum acceptable capability of every torque from 100 N-m to 400 N-m, as shown in Figure 12. The same specification for a transmission restricted to have a ratio between 30 to 100 only, where a ratio of 60 is preferred, as shown in Figure 10.
A feasible motor must at least provide torque in the labeled \( \alpha \)-cut \([R\ every \ T_m \ 3.3\ 4.0\ \mu =0]\). The motor must have a capacity of 3.3 N-m when combined with a transmission ratio of 30 to satisfy the requirement of actuator torque of 100 N-m, and a capability of 4 N-m combined with a transmission ratio of 100 to satisfy the requirement of actuator torque of 400 N-m. The most preferred motors (\( \mu =1 \)) are those with the capability of providing every torque up to 10 N-m, which in combination with the most preferred transmission ratio of 60 results in the most preferred actuator capability of torque up to 600 N-m.

The representation of parametric possibility, necessity, and preference in Labeled Fuzzy Sets, and their mathematical manipulation with the Labeled \( \alpha \)-cut Algorithm results in the unification of the LIC and the MI in a new method, the \textit{Labeled Fuzzy Sets} method. The Labeled Fuzzy Sets method allows the simultaneous filtering of the Necessary and Desirable Conditions of the Sequential Filters approach. The resultant design approach of the Labeled Fuzzy Sets method is referred here as the \textit{ Concurrent Filter} approach.

5.4 Concurrent Filter

The Labeled Fuzzy Sets method developed in the previous section allows the unification of the Sequential Filters in a single Concurrent Filter (CF). The resultant CF approach is based on the concurrent screening of design possibilities considering simultaneously necessary and desirability conditions. The CF approach allows the simultaneous manipulation of the necessary and preference constraints and requirements. A schematic plot of the CF strategy is shown in Figure 14.
Conclusions

The labeled interval language for the representation of parametric possibility and necessity in the Labeled Interval Calculus, and the fuzzy sets for preference for the representation of possibility and preference in the Method of Imprecision are unified in a new form of parametric representation, the Labeled Fuzzy Set. The mathematical operations of the Labeled Interval Calculus and the Level Interval Algorithm of the Method of Imprecision are unified in the Labeled α-cut Algorithm. These developments allow the unification of the Labeled Interval Calculus and the Method of Imprecision of the sequential engineering approach into a concurrent engineering approach for design. The unified representation, manipulation and design approach are integrated in a unified method for parameter design, the Labeled Fuzzy Sets method.

References