Geometric Tolerancing in Mechanical Design Using Fractal-Based Parameters

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The submacrogeometric (tolerance) variations of mechanical components have an important impact on product function. The problem of combining tolerance assignment in engineering design with tolerance control in manufacturing and quality assurance presents a continuing challenge to researchers. This paper illustrates the feasibility of using fractal-based methods for the problem of tolerance specification in engineering design. Error data are generated as a function of the fractal dimension using the fractional Brownian motion model and are superposed on an ideal profile of a slider bearing. The consequent changes in performance parameters are studied and the detrimental effect of large variations perceived. This simple case study indicates the potential of the method to be extended for more complex problems.

1 Introduction

In this paper, the feasibility of a new theory for combining manufacturing information with design choices is described. This theory is based on the concept of representing the pattern of manufacturing variations with fractal-based parameters and relating these parameters explicitly to performance parameters that describe a design. Geometry is the usual language used for such representations. Geometry can be classified as *macrogeometry*, *submacrogeometry* and *microgeometry*. Macrogeometry and microgeometry dictate the overall shape or form of a component and the surface finish, respectively. The focus of this paper is on the remaining category, submacrogeometry. These are low frequency variations on a component's nominal geometry and are usually characterized by size and geometric tolerances in mechanical design (ANSI-Y14.5M, 1982).

Submacrogeometry, or tolerance scale variations, can govern the function of many devices. For example consider the variations associated with thrust or journal bearings. If such bearings are misaligned or have poor clearance due to manufacturing errors, excessive component wear can result, even to the point of bearing seizure. For this reason and other associated issues like cost, assemblability, etc., submacrogeometry must be related directly to function, especially early in the design process when initial mechanical component choices are being made.

1.1 Related Research. The tolerancing problem has attracted considerable research in the past decade. In particular, research efforts have focused on tolerance representation (Lin et al., 1981; Requicha, 1983; Jayaraman and Srinivasan, 1989; Martino and Gabriele, 1989; Turner and Wozny, 1990), analysis (Siddall, 1983; Chase and Greenwood, 1987; Bjorke, 1989), control (Taguchi, 1986), and cost functions (Ostwald and Huang, 1977; Vasseur et al., 1993). While these efforts have produced important results, the area of performance-tolerance interaction, in spite of being recognized as critical, has not been the focus of intensive research work (Finger and Dixon, 1989). A new fractal-based theory for geometric tolerance estimation is outlined in the next section to address this documented problem.

2 A Philosophical Basis for Fractal Tolerancing

The primary hypothesis for this research is that there is a distinguishable *structure* or *pattern* in the geometric and size errors associated with a manufacturing process, operating under specified process conditions (Srinivasan and Wood, 1992a). It is proposed to extract and embody this error pattern in a noninteger, fractal dimension. Fractal dimensions, in this sense, are metrics that provide a general and uniform measure of submacrogeometric variations over the set of possible manufacturing processes and process parameters.

Fractals are a novel class of mathematical functions, developed by Mandelbrot (Mandelbrot, 1983) and are used to model irregular geometric forms. The concept of physical fractals (Jaggard and Kim, 1987) is adopted to develop fractal-based parameters for tolerances. Physical fractals, e.g., natural coastlines, possess statistical self-similarity, are differentiable over specific scale ranges, and have an upper and lower limit on the scales over which the fractal representation is valid. This flexibility in the definition of physical fractals affords a valuable tool to analyze both artificial and natural systems (Mandelbrot, 1983). The additional space-filling and nonstationary properties of physical fractals suggest that a fractal model for tolerances will be, in general, more representative of manufacturing profiles, compared with other classical methods, such as Fourier transforms or statistical models (Srinivasan and Wood, 1993).

3 Fractal Dimension Based Profile Generation

Physical fractals generally have a random component and are hence sometimes called random fractals. One type of physical fractals, called *fractional Brownian motion* or fBm (Mandelbrot, 1983), is used in the present development of one dimensional error profiles. The reasons for this choice are: (*1*) machining processes are characterized by nonstationarity (Peklenik and Jerele, 1992), i.e., their stochastic properties are dependent on time. fBm is a proven model for nonstationary random processes (Peitgen and Saupe, 1988); (*2*) fBm traces are very similar, graphically, to machined profiles with form errors; and (*3*) fBm is a starting point, algorithmically, for demonstrating the feasibility of generating realistic profiles from fractal dimensions.

3.1 Spatial Approximation of fBm and Modified Midpoint Displacement. Fractional Brownian motion usually considers a dependent-variable trace as a function of time t. For one-dimensional tolerance profiles, we introduce a spatial variable x. In this context, fBm is a function $y_H(x)$. The parameter H controls the "smoothness' of the trace and is related to the fractal dimension D_f of the profile as: $H = 2 - D_f$, where $0 \leq H \leq 1$. An increment in $y_H(x)$ is related to the change in x by

$$\Delta y_H(x) \propto \Delta x^H, \tag{1}$$

where $H = 0.5$ corresponds to the classical Brownian motion trace.

The spatial approximation method for fBm, in particular a modified version of the midpoint displacement method (Peitgen and Saupe, 1988), is used to generate profile errors. It is a recursive generating technique. Considering a fBm function $y_H(x)$, let the mean square increment in $y_H(x)$ for points a distance $\Delta x = 1$ from one another be σ^2, where σ is the standard

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deviation of a zero mean Gaussian distribution. σ, the random component, and D_f, the fractal structure, fully specify a fBm tolerance profile.

From Eq. (1), for points separated by a distance x,

$$E[(y_H(x) - y_H(0))^2] = x^{2H}\sigma^2, \quad (2)$$

where $E[.]$ denotes the expected value. To assure that the profile errors fluctuate about a mean value of zero (for manufacturing processes), we introduce the boundary conditions $y_H(0) = y_H(1) = 0$. Using these boundary conditions, the first level of recursion is set as

$$y_H(0.5) = 0.5\,(y_H(1) + y_H(0)) + \delta_1, \quad (3)$$

where δ_1 is a Gaussian random variable with mean zero and variance $(\sigma_1)^2$, calculated from the condition of Eq. (2).

The procedure is performed recursively, until a preset maximum level is reached. The variance at each step is related to the initial variance σ^2 as

$$(\sigma_n)^2 = \frac{\sigma^2}{(2^n)^{2H}}\,[1 - 2^{2H-2}]. \quad (4)$$

The $y_H(x)$ values so calculated give the error values, for a given manufacturing process, at the respective x positions. These values can be superimposed on an ideal profile and related to design function for tolerance assignment, process selection, etc. In the above model, the fractal dimension specifies a constraint on the irregularities of the surface. Random elements are also introduced in the model via the initial standard deviation. Table 1 lists the estimated fractal dimensions for selected manufacturing processes and the corresponding tolerance zones from the extended midpoint displacement method.

4 Case Study: Tolerance Design for Bearings

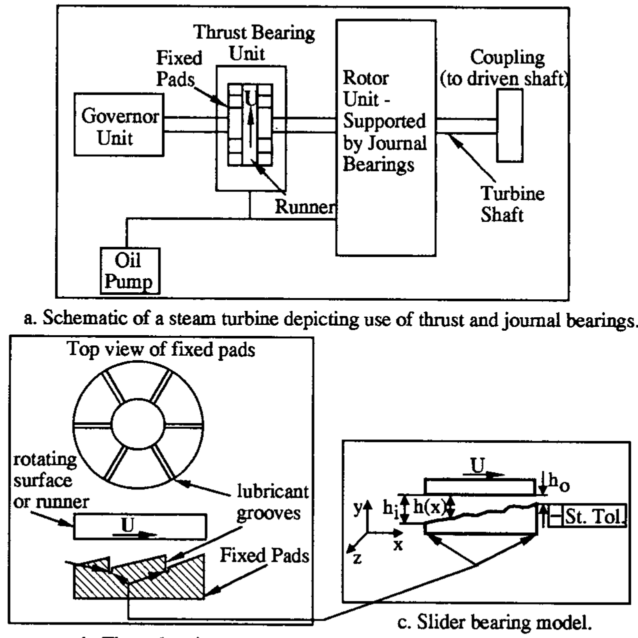
This section illustrates the use of fractal-based techniques in the tolerance design of thrust bearings. The performance of fluid-film bearings is affected by surface roughness and waviness. This problem has been the subject of past and ongoing research (Christensen, 1969; Hargreaves, 1991). While roughness represents high frequency errors, waviness constitutes low frequency errors in the straightness of the profile and is considered as a geometric tolerance characteristic. However, manufactured parts rarely exhibit isolated periodic waviness; it is more likely to be found in combination with other, randomly induced form errors (composite straightness error).

The example problem considered is the design (emphasizing tolerances) of a fixed pad type thrust bearing used in a steam turbine (Fig. 1). Tolerance design is critical because fluid film bearings operating under high loading conditions have minimum film thicknesses on the order of the accuracy obtainable from most general purpose machine tools. In the present example, the pads are assumed to be manufactured with such machine tools, and it is required to assign a suitable straightness tolerance to the pads so that the bearing will have a prescribed load carrying capacity.

4.1 The Bearing Subproblem. The performance of the thrust bearing can be estimated by simplifying the problem as follows: considering a single pad of length L and width B, and assuming the curvature is negligible, we can use the techniques developed for the analysis of slider bearings. Central to the hydrodynamic theory of fluid film bearings is the Reynolds' equation:

$$\frac{\partial}{\partial x}\left(h^3\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial z}\left(h^3\frac{\partial p}{\partial z}\right) = 6U\mu\frac{\partial h}{\partial x}, \quad (5)$$

based on lubricant pressure p, fluid film height h, coordinates x and z, Fig. 1(c), relative bearing velocity U, and lubricant viscosity μ. The film height h is a function of the distance in the direction of flow, and the profile error, representable by



a. Schematic of a steam turbine depicting use of thrust and journal bearings.

b. Thrust bearing.

c. Slider bearing model.

Fig. 1 Fixed pad thrust bearing: application and model

a fractal dimension D_f and is expressed as $h = h(x, D_f)$. The overall profile can be considered to be the straightness errors $y_{D_f}(x)$, superimposed on the smooth, inclined profile.

$$h(x,D_f) = h_i - (h_i - h_o)\,\frac{x}{L} + y_{D_f}(x). \quad (6)$$

Introducing normalization in Eq. (6), i.e., dividing by h_o, refer to Fig. 1(c),

$$\bar{h} = H_x - (H_x - 1)\bar{x} + \bar{y}_{D_f}(\bar{x}). \quad (7)$$

Replacing, the variables in the Reynolds' Eq. (5) by their normalized counterparts (Hargreaves, 1991), we have:

$$\frac{\partial}{\partial \bar{x}}\left(\bar{h}^3\frac{\partial \bar{p}}{\partial \bar{x}}\right) + \left(\frac{L}{B}\right)^2\frac{\partial}{\partial \bar{z}}\left(\bar{h}^3\frac{\partial \bar{p}}{\partial \bar{z}}\right) = 6\,\frac{\partial \bar{h}}{\partial \bar{x}}, \quad (8)$$

where the normalized pressure $\bar{p}$ is given by $ph_o^2/\mu UL$.

4.1.1 Synthesis of the Irregular Profile. The extended midpoint displacement algorithm is used to generate the error profiles; however, the choice of the initial standard deviation σ (Gaussian) influences the extent of variability of the profile from the nominal. For the purposes of slider bearing design, the selection of σ is motivated from a physical basis, satisfying the most general operational requirement, viz., lubricant flow. The reasoning is as follows: for the normalized film, the minimum film thickness is 1.0. If the errors are greater than or equal to one (in the worst case) along the bearing, the profile will fill the two-dimensional bearing gap and prevent lubricant flow. In this case, assuming that the maximum positive value of the normal distribution is given by 4σ, σ[3] can be calculated from the relation:

$$max(error) = 1.0 = 4\sigma. \quad (9)$$

Invoking the packing density/space filling property of fractals,the extent of the errors are limited by

$$\sigma = 0.25\,(D_f - D_t), \quad (10)$$

where $D_t = 1$ is the topological dimension of the profile. Equation (10) may be checked for conformance with the extrema, i.e., $\sigma = 0$ when $D_f = 1$ (no errors) and $\sigma = 0.25$ when

[3]The area under the standard-normal curve between $\pm 4\sigma$ is 0.99994.

Table 1 Tolerance zones (h_o = 0.5 mm)

D_f	Manufacturing Process	Maximum $\Delta[y_H]$ (mm)
1.05	Lapping	0.0016
1.10	Finish Grinding	0.0044
1.20	Coarse Grinding	0.0090
1.35	Finish Milling	0.0309
1.50	Rough Milling	0.0821

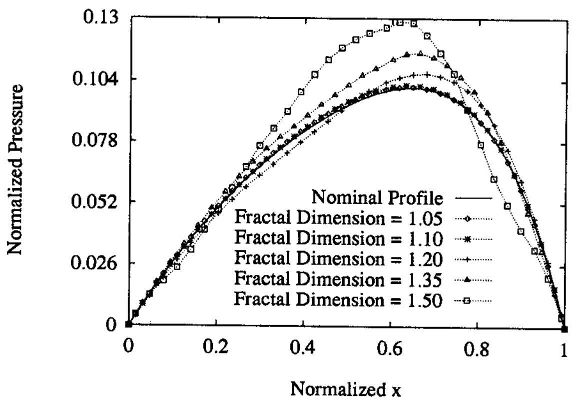


Fig. 2 Maximum pressure distribution at bearing centerline: H_x = 1.5

$D_f = 2$ (maximum error). An intermediate value of D_f and σ corresponds to a certain manufacturing process, at certain operating conditions (process parameter settings), such as that shown in Table 1.

4.2 Pressure Calculations. The error profile, $\bar{y}_{D_f}(\bar{x}) \equiv \bar{y}_H(\bar{x})$, is calculated using the recursive displacement procedure. The actual bearing profile is calculated by numerically superimposing the ideal profile and the error values as shown in Eq. (7). A finite difference approximation procedure is used, with a Gauss-Seidel iterative scheme, to solve the resulting system of equations for the pressure distribution. Example (maximum) centerline pressure distributions for $H_x = 1.5$, are depicted in Fig. 2. Another set of simulations yield pressures below nominal (Srinivasan and Wood, 1992b). Similar results are obtained for $H_x = 2.0$. As shown in the figure, for $D_f \leq 1.20$, the pressure distributions deviate little from the nominal. However, for $D_f \geq 1.35$, there is a significant change in the pressure and load carrying capacity (as shown in Table 2), either greater or less than the nominal. This implies that manufacturing processes corresponding to these fractal dimensions will have a significant impact on the function of the bearing.

5 Simulation and Discussion

Using the methodology described above, the normalized slider bearing problem is simulated. The parameter values used in the simulation and some representative results are summarized in Table 2 and discussed below. For this study, only two performance parameters, load capacity, and friction force are discussed, based on the pressure distribution calculations (Hargreaves, 1991; Srinivasan and Wood, 1992b).

From Table 2, the load capacity remains almost constant or increases marginally for less irregular profiles ($D_f \leq 1.20$), but exhibits unpredictable behavior for more irregular profiles, consistent with the observations of Christensen for surface roughness effects (1969). There are also cases when the load carrying capacity is larger than that for a smooth bearing. This behavior may be compared to the findings of Hargreaves (1991) for surface waviness effects on fluid film bearings. In Hargreaves' experimental and analytical study, ideally periodic surface waviness is shown to increase the load capacity of slider bearings. Based on these results, it is conjectured that the presence of discernible periodicities in the error pattern will govern the load carrying capacity. If a manufacturing process can be "set" with a deterministic structure that has a relative regular period (where purely random, irregular effects are much smaller in magnitude), the load carrying capacity will be assured to increase over the nominal geometry, compared to other processes. However, for the processes (fractal dimensions) considered, a fBm profile does not consistently exhibit such structure. Thus, the designer must choose a process from a fractal dimension, where the requirement on *minimum* load capacity is not violated.

Table 2 Simulated performance data

Profile	D_f	H_x	Load Capacity		Friction Force
			max	*min*	
Nominal	1.00	1.5	0.0477	0.0477	0.8224
		2.0	0.0628	0.0628	0.7242
Irregular	1.05	1.5	0.0479	0.0476	0.8224
		2.0	0.0630	0.0627	0.7246
Irregular	1.10	1.5	0.0481	0.0478	0.8224
		2.0	0.0628	0.0626	0.7241
Irregular	1.20	1.5	0.0484	0.0432	0.8222
		2.0	0.0630	0.0621	0.7241
Irregular	1.35	1.5	0.0516	0.0432	0.8213
		2.0	0.0655	0.0615	0.7248
Irregular	1.50	1.5	0.0524	0.0377	0.8203
		2.0	0.0656	0.0595	0.7206

The friction force on the other hand, remains relatively insensitive to the superposition of form errors on the smooth profile (Table 2), consistent with the observations of Christensen (1969) for surface roughness effects. The design conclusion is thus straightforward: the friction force is insensitive to the set of manufacturing processes (fractal dimensions) considered. Likewise, the friction force is decoupled (not functionally coupled) to the load capacity in terms of the manufacturing process choice.

From a manufacturing viewpoint, the errors generated with different fractal dimensions is examined for the extent of variability $\Delta[y_H]_{max}$ to decide on manufacturing process selection (Table 1). The magnitude of the errors can give us the relative accuracies and indicate comparable manufacturing processes. Based on the simulation results, coarse grinding (corresponding to a fractal dimension of $D_f = 1.20$), and finer processes are appropriate choices for consistent load carrying capability.

6 Conclusion

An important subset from the plethora of open questions in tolerancing mechanical components is addressed in this paper. The utility of tolerances as communication media between design and manufacturing is the focal point of the overall research. Geometric tolerances are modeled using fractal-based concepts. The underlying motivation arises from the conceptual equivalence of the tolerance zones prescribed by geometric tolerances, which constrain the component profile/surface to specific regions in space and the area/space filling properties of fractal objects. The results from concomitant research on associating the accuracy of manufacturing errors with a fractal dimension, are expected to enhance the integrated design process, providing specific guidelines for process selection, machine tool characteristics, and related issues.

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References

ANSI-Y14.5M, 1982, "Geometric Dimensioning and Tolerancing," ANSI Standard Y14.5M-82.

Bjorke, O., 1989, *Computer-Aided Tolerancing*, ASME Press, New York.

Chase, K. W., and Greenwood, W. H., 1987, "Design Issues in Mechanical Tolerance Analysis," *Advanced Topics in Manufacturing Technology*, pp. 11-26, ASME Winter Annual Meeting, Boston, MA.

Christensen, H., 1969, "Stochastic Models for Hydrodynamic Lubrication of Rough Surfaces," *Proc. of the Instn. of Mech. Engrs.*, Vol. 184, Pt. 1, No. 55, pp. 1013-1026.

Finger, S., and Dixon, J. R., 1989, "A Review of Research in Mechanical Design. Part II: Representations, Analysis and Design for the Life Cycle," *Research in Engineering Design*, Vol. 1, No. 2, pp. 121-137.

Hargreaves, D. J., 1991, "Surface Waviness Effects on the Load-Carrying Capacity of Rectangular Slider Bearings," *Wear*, Vol. 145, No. 1, pp. 137-151.

Jaggard, D. L., and Kim, Y., 1987, "Diffraction by Band-Limited Fractal Screens," *Journal of the Optical Society of America A*, Vol. 4, No. 6, pp. 1055-1062.

Jayaraman, R., and Srinivasan, V., 1989, "Geometric Tolerancing: Parts I and II," *IBM Journal of Research and Development*, Vol. 33, No. 2, pp. 90-124.

Lin, B. C., Gossard, D. C., and Light, R. A., 1981, "Variational Geometry in Computer-Aided Design," *Computer Graphics*, Vol. 15, No. 3, pp. 171-177.

Mandelbrot, B. B., 1983, *The Fractal Geometry of Nature*, W. H. Freeman and Co., San Francisco.

Martino, P. M., and Gabriele, G. A., 1989, "Application of Variational Geometry to the Analysis of Mechanical Tolerances," ASME *Advances in Design Automation*, Ravani, B., ed., pp. 19-27.

Ostwald, P. F., and Huang, J., 1977, "A Method for Optimal Tolerance Selection," ASME *Journal of Engineering for Industry*, Series B, Vol. 99, No. 3, pp. 558-565.

Peitgen, H. O., and Saupe, D., 1988, *The Science of Fractal Images*, Springer Verlag, New York.

Peklenik, J., and Jerele, A., 1992, "Some Basic Relationships for Identification of the Machining Processes," *Annals of the CIRP*, Vol. 41, No. 1, pp. 155-159.

Requicha, A. A., 1983, "Toward a Theory of Geometric Tolerancing," *The International Journal of Robotics Research*, Vol. 2, No. 4, pp. 45-60.

Siddall, J. N., 1983, *Probabilistic Engineering Design*, Marcel Dekker Inc., New York.

Srinivasan, R. S., and Wood, K. L., 1992a, "A Computational Investigation into the Structure of Form and Size Errors Based on Machining Mechanics," ASME *Advances in Design Automation 1992*, Vol. 1, pp. 161-171, Phoenix, AZ.

Srinivasan, R. S., and Wood, K. L., 1992b, "Fractal-Based Geometric Tolerancing in Mechanical Design," ASME *Proceedings of the 1992 ASME DTM Conference*, pp. 107-115, Phoenix, AZ.

Srinivasan, R. S., and Wood, K. L., 1993, "Wavelet Transforms in Fractal-Based Form Tolerancing," ASME *Design Theory and Methodology-DTM '93*, Hight, T. K., ed., pp. 51-63, Albuquerque, NM.

Taguchi, G., 1986, *Introduction to Quality Engineering*, Asian Productivity Organization, Unipub, White Plains, NY.

Turner, J. U., and Wozny, M. J., 1990, "The M-Space Theory of Tolerances," *Advances in Design Automation, 1990*, Vol. 1, Ravani, B., ed., pp. 217-225.

Vasseur, H., Kurfess, T., and Cagan, J., 1993, "Optimal Tolerance Allocation for Improved Productivity," *Proceedings of the 1993 NSF Design and Manufacturing Systems Conference*, pp. 715-719, Charlotte, NC. NSF, SME.

Optimally Directed Truss Topology Generation Using Shape Annealing

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This paper presents a technique for the generation and optimization of truss structure topologies based on the shape annealing algorithm. Feasible truss topologies are generated through a shape grammar in an optimally directed manner using simulated annealing for minimum weight, subject to stress and Euler buckling constraints.

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1 Introduction

The advent of computer methods of analysis and design have propelled intensive research in the field of structural optimization over the last three decades. The basic structural optimization problem is to minimize an objective function, such as the weight or cost of the structure, subject to a set of structural constraints: stress constraints, buckling constraints, and geometric constraints such as location of loads, support points, and obstacles. Shape optimization of such a problem is rather well understood, although not necessarily easy, for a given topology; generation and optimization of the topology itself is a difficult problem. This paper proposes a systematic way of generating truss topologies and integrating this process with shape optimization.

Previous efforts in topology optimization go back to the early 1900s (Michell, 1904). Recent efforts have taken three approaches: selection from predefined grid or node layouts (e.g., Hemp, 1973; Dorn et al., 1964; Pedersen, 1992; see Kirsch, 1989), heuristic approaches (Spillers, 1985; Shah, 1988; Rogers et al., 1988; Lakmazaheri and Rasdorf, 1990) and material distribution approaches (homogenization: Bendsøe and Kikuchi, 1988; Diaz and Belding, 1993; Rodrigues and Fernandes, 1993; genetic algorithms: Chapman et al., 1993; combinatorial particle/element approach: Anagnoustou et al., 1992). In these latter approaches the structure emerges from within the predefined acceptable geometric bounds; however the resulting material must be further interpreted to determine the useful structure generated (e.g., Papalambros and Chirehdast, 1990).

In this paper we propose an algorithm for truss structural topology optimization based on the shape annealing algorithm introduced by Cagan and Mitchell (1993). The shape annealing algorithm provides a systematic way of generating truss topologies, integrated with shape optimization. This method combines the advantages of optimization-based techniques and production system-based techniques to generate optimally directed topological configurations.

2 Shape Annealing for Truss Generation

In shape annealing, knowledge of components and connectivity between components is modeled through shapes and their grammars (Stiny, 1980). A *shape* is a limited arrangement of straight lines which form an entity upon which Boolean operations can be applied; distinguishing information about the shape is defined through *labels*. A set of grammatical rules called *shape rules* are defined on the set of shapes to map one shape or sub-shape onto another shape or sub-shape. Optimally directed search of the resulting design space is accomplished through the stochastic optimization technique of simulated annealing (Kirkpatrick et al., 1983). Given a current design state, an eligible rule selected at random from the shape grammar is applied to the design. If the new design produced does not violate any constraints, it is evaluated and compared to the old design; acceptance is determined by the simulated annealing algorithm (it is accepted as the new current design if better, and accepted with a probability, which decreases over time, if worse). If the new design state violates any constraint then the old state is maintained as the current one. To allow for backtracking out of local minima, any discrete rule is defined with a counterpart rule that removes the effects of the original rule when applied successively. This paper demonstrates how shape