Investigation of Characteristic Measures for the Analysis and Synthesis of Precision-Machined Surfaces [1]

Irem Y. Tumer, R.S. Srinivasan, and Kristin L. Wood, Mechanical Engineering Department, The University of Texas at Austin, Austin, Texas

Abstract

Error prediction and control are key factors in precision machining. These factors rely on the development of formal approaches for analyzing and characterizing error sources in manufacturing. One such approach is the development of mathematical measures of precision, where precision, in this context, is defined as surface variations of manufactured part profiles. In this paper, we discuss a novel investigation of four mathematical measures. These four methods, namely the autocorrelation function, the Fourier spectrum, a fractal-wavelet representation, and the Karhunen-Loève expansion, are applied to surfaces produced from grinding processes. The first two methods provide a basis for the investigation, as they are commonly used in the literature for qualitative signal characterization of manufacturing surfaces. However, the fractal-wavelet method and Karhunen-Loève expansion have never been applied to the analysis and synthesis of surface variations. While other fractal methods have been used to characterize surface-finish variations, a wavelet formalism is a new approach, especially at the scales of *both* surface finish and tolerances. A combination of the first three techniques is shown to give a proper minimum set of characteristic precision measures for representing grinding surfaces. This combination is a clear contribution to the field of analysis of surface characteristics. It is also shown that the Karhunen-Loève technique is a novel alternative to represent surface errors. The existence of characteristic measures of surface precision should aid designers in choosing process and design parameters and in comparing the precision between competing machining processes.

Introduction

Precision Machining: Background and Motivation

The particular interest of this work is the development of precision measures for metal removal processes using solid tools, such as grinding. The main objective of efforts in precision engineering is to design machines with high and predictable work-zone accuracies [1]. These efforts lead to the need to understand the factors that affect machine performance, as well as the basic physics that characterizes a machine component or system. This is an essential step for the development of superior designs and proper selection of components. The design of quality precision machines depends primarily on the ability of engineers to predict machine performance and monitor process variations [1, 2]. In this work, process variations are studied as surface profile error of a manufactured part. Only profile height variations are considered for manufacturing surfaces. These variations correspond to geometric tolerances in the significant class of straightness, circularity, and profile errors. The methods are intuitively extendible to higher-order variations, such as flatness, but such extensions are beyond the scope of this paper and remain as future work. In the following sections, the concept of precision measures is introduced as a tool to aid in surface error prediction and process monitoring.

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Characteristic Measures

A measure is defined in this paper as a quantitative assessment of physical phenomena, represented mathematically and concisely as a minimum set of real-valued numbers. In terms of precision manufacturing, measures provide the potential of detecting and improving surface errors in high-precision product geometry. A measurement approach, of this form, will minimize the possibility of misinterpretation of physical data, thus introducing a formal representation. Mathematical measures will also help eliminate the exclusively empirical nature of manufacturing, and hence facilitate the integration of design and manufacturing [3]. By so doing, a mathematical measure has the potential to be used in conjunction with computational methods, enabling automation of the production and design cycle with minimal ambiguity [4].

A formal measurement of a surface profile can be accomplished by means of real-valued measures [5, 6]. The criteria from formal measurement theory dictate the need for a base point and a finite range. A base point is required as a reference from which surface coordinates are measured relatively. An upper limit to the measure, combined with the base point, defines the range. A finite range is required to compare the deviation of each surface coordinate from the base point.

In addition to these mathematical requirements, characteristic measures must satisfy a number of pragmatic needs in manufacturing. For example, process monitoring requires a means to *analyze* machined surfaces. Such analysis provides the necessary representation for determining the state of a process and the machine modes or sources that are causing error. Error prediction, on the other hand, implies the ability to *synthesize* surfaces. Synthesized surfaces may be used in design analysis or for comparing competing processes.

Goals and Current Focus

Measures of manufacturing precision will potentially achieve a number of goals. One possible use of measures is to select proper processes and parameters by matching specified precision requirements in the expected machine performance. Another foreseeable use of precision measures is control of a machining process, by monitoring and identifying errors, and compensating for them. A valuable use of measures in design is to identify performance limits and indicate allowable variations, thus serving as a tool of communication between design and manufacturing. For example, given a type of process and system parameters, measures can be used to generate the expected surface profile variations, and generate mating surfaces to verify assemblability. Another valuable use in design is to improve machine design by comparing the machine precision to the precision of a component surface, and varying the design parameters until a match is found. Finally, precision measures can be used as a quality control tool to ensure proper performance of the part. The ultimate goal in the project is to provide a link between manufacturing and design, working towards integrating the two fields.

To meet this goal, we seek to develop a method to characterize surfaces from manufacturing processes, and derive measures that describe the precision of surfaces. Methods commonly used in the literature to quantify surface errors, in addition to other signal analysis methods, are investigated as possible candidates to represent surface precision. Four methods form the basis for this investigation, namely the autocorrelation function, the Fourier spectrum, a fractal-wavelet representation, and the Karhunen-Loève expansion. The applicability of these methods to machining processes is then studied, with a focus on a precision grinding process. Subsequently, the use of measures is verified by applying them to a design example. The design example consists of machine elements, namely gears, whose functional requirements are directly influenced by surface irregularities.

Surface Analysis and Synthesis Methods

There exist several methods that can be applied to the analysis and synthesis of machined surface profiles. For example, the autocorrelation function and Fourier spectrum have been widely applied to qualitatively distinguish among different profiles and to monitor signals in manufacturing processes [7, 8, 9, 10]. Likewise, the surface finish of parts from manufacturing processes has been studied using fractal dimensions [11, 12]. These methods form the foundation of our investigation, with two significant extensions. First, we introduce a novel fractal-based representation of surface profiles [13, 14]. This representation is based on fractal-wavelet mathematics and is applied at *both* surface finish and tolerance scales. While wavelet theory has been used for signal processing purposes [15, 16], it has never been applied to the study of surface characteristics of parts from manufacturing, considering both surface finish and tolerances scales together. As a second extension, the Karhunen-Loève expansion is formulated as an alternative representation approach. While the Karhunen-Loève expansion has been used in speech and pattern recognition applications [17, 18], as well as in characterizing turbulent fluid flow [19], it also has never been applied to characterize machining surfaces.

Since the autocorrelation function and Fourier spectrum techniques are widely applied to manufacturing surfaces, the details of the mathematics and the interpretation of the methods are available in the literature. As a result, only a brief summary of these two methods will be presented here. However, the fractal-wavelet approach is a novel approach in the field and thus needs to be explained in more detail. Finally, the Karhunen-Loève technique has never been applied to manufacturing and hence requires more detail as well. The fundamentals of the last two methods are presented below in the context of precision machining.

Autocorrelation Function

The autocorrelation function (ACF), denoted $\rho(h)$, is a measure of the dependence structure in a profile; i.e., it indicates the degree of similarity between a profile, and a copy of itself, translated by h units (h is referred to as the lag). Equivalently, $\rho(h)$ is interpreted as a measure of the dependence of the profile value at a given location, on the profile value h units downstream [20, 21].

The ACF can reflect specific properties of a data set, such as periodicity, randomness, and existence of trends. In theory, synthesis from this method can be accomplished by fitting a model to the data, based on the information extracted from the ACF plot. In other words, if we can obtain the proper information from this method, such as the frequency and amplitude of the periodic component, or the amplitude of a constant trend, then a deterministic model can be fit to this information to synthesize a profile.

Fourier Spectrum

The Fourier spectrum is the frequency domain counterpart of the autocorrelation function [21]. The power spectrum is the square of the Fourier Transform (amplitude) per unit length [20, 22]. It can be interpreted as a measure of the energy per unit length (or the power) contained in a signal as a function of spatial frequency.

The power spectrum can reveal the presence of offsets, or periodic structures in a data set. It is also used to obtain a preliminary estimate of the frequency, before using more rigorous computations to determine the exact parameter values. If we can extract proper information about surface characteristics from this method alone, e.g., the frequency and amplitude of the main modes of profiles, then synthesis of the profiles is plausible. This might be accomplished by fitting a deterministic model to the data, using the frequency and amplitude information provided.

The Fractal-Wavelet Representation

A fractal representation of irregular objects was first introduced by Mandelbrot [23]. The concept is quantified by means of a fractal dimension [11]. The fractal representation of irregular surfaces is based on the idea of intermediate dimensions, i.e., the fractal dimension, which describes irregular geometries that deviate from the ideal dimensions of 1 for a line, and 2 for a surface, and so on.

In order to enable the forward and inverse mapping of fractal-based error information, the use of Wavelet transforms are introduced as a suitable model for the analysis and synthesis of profile errors [21]. Because fractals and wavelets share two properties of primary interest in machining processes, namely non-stationarity and statistical self-similarity, the mathematical theory of wavelets is shown to be directly applicable to entities possessing fractal structure [21]. The link between fractals and wavelets have been researched before [15, 16]. However, the Wavelet transforms in conjunction with fractals have never been applied to analyzing and synthesizing surfaces from manufacturing processes. As a result, the mathematics of this novel approach are presented in more detail in the following sections.

Based on the use of wavelet theory for multiresolution signal decomposition [15, 21], the approximation and detail spaces of surface profile signals are developed. The necessary mathematical operations for this task are based on Hilbert spaces. The approximation space is the minimum information needed to represent a signal, whereas the detail space is the additional information needed to exactly reconstruct a signal. Multiresolution analysis implies the study of a physical profile structure at different resolutions, thus enabling the study of different scales of errors. The central idea is to examine a given signal (error profile) as successive approximations with an increased degree of smoothing. Successive approximations correspond to different resolutions; the difference between two approximations is called the detail [21]. As a final step, the power spectra of fractal surfaces are used to derive a relationship between wavelets and fractals. From this relationship, two measures of precision, namely the fractal dimension and the magnitude factor, are derived. From this fractal-wavelet relationship, profiles of manufactured surfaces can be directly synthesized [21].

Approximation

The discrete approximation is interpreted in terms of filtering the signal with a low-pass filter (approximation). Consider a discrete filter H with the following impulse response:

$$h_k = (\phi_{-1}(u), \phi(u-k)), \qquad k \in \mathbf{Z}. \tag{1}$$

The mirror filter [15] $\tilde{H}$ is defined as having the impulse response $\tilde{h}_k = h_{-k}$. Using this notation, the discrete approximation at the resolution 2^m can be written as follows:

$$(A_m^k f)_d = (f(u), \phi_m(u - 2^{-m}k)) = \sum_{n=-\infty}^{\infty} \tilde{h}_{2k-n} \, (A_{m+1}^n f)_d. \tag{2}$$

This implies that the approximation at resolution 2^m is obtained by filtering the approximation at resolution 2^{m+1} with the filter $\tilde{H}$ and retaining every other data point [21].

Detail

The discrete detail at resolution 2^m can be written with the filter interpretation as follows: let g_k be the impulse response of the discrete filter G [21]:

$$g_k = (\psi_{-1}(u), \phi(u-k)), \qquad k \in \mathbf{Z}. \tag{3}$$

Defining a corresponding mirror filter $\tilde{G}$ with impulse response $\tilde{g}_k = g_{-k}$, the discrete detail is given by:

$$(D_m^k f)_d \;=\; (f(u), \psi_m(u - 2^{-m}k)) \;=\; \sum_{n=-\infty}^{\infty} \tilde{g}_{2k-n}\; (A_{m+1}^n f)_d. \tag{4}$$

Derivation of Measures from the Power Spectrum Approach

Fractal profiles and surfaces are characterized by power spectra $S(\xi)$ of the form:

$$S(\xi) \;\propto\; \xi^{-\beta(D_f)}, \tag{5}$$

where ξ is the frequency and D_f is the fractal dimension, and β is the spectral exponent. Consider the discrete details of a fractal profile. As the power spectrum is related to the autocorrelation function, and hence the variance [22], Wornell [16] presents a scaling argument in terms of the variance of the discrete detail signals $(D_m^k f)_d$:

$$\sigma^2[(D_m^k f)_d] = V_0\; 2^{\beta m}, \tag{6}$$

where V_0 is a constant. This parameter plays a crucial role in determining the magnitude of variation. With this equation, the fractal dimension is calculated from the slope of the log-log plot of the variance versus the scale 2^m.

Reconstruction and Synthesis of Profiles

Following the implementation of the approximation and detail operators for surface profiles, a method is needed for reconstructing the profile signal. Since the approximations and details at the coarser resolutions are known, the reconstruction yields the requisite approximation at finer resolutions. The reconstruction formula is written in terms of the filtering operation by simplification and by using the filter definitions in Equations 1 and 3:

$$(A_{m+1}^k f)_d \;=\; 2\sum_{n=-\infty}^{\infty} h_{k-2n}\; (A_m^n f)_d \quad + 2\sum_{n=-\infty}^{\infty} g_{k-2n}\; (D_m^n f)_d. \tag{7}$$

In the synthesis of profiles, details $(D_m^n f)_d$ are derived based on the power spectrum relationship between the fractal measures and the details (Equation 6) [21] .

The Karhunen-Loève Representation

Another possible representation of surfaces can be achieved in terms of a Karhunen-Loève (K-L) expansion, also called principal-components analysis in statistical literature. An example application of the K-L expansion method is in the efficient encoding of speech spectral data, successfully modeling the underlying structure of the spectral data [18]. Other applications are in the fields of pattern recognition [17], successfully reproducing faces using the major dominant modes, and in the detection of turbulent flow in the area of fluid dynamics [19]. Given an ensemble of patterns, the technique yields an optimal orthogonal basis for the representation of the ensemble. The K-L basis vectors are the m eigenvectors of the covariance matrix corresponding to the m largest eigenvalues of the matrix [18]. The K-L technique has never been considered for analyzing surface errors from manufactured parts. The technique also yields a measure of the relative contribution of each basis function to the total energy of the ensemble. The average energy is given by the eigenvalues of the covariance matrix. The covariance matrix contains the statistical properties of the original data set.

Analysis of Surfaces

The profile data is generated by simulation or experimentation [21]. A large number of such profiles must be generated to study the nature of the errors and to model them. Considering an ensemble of M profiles, each error profile is represented by a vector $\vec{X}_m = [x_m^1, x_m^2, ..., x_m^i, ..., x_m^N]^T$, where, i indicates the number of simulated/discretized points on the profile, and $m = 1, 2, ..., M$ is the identification number of the profile. The total number of data points (N) is called the dimensionality of the profile. The departure, $\vec{y}_m$, or deviation of the individual error data from the mean ($\overline{\vec{X}}$) is first computed. Next, the covariance matrix for the deviation ensemble is calculated as:

$$\vec{C} = \frac{1}{M} \sum_{m=1}^{M} \vec{y}_m \; [\vec{y}_m]^T. \tag{8}$$

The covariance matrix $\vec{C}$ is symmetric and non-negative [17].

For a given $\vec{C}$, the K-L basis functions are the eigenvectors $\vec{u}_i$ of $\vec{C}$ [24, 17]:

$$\vec{C} \; \vec{u}_i = \lambda_i \; \vec{u}_i \qquad\qquad 1 \leq i \leq N. \tag{9}$$

Since $\vec{C}$ is an $N \times N$ matrix, there exist N eigenvalues and eigenvectors. This K-L basis is optimal in the sense that a *subset* of these N eigenvectors (say $n, n < N$) are used to generate a truncated representation, such that the mean square error induced by the truncation is a minimum; i.e., the truncated representation closely resembles the original. This subset of eigenvectors corresponds to the n largest eigenvalues of $\vec{C}$ [17]. In other words, an important step in the optimal representation of the profile data is the computation of the eigenvalues of $\vec{C}$, and ranking them in an ascending order.

Reconstruction and Synthesis of Profiles

Any particular profile in the ensemble can be reconstructed from the significant eigenvectors [21]. Assuming all the eigenvectors $\vec{u}_i$ have been calculated, any deviation profile $\vec{y}$ is represented as:

$$\vec{y} = \sum_{i=1}^{N} a_i \vec{u}_i, \tag{10}$$

where a_i is the decomposition coefficient or component corresponding to the deviation $\vec{y}$, and the eigenvector $\vec{u}_i$, and is calculated using $a_i = [\vec{y}]^T \vec{u}_i$. As the eigenvalues are calculated, the decomposition coefficients are also computed. Then the profile is reconstructed from a truncated version of Equation 10. The optimality of the basis requires only the n significant eigenvectors to be considered in the summation, instead of all N terms. The complete (approximate) reconstruction (synthesis) of a profile is given by:

$$\vec{X} = \overline{\vec{X}} + \sum_{i=1}^{n} a_i \vec{u}_i. \quad (11)$$

A Machining Application: Error Profiles from Grinding

Following the preliminary discussion of the four methods investigated in this paper, this section presents a machining application to investigate the applicability of the methods. Recall that, in this work, given a machining process, i.e., precision grinding, the ability of each method to monitor changes in selected system parameters is verified.

Grinding is the most accurate of the common manufacturing processes and one of the most frequently used in manufacturing precision machine components [25]. Accuracy of the grinding process is a function of a number of factors, such as accuracy of the machine, dressing of the wheel, temperature control of the coolant, fixturing method, and feed rate. Desired precision accuracies can be reached when these factors are properly controlled. Consequently, it becomes crucial to monitor the process, and to predict the error profile given the machine and process parameters. In this work, the focus is on surface grinding, which is designed to produce high tolerance, low surface roughness, and flat planar surfaces [26]. The horizontal-spindle reciprocating-table surface grinder is noted for its precision. The amplitude of vibration of a grinding system does not have to be very large to affect process performance.

In this section, experimental data from a surface grinding process are analyzed using the four methods discussed above. The experimental data contain profiles with high and low values of the grinding wheel speed and workpiece speed. These factors result in surface variations, which must be detected and monitored by means of measures, as well as predicted given the machine and process parameters. For high surface precision, the workpiece speed is the most important parameter. In addition, first order interactions with the workpiece speed and the combined effect of workpiece speed with other parameters are also significant. High surface precision in surface grinding requires a low workpiece speed, a high wheel speed, and a low infeed rate [27].

The questions that must be answered are as follows: Can these methods detect the changes in the surface precision caused by varying the two parameters (wheel and workpiece speeds)? Can the methods be used as a tool for analysis and synthesis of error profiles? Can the methods provide measures that satisfy the set of criteria established previously?

Experimental Setup

The experiments are run on a surface grinder, with a medium grit size and hardness grinding wheel. The wheel is dressed before each experiment. The radial depth of cut (infeed) on the workpiece surface is 0.0127 mm (0.0005 $in.$). The parameters that are controlled in the experiments are the grinding wheel speed and the workpiece speed. Two runs (I and II) and two measurements for each run (a and b) are performed using the same combination of wheel and workpiece high and low speeds. These combinations are specified with the letters A to D. The high and low wheel speed values are 31.8 m/s (6259.0 ft/min) and 17.17 m/s (3500 ft/min). The high and low workpiece speed values are 0.27 m/s (53.6 ft/min) and 0.03 m/s (6.1 ft/min).

Autocorrelation Function Method

To compare surface precision trends using the autocorrelation function (ACF) technique, simulations are run to compute and graph the ACF of each of the grinding profiles. The autocorrelation function is then plotted versus the lag component, as explained previously. A sample ACF plot is shown in Figure 1. The ACF plots show that the experimental profiles have a dominant stochas-

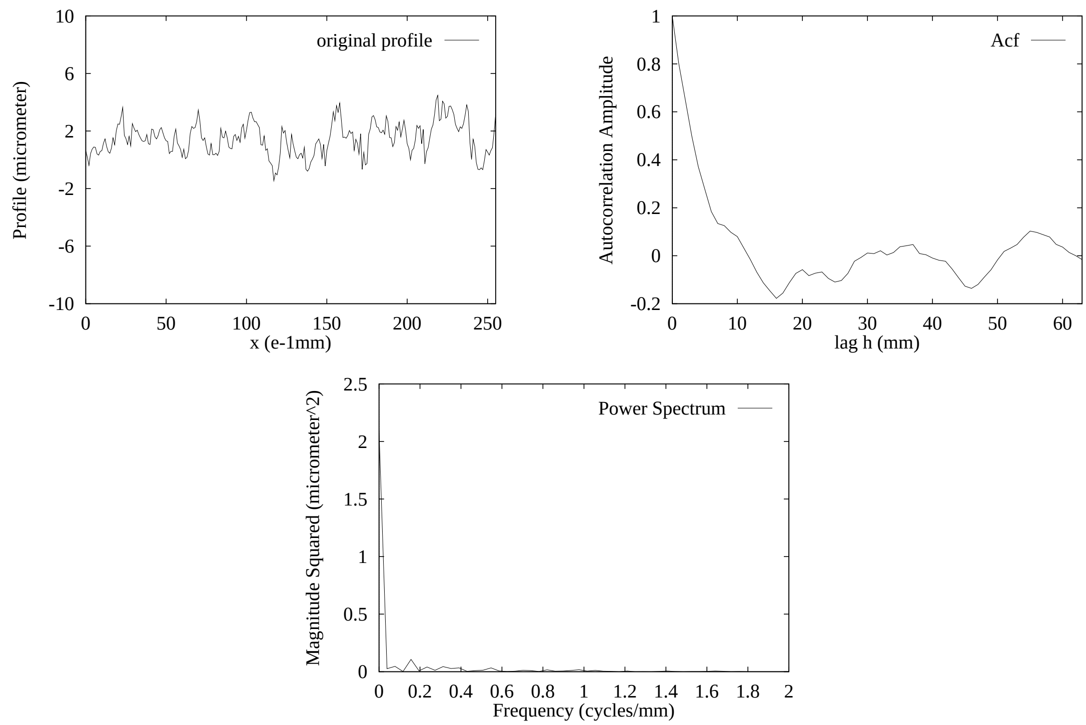

Figure 1: Grinding Profile, Autocorrelation Function, and Power Spectrum (grindII.Db).

tic component, and no significant periodicities. In addition, the experimental profiles have a trend, shown as a slow decay exhibiting a slope. The autocorrelation method has been used quite frequently in the literature [28]. From the examples found in literature, a successful application of the ACF method is when changes in machine and process parameters are monitored on a qualitative basis. The method is helpful for a broad classification of the surface profiles and the estimation of the presence and absence of features [9].

However, when applied to random grinding data, the method cannot provide a precise quantitative measures. There are some inherent problems with the method. Most data from machining processes have time-varying statistics, and are therefore nonstationary. Nonstationary effects may be due to statistically unpredictable freak marks on the surface due to inhomogeneity of the material, chip damage, a deep scratch, and so on. Such effects can cause corrosion and potential failure and should be detected and quantified as they occur. Accounting for nonstationarities in the profile is difficult with ACF, since they tend to integrate out from position to position [28], giving an average representation of the surfaces. Although several parameters can be extracted from the ACF plots, none of them gives a proper mathematical description of the surfaces. Due to the high stochastic component shown on the plots such as Figure 1, when plots for different data sets are compared, a distinguishing characteristic is not detectable. This difficulty in extracting characteristic measures makes an accurate synthesis of profiles from the ACF method alone impossible. Therefore, the autocorrelation method alone is not an effective tool for monitoring and predicting surface precision for grinding.

Fourier Spectrum Method

Similarly, simulations are run to compute and plot the Fourier spectrum of each experimental grinding profile. Figure 1 shows a sample spectrum plot of a ground surface profile.

Note that the spectrum method has been used widely in the literature [7, 10]. The Fourier spectrum of a surface is an efficient tool in observing trends and changes in a process, as well as identifying the modes and dominant frequencies of surfaces. For example, to reveal tool wear, the relative change in the magnitude of the harmonics of a spectrum is compared to the fundamental frequency component (e.g., feed). As more wear scars appear on the tool, more harmonics are added to the clean spectrum, hence increasing the relative harmonic to fundamental ratio. So, changes in the system conditions can be detected by monitoring for the emergence of new harmonics, and the change in their amplitudes [7, 10].

However, this method also has some inherent problems associated with it. The frequencies are not meaningful when studying highly random surfaces like grinding. As is shown in the example (Figure 1), the method only shows low frequency components, and does not provide a proper measure of the randomness characteristic of surfaces from grinding. In addition, there is some detail information lost due to the property of averaging over time, making detection of smaller features on the surface difficult. Furthermore, nonstationary profiles cause problems with this method as well [28]. Any difference in the nature of the data function is smoothed out because the total signal is encompassed under the integral sign [28]. Finally, when comparing ground surface profiles, such as the one shown in Figure 1, it is very difficult to distinguish between different grinding profiles. As a result, the method cannot be used to monitor the grinding process quantitatively, or to synthesize random surfaces effectively.

Fractal-Wavelet Method

The fractal-wavelet technique computes the fractal dimension and the magnitude factor of the machined profiles. When this method is applied to profiles from grinding, changes in parameters are reflected in the measures. The fractal-wavelet method appears to give more encouraging results than the first two methods. The fractal dimension reflects the structure of the error profile, and the magnitude factor captures the size of the variations on the surface. Note that the fractal dimension and the magnitude factor are real-valued and belong to a finite range. Furthermore, a change in the fractal dimension and the magnitude factor occurs when the wheel speed and the workpiece speed vary. Since it is possible to distinguish between different profiles using this technique, synthesis of profiles based on the fractal dimension is promising.

However, a problem is encountered when the measures are compared between replicates for a grinding process. For example, the profiles with the smallest fractal dimension from matching experiments from the two replicates should compare. However, the profile with the high wheel speed

Principal Component	*Eigenvalue (Replicate I)*	*Eigenvalue (Replicate II)*
1	2869.44	1812.1
2	179.37	126.29
3	125.53	88.22
4	102.98	65.92
5	86.1	56.56
6	61.64	40.48
7	27.5	31.19

Table 1: Principal KL Eigenvalues from the Grinding Profiles.

and high work speed, (I.Da), has the smallest fractal dimension in replicate I ($D_f = 1.52$), whereas, in replicate II, the profile with the low wheel speed and high work speed, (II.Ba), has the smallest fractal dimension ($D_f = 1.57$). Another problem emerges when the fractal measures are compared based on the expected trends from the physics of the grinding process. As discussed above, the experiment with the highest wheel speed and the lowest work speed should give the best surface, and hence the smallest fractal dimension. If we look at the profiles from the first replicate, the smallest fractal dimension is $D_f = 1.52$ from the profile with high wheel speed and high work speed (I.Da), whereas the profile that was expected to show the smallest dimension, with high wheel speed and low work speed, (I.Ca), has a dimension of $D_f = 1.706$, one of the highest values. Finally, the magnitudes of the fractal measures are in general too high, contrary to what is expected for a grinding process [21]. In conclusion, although this method gives promising results, the measures give inconsistent results, and thus the method requires further investigation. Furthermore, profiles synthesized using the fractal component only show a lack of certain information such as trends and periodic components.

Karhunen-Loève Method

The K-L expansion technique is applied to the ensemble of error profiles from surface grinding. Eight profiles from each measurement, with 256 sample points each, are assembled into a matrix, which is then fed into an algorithm to compute the covariance matrix $\vec{C}$ and its 256 eigenvalues and eigenvectors [29]. From these 256 eigenvalues, only seven have significant values. These seven eigenvalues represent the majority of the energy present in the signal, compared to the total set of 256 eigenvalues. Reconstruction of the profiles using these principal modes should give realistic results.

Analysis of Grinding Profiles

The seven eigenvalues, shown in Table 1, correspond to the principal components (modes) of the error profiles. Using this technique, the major modes are easily identified. The main four eigenvectors for Replicate I are shown in Figures 2. Note that the eigenvectors from Replicate II show similar trends. Note that the eigenvalues are always positive and real-valued, and when normalized by the total energy of the system, belong to a finite range between 0 and 1. As a result, the main criteria for mathematical measures are satisfied. In addition, the method can efficiently identify the dominant error modes, which correspond to the highest eigenvalues in the solution set of eigenvalues of the covariance matrix. Note that the method has not yet been investigated to verify whether changes in parameters are reflected in the minimum set of eigenvalues. In theory, the decomposition coefficients

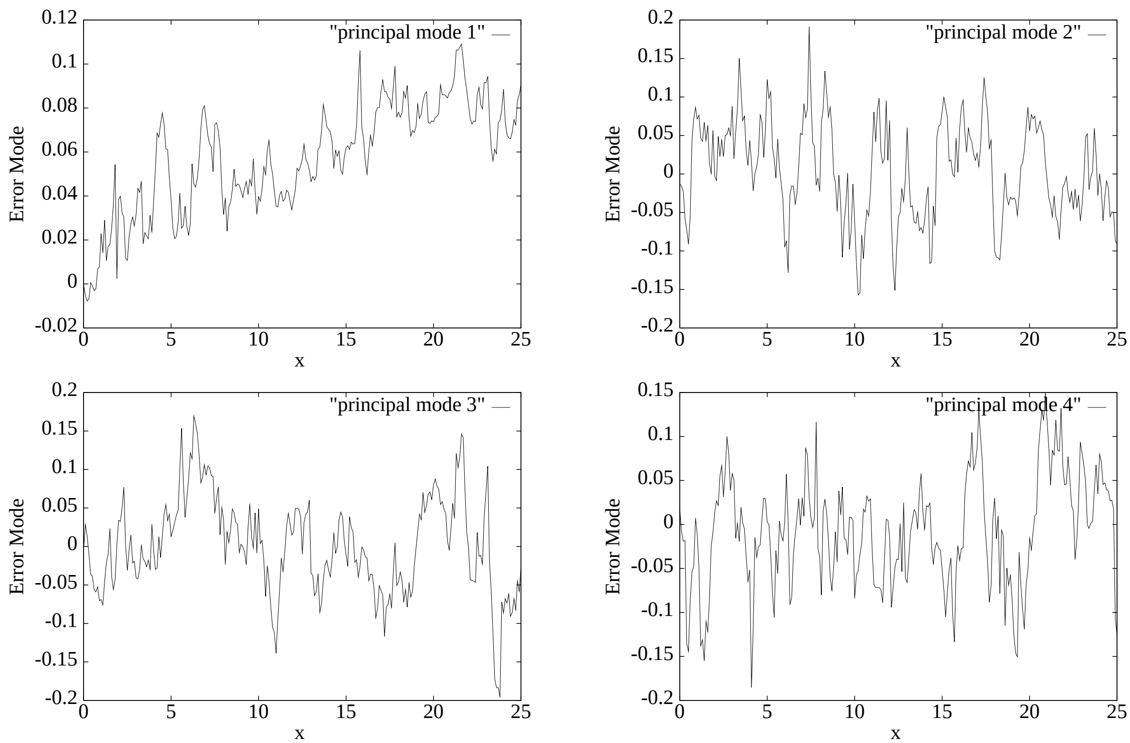

Figure 2: Main Four Modes for Grinding, Replicate I.

a_i corresponding to each deviation $\vec{y}$ and eigenvector $\vec{u_i}$ should be different for each set of profiles, thus allowing a comparison between different surfaces. However, the main question is whether these coefficients will correspond to actual physical factors. Further research remains to be performed on this aspect of the technique. The K-L technique described can nevertheless be used as an effective technique for analysis and synthesis of machining profiles.

Synthesis and Comparison

The synthesis procedure for K-L is tested using the grinding profile with low wheel speed and low work speed (I.A). The seven principal eigenvectors are used in Equation 11, and the reconstructed profile, along with the actual experimental profile, is shown in Figure 3. The reconstruction is nearly perfect, and achieved by using just seven eigenvectors instead of 256, thanks to the optimal representation properties of the K-L approach.

There are some attractive features of the Karhunen-Loève method, which can be used advantageously in the analysis of machined profiles. A paramount advantage in the use of this method is the ability it extends to isolate the primary design and manufacturing components contributing to the errors, even without prior knowledge of their identity. Each significant eigenvector or mode is interpreted as a factor influencing the profile generation process. However these are abstract factors, and are usually not directly interpretable in terms of physical factors [30]. *Target transformations* are prescribed to transform the abstract factors to real, physical factors [31]. However, this is a non-trivial problem. A joint inference from ANOVA and Karhunen-Loève decomposition is potentially

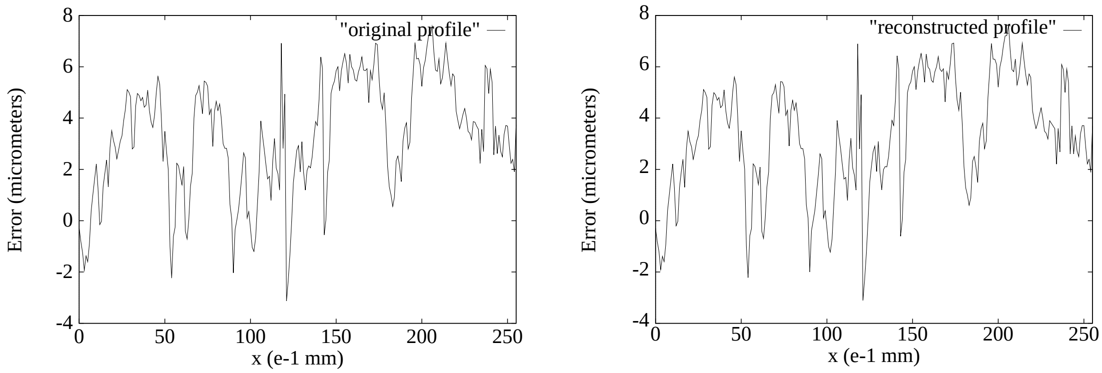

Figure 3: Grinding Experimental and Reconstructed Profiles with K-L approach (GrindI.Aa).

valuable information to determine these transformations.

While the K-L method is an extremely useful tool for studying manufacturing processes, it has its drawbacks [30]. The transformation is computationally burdensome. For instance, in the grinding experiments, the size of the covariance matrix is 256×256. Further, the transformation yields abstract factors, which in general have no direct physical connotations. While the number of eigenvectors is reduced drastically (e.g, from $N = 256$ to $n = 7$ in grinding), each eigenvector still must be expressed at N discrete points. This is a large quantity of information, and the abstraction is not as concise as the fractal parameters. As a result, another method for the analysis and synthesis of error profiles is sought, presented in the next section.

A Novel Method for the Analysis and Synthesis of Surfaces

In this section, the feasibility of a concise synthesis and analysis method is investigated. Having identified the shortcomings of each method, the idea is to combine the strong points of the methods presented above and develop a concise approach. Note that the K-L method is excluded from this investigation. Any attempts to combine the K-L method with other methods is left as part of future work. A hybrid method, combining the ACF, Fourier spectrum, and fractal-wavelet methods, along with a regression analysis of the data, is developed [21, 32] and presented next.

Subcomponents of Surface Profiles

Any surface from a machining process will have deterministic, as well as random characteristics. The deterministic components include trends and periodicities in a profile. The random component is best represented by the fractal model of the irregular surface. The fractal-wavelet representation handles nonstationary effects caused by the randomness of the grinding process, whereas the previous two mappings handle the deterministic trends and periodicities. In the following sections, the different components of a surface are identified first, and then combined using a superposition model.

In analyzing a given experimental surface profile from a precision grinding process, first a linear regression analysis is carried out on the profile data to obtain the trend information. In this case, all grinding profiles have an intercept and a slope, as can be inferred from the profile plots, and more

Profile (Rep.I)	Slope	Intercept (μm)	Profile (Rep.II)	Slope	Intercept (μm)
I.Aa	0.012156	1.752679	II.Aa	-0.003322	-1.894023
I.Ab	0.019312	1.748268	II.Ab	0.012707	0.121255
I.Ba	0.020890	-0.885780	II.Ba	0.023930	0.278721
I.Bb	0.011962	0.713972	II.Bb	-0.019406	1.086330
I.Ca	-0.011039	-0.815722	II.Ca	-0.029827	-0.178365
I.Cb	-0.022987	-2.522975	II.Cb	0.006601	0.821609
I.Da	-0.385428	1.496771	II.Da	0.009690	-0.045278
I.Db	0.011787	0.588932	II.Db	0.002408	1.142413

Table 2: Trend Parameters for Grinding.

definitively from the power spectra and the ACF plots. The ACF plot indicates the presence of a slope, by exhibiting slow decay of $\rho(h)$, as the lag h increases [20]. The power spectrum of a profile with an offset shows a peak at zero frequency. The trend $y_t(x)$ can be approximated by a straight line, and isolated by using linear regression techniques [33, 21]. This procedure compresses the trend information into two parameters, i.e., an intercept y_{t0}, and a slope s_t.

$$y_t(x) = y_{t0} + s_t \cdot x \qquad\qquad 0 \leq \ x \ \leq \ L. \tag{12}$$

The linear regression analysis is carried out for the grinding data and is shown in Table 2 [21, 32].

The periodic component is estimated from the surface profile by using a nonlinear regression procedure, providing three additional measures, namely frequency, amplitude, and offset [34, 33]. For the case of precision grinding, since neither the grinding profiles nor the ACF and Fourier spectrum plots indicate the presence of a periodic component, the nonlinear regression analysis is not carried out [21].

After detrending the grinding profiles, the fractal-based method is used to extract the fractal dimension and magnitude factor. The fractal dimension effectively describes the structure resulting from complex processes [23]. As a result, the fractal component takes into account effects that cannot be represented with the other two components. The results from this technique, shown in Table 3, follow expected trends [21, 32]. First, the parameter changes are reflected by the fractal measures, and hence the profiles are shown to be fractal. Second, the fractal measures have smaller values, hence corresponding to expected precision levels from a grinding process. Finally, in Replicate I, as expected, the profile with the high wheel speed and low workpiece speed has the smallest fractal measure. Recall that the fractal-wavelet method had some problems associated with it. These problems are handled by first removing the deterministic trends, such as linear trends and periodicities [21].

Synthesis and Comparison of Precision-Ground Surfaces

Once the measures are known, the wavelet reconstruction procedure is used to synthesize the fractal component of the surface, based on the idea that the details from the wavelet analysis are related to the fractal information [21, 16]. This reconstruction step is followed by invoking respective models to synthesize the deterministic components of the surface profiles. As a final step, a superposition model is used to reconstruct the entire surface [21, 32]. Assuming independence of the three components identified above, a superposition model is defined as follows [21, 32]:

$$y(x) = y_t(x) + y_p(x) + y_f(x), \tag{13}$$

REP.No	Wheel Speed	Workpiece Speed	Spectral Exponent	Fractal Dimension D_f^{Δ}	Magnitude Factor (μm^2)
I.Aa	low	low	0.456197	1.271902	0.158817
I.Ab			0.508043	1.245979	0.188157
I.Ba	low	high	0.640279	1.179861	0.103456
I.Bb			0.425596	1.287216	0.120276
I.Ca	high	low	0.823478	1.088261	0.136429
I.Cb			0.696449	1.151776	0.110229
I.Da	high	high	0.185799	1.407101	0.062795
I.Db			0.162700	1.418650	0.065793
II.Aa	low	low	0.464151	1.267925	0.079817
II.Ab			0.130880	1.143456	0.073059
II.Ba	low	high	0.035566	1.482217	0.239665
II.Bb			0.102393	1.448804	0.244190
II.Ca	high	low	0.494507	1.252747	0.065129
II.Cb			0.427942	1.286029	0.067165
II.Da	high	high	0.508013	1.245994	0.030870
II.Db			0.504878	1.247561	0.057347

Table 3: Fractal Parameters for Grinding.

where $y_t(x)$, $y_p(x)$, and $y_f(x)$ are the trend, periodic, and fractal components respectively.

An example of an actual grinding profile, and the corresponding synthesized version is shown in Figure 4. A preliminary visual comparison proves how well the two plots conform. Here, the fractal parameters are obtained using the analysis method described above, and fed into the wavelet reconstruction algorithm to synthesize an error profile [21]. In actuality, the fractal parameters could, for example, come from the specifications of the part. The error profile can then be synthesized prior to machining, and used by the engineer to make *a priori* design decisions about the part.

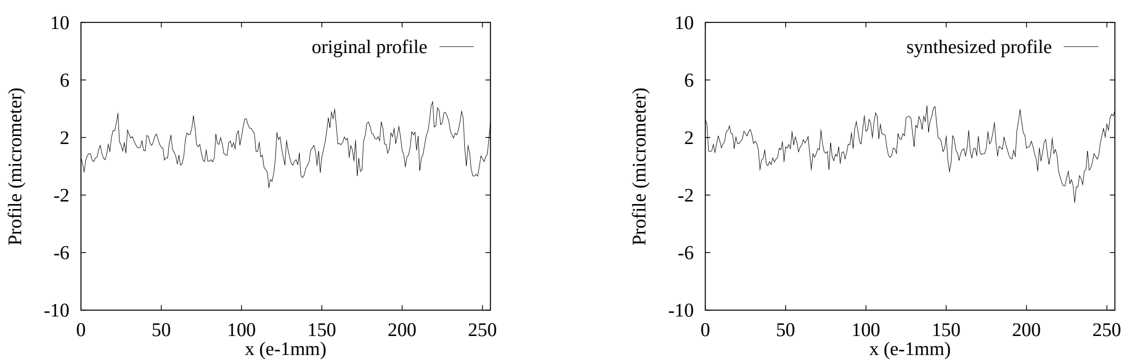

Figure 4: Experimental and Synthesized Profiles: Grind-II.Db.

Precision Measures from the Hybrid Method

Using the hybrid method discussed above, a minimum set of measures for any surface profile from a manufacturing process includes the following parameters: fractal dimension and magnitude factor from the fractal-wavelet method; intercept and slope from the linear regression analysis; and frequency, amplitude, and offset from the nonlinear regression analysis. In the case of precision grinding, no periodic component is detected. As a result, the set of measures includes the first four measures only, listed above. All four of these parameters satisfy the mathematical criteria established for measures. The fractal dimension and magnitude factor for the grinding data are shown in Table 3. They are both positive and real values. The fractal dimension has a base point of 1 and belongs to the finite range between 1 and 2. The magnitude factor has a base point of 0 and can be normalized to be reduced to a finite range. The intercept and slope for grinding are shown in Table 2. They are both positive and real values. The fractal dimension has a base point of 1 and belongs to the finite range between 1 and 2. The magnitude factor has a base point of 0 and can be normalized to be reduced to a finite range. The intercept and slope for grinding are shown in Table 2. They are both real-valued measures with a base point of 0. The values deviate from the base point in both the negative and positive directions, and can be reduced to belong to a finite range when normalized. Recall that this minimum set of measures can now be used to effectively characterize the precision of surfaces from grinding processes. These measures can potentially be used to monitor surface errors produced during manufacturing processes, as well as to predict surface errors prior to machining.

Application to a Design Example

This section investigates whether the method discussed in the previous section can be applied to a design application, involving surface synthesis. The design application must involve functional parameters that are directly affected by surface irregularities caused by machining errors. An effective way to judge the validity of the methods is to compare functional parameters from experimental profiles with functional parameters from profiles synthesized using the methodology described above. In the following sections, the combined method of the previous section, along with the experimental data from grinding, are applied to a design example (gears).

Precision of Gear Teeth and Transmission Error

For good gear performance, the toothed wheel has its teeth arranged so that, when they are meshing with the teeth on another toothed wheel, motion is transmitted. This transmission can, however, be affected when the teeth are not machined accurately to the required precision levels. Many high-precision, fine-pitch gears have their teeth ground from the blank, either by form-grinding or by generating grinding [35]. Consequently, the degree of precision of ground gear teeth becomes a very desirable piece of information.

In precision gear applications, the transmission of motion from shaft to shaft must have a high degree of linearity. Theoretically, involute gears will function perfectly. In reality, however, there are deviations from ideal motion transmission as a result of involute profile variations and machining errors [35], described by an expression of transmission error [36]. Transmission error is defined as the deviation in the position of the driven gear for any position of the driving gear, relative to the position the driving gear would occupy if both gears were geometrically perfect and undeformed [37].

To facilitate the assessment of machining errors on performance, defined by the transmission error, the problem is presented as follows. The transmission error, $TE(x)$, is expressed as a function of the local composite error, $\epsilon_i(x)$, and the local stiffness, $k_i(x)$, of the ith tooth pair, and the normal transmitting force in the transverse section of gears (W) [38]:

$$TE(x) = \frac{W + \sum_{i=1}^{n} \epsilon_i(x) k_i(x)}{\sum_i^n k_i(x)}, \qquad 0 \le x \le L \tag{14}$$

where n is the total number of tooth pairs in the zone of contact.

Comparison of Experimental and Synthesized Gear Profiles

The parameters for the gear pair used in this study are given in [39, 21]. The length of contact L is calculated as [36]:

$$L = \sqrt{R_{tg}^2 - R_{bg}^2} + \sqrt{R_{tp}^2 - R_{bp}^2} - \tan\phi(R_{bg} + R_{bp}), \tag{15}$$

where R_{bg} is the radius of the base circle for the gear, and R_{tp} is the outside circle radius of the pinion, and so on. As the contact ratio is unity, the expression for transmission error becomes:

$$T.E.(x) = \frac{W + \epsilon(x)k(x)}{k(x)}, \qquad 0 \le x \le L. \tag{16}$$

The stiffness values $k(x)$ are calculated by assuming $k(x) = k_0 \cdot x$, where k_0 is a constant stiffness [38].

Two grinding profiles (profiles I.Da and I.Db) are used as error profiles on a pair of meshing gear teeth. Equation 14 is used to calculate "experimental transmission error", and then the calculation is repeated with the corresponding synthesized errors, to obtain "synthesized transmission error" [21, 32]. The plots of these errors for one mesh cycle are shown in Figure 5. Note that the overall pattern of the error profiles and the maximum and minimum values compare well. From the definition of the transmission error, (Equation 16), the component due to the compliance, i.e., $W/k_i(x)$, decreases with the progression of meshing [38]. The component due to the profile errors is called the transmission error of "unloaded gears" [39]. For a perfect gear pair, this component is zero. However, the total transmission error for both the experimental and synthesized profiles is increasing in a mesh cycle; the magnitude of the error, $10\mu m$ could be acceptable, for example in a concrete mixer. For more critical applications, like machine tool gears, this error is unacceptable, indicating the need for more precise gear finishing methods.

This example establishes the applicability of the proposed methodology in synthesizing realistic part models. Using similar results, the designer can choose the proper process and machine parameter combinations to reach a compromise between performance and time of manufacture. For example, the designer can vary process parameters and compare system performance [32]. Is the performance acceptable if higher speeds (required for higher precision) are used, thus reducing manufacturing time?

Conclusions and Future Work

In this paper, we investigate the feasibility of finding characteristic measures of precision for precision-ground surfaces. Four methods are investigated based on the objectives and criteria established to allow for monitoring and prediction of the surface errors. The methods are applied to

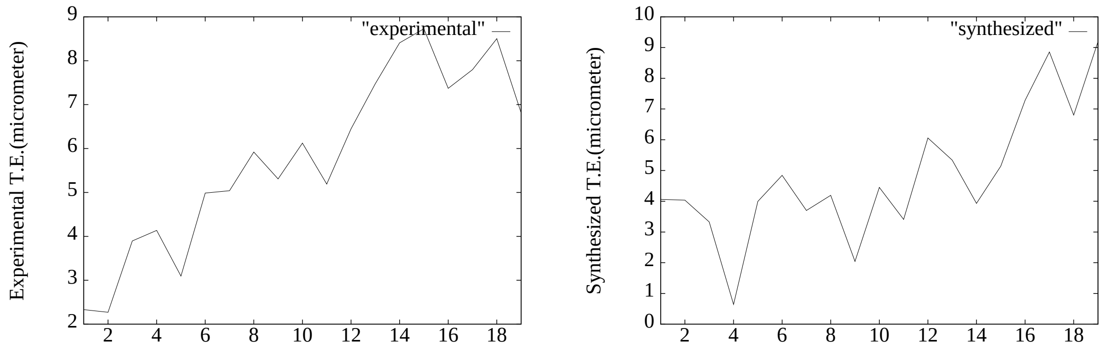

Figure 5: Transmission Error: Experimental & Synthesized Grinding Profiles.

machined surfaces from a surface grinding experiment. The results are used to identify the shortcomings of each method. Based on the results, a novel method that incorporates the autocorrelation function, power spectrum, fractal-wavelet method, and regression analysis is presented, and a minimum set of measures is derived. The method is then applied to a design problem to verify that functional parameters from experimental surfaces and from synthesized surfaces compare effectively.

The Karhunen-Loève technique is presented as an alternative to the hybrid fractal-wavelet method. However, there are currently limitations in determining the physical significance of the measures from the K-L method, as well as difficulties due to the computational complexities. Future work will explore the extensions to this alternative.

Based on this summary, contributions of this work are stated as follows. First, we present a clear and objective comparison of surface representation techniques based on measurement criteria. In addition, we present the development of a novel application of the fractal-wavelet method to surface variations at both tolerance and surface finish scales. More specifically, we present the development of a superposition model, combining individual deterministic and stochastic errors, for the *analysis* and *synthesis* of manufacturing surfaces, and validate this model with experimental results. The developed approach is then applied to a common design example to illustrate a possible use in the real world. Finally, we illustrate a novel application of the Karhunen-Loève technique to the analysis of error modes on surfaces from manufacturing and to the synthesis of surface variations based on an optimal set of error modes.

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