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CHARACTERISTIC MEASURES FOR THE REPRESENTATION OF MANUFACTURED SURFACE QUALITY

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ABSTRACT

In this paper, we investigate four methods that yield mathematical measures to analyze the precision of surfaces of manufactured parts. These four methods, namely the autocorrelation function, the Fourier spectrum, the Karhunen-Loève expansion, and a fractal-wavelet representation, are applied to surfaces produced from grinding processes. The first two methods are standard methods used in the surface analysis literature for qualitative signal characterization. The Karhunen-Loève expansion method, used in various signal processing applications, has never been applied to the field of surface characterization and representation. The fractal-wavelet representation has been previously proposed by the authors; its suitability to generate characteristic measures is investigated in this paper. The existence of characteristic measures of surface precision should aid designers in choosing process and design parameters and in comparing the precision between competing machining processes. The use of such measures is essential in taking a forward step towards integrating the fields of design and manufacturing.

INTRODUCTION

Errors in manufacturing systems manifest themselves in the form of deviations on the surfaces of manufactured parts, and hence are often studied so that they can be eliminated from the part production process. The manufacturing of accurate parts is dependent upon being able to predict and control the errors that occur in manufacturing machines. To enable error prediction and control, methods to "characterize" the structure and "measure" the severity of these errors are required. In this work, we investigate two standard methods of characterizing errors on manufactured parts, and compare them to two novel methods developed by the authors. This comparison is done based on parts generated using a factorial experimental design, by varying the selected process variables in a surface grinding process and "measuring" the error on their surfaces.

Precision of Machines: Background and Motivation

The particular interest of this work is the development of precision measures for metal removal processes using solid tools, such as grinding. The strict requirements to produce high precision parts leads to the necessity to design machines with high and predictable work-zone accuracies (McKeown, 1987). These efforts lead to the need to understand the factors that affect machine performance, as well as the basic physics that characterizes a machine component or system. The design of quality precision machines depends primarily on the ability of engineers to predict machine performance and monitor process variations (McKeown, 1987). In this work, surfaces are analyzed in order to study the effect of process variations on the surface profile errors of a manufactured part. Surface analysis relies on the assumption that the surface geometry irregularities can be used as a fingerprint of the process and machine tool (Whitehouse, 1994). The slightest change in the process parameters or the condition of the machine tool will manifest itself as changes in surface geometry, in size, texture, form, or a combination (Whitehouse, 1994). In this paper, only profile height variations are considered for manufacturing surfaces. In the following sections, the concept of precision measures is introduced as a tool to aid in surface

error prediction and process monitoring.

Why Characteristic Measures?

A measure is defined in this paper as a quantitative assessment of physical phenomena, represented mathematically and concisely as a minimum set of real-valued numbers. In terms of precision manufacturing, measures provide the potential of detecting and improving surface errors in high-precision product geometry. A measurement approach, of this form, will minimize the possibility of misinterpretation of physical data, thus introducing a formal representation. Mathematical measures will also help eliminate the exclusively empirical nature of manufacturing, and hence facilitate the integration of design and manufacturing (Delbressine and VanderWolf, 1990). By so doing, a mathematical measure has the potential to be used in conjunction with computational methods, enabling automation of the production and design cycle with minimal ambiguity (Wirtz et al., 1993).

A formal measurement of a surface profile can be accomplished by means of real-valued measures (Krantz et al., 1971; Otto, 1993). The criteria from formal measurement theory dictate the need for a base point and a finite range. A base point is required as a reference from which surface coordinates are measured relatively. An upper limit to the measure, combined with the base point, defines the range. A finite range is required to compare the deviation of each surface coordinate from the base point.

In addition to these mathematical requirements, characteristic measures must satisfy a number of pragmatic needs in manufacturing. For example, process monitoring requires a means to *analyze* machined surfaces. Such analysis provides the necessary representation for determining the state of a process and the machine modes or sources that are causing error. Error prediction, on the other hand, implies the ability to *synthesize* surfaces. Synthesized surfaces may be used in design analysis or for comparing competing processes.

Overall Goals and Current Focus

The goal in the project is to provide a link between manufacturing and design, working towards integrating the two fields. To meet this goal, we seek to develop a method to characterize surfaces from manufacturing processes, and derive measures that describe the precision of surfaces. Ultimately, we will develop methods to detect and quantify all of the significant faults that occur during the manufacturing of a part. The purpose is to assure the precision of a manufactured part by monitoring the faults in the manufacturing machine; automated detection and quantification of faults will help eliminate the production of poor quality parts.

Methods commonly used in the literature to quantify surface errors, in addition to other signal analysis methods, are investigated as possible candidates to represent surface precision. Four methods form the basis for this investigation, namely the autocorrelation function, the Fourier spectrum, the Karhunen-Loève expansion, and the fractal-wavelet representation. The applicability of these methods to machining processes is then studied, with a focus on a precision grinding process. Subsequently, the use of measures is verified by applying them to a design example. The design example consists of machine elements, namely gears, whose functional requirements are directly influenced by surface irregularities.

SURFACE CHARACTERIZATION METHODS

There exist several methods that can be applied to the analysis and synthesis of machined surface profiles. The first two methods, namely the autocorrelation function and the Fourier spectrum representations, are standard random process analysis tools used to extract the most useful information from surface profiles (Hingle, 1987; Whitehouse, 1994). The Karhunen-Loève representation is formulated as an alternative approach. While the Karhunen-Loève expansion has been used in speech and pattern recognition applications (Sirovich and Kirby, 1987; Zahorian and Rothenberg, 1981), as well as in characterizing turbulent fluid flow (Ball et al., 1991), it also has never been applied to characterize machining surfaces. Finally, the fourth method, namely, the fractal-wavelet representation, incorporates fractals and wavelets to form a unique approach to surface quality representation. The surface finish of parts from manufacturing processes has been studied using fractal measures (Grosser et al., 1992). The authors have proposed a comprehensive method that uses fractal measures in conjunction with a wavelet decomposition of surface profiles in (Srinivasan et al., 1995; Tumer et al., 1995). This representation is based on fractal-wavelet mathematics and is applied at *both* surface finish and tolerance scales. While wavelet theory has been used for signal processing purposes (Mallat, 1989; Wornell, 1991), it has never been applied to the study of surface characteristics of parts from manufacturing, considering both surface finish and tolerances scales together. The fundamentals of these methods are presented in the following subsections.

Autocorrelation Function Representation

The autocorrelation function (ACF) is a measure of the dependence structure in a profile, indicating the degree of similarity between a profile, and a copy of itself, translated by a fixed amount (Bendat and Piersol, 1986). The time structure of a signal in the time (lag) domain can be characterized by computing the average dependence between two specific times, called lag.

The ACF can reflect specific properties of a data set, such as periodicity, randomness, and existence of trends.

The autocorrelation is not invertible. However, in theory, synthesis from this method can be accomplished by fitting a model to the data, based on the information extracted from the ACF plot. In other words, if we can obtain the proper information from this method, such as the frequency and amplitude of the periodic component, or the amplitude of a constant trend, then a deterministic model can be fit to this information to synthesize a profile.

Fourier Spectrum Representation

The Fourier spectrum is the frequency domain counterpart (Fourier transform) of the autocorrelation function (Bendat and Piersol, 1986). Strictly speaking, the Fourier transform (FT) of a random process $x(t)$ does not exist, since the random process is of infinite duration and, hence, not absolutely integrable (Bendat and Piersol, 1986). Instead, the FT of the correlation function is used, which corresponds to the power spectral density function, and describes how the power in a signal is distributed over frequency (Bendat and Piersol, 1986).

The power spectrum can reveal the presence of offsets, or periodic structures in a data set. If we can extract proper information about surface characteristics from this method alone, e.g., the frequency and amplitude of the main modes of profiles, then synthesis of the profiles is plausible. This synthesis might be accomplished by fitting a deterministic model to the data, using the frequency and amplitude information provided. However, the reconstruction of the stochastic data using this method is not possible.

The Karhunen-Loève Representation

Another possible representation of surfaces can be achieved using a Karhunen-Loève (KL) expansion, also called principal-components analysis in statistical literature. An example application of the KL expansion method is in the efficient encoding of speech spectral data, successfully modeling the its underlying structure (Zahorian and Rothenberg, 1981). Other applications are in pattern recognition (Sirovich and Kirby, 1987), successfully reproducing faces using the major dominant modes, and in the detection of turbulent flow in the area of fluid dynamics (Ball et al., 1991). Given an ensemble of patterns, the technique yields an optimal orthogonal basis for the representation of the ensemble. The KL basis vectors are the m eigenvectors corresponding to the m largest eigenvalues of the covariance matrix (Zahorian and Rothenberg, 1981). The KL technique has never been considered for analyzing surface errors from manufactured parts. The technique also yields a measure of the relative contribution of each basis function to the total energy of the ensemble. The average energy is given by the eigenvalues of the covariance matrix. The covariance matrix contains the statistical properties of the original data set.

Analysis of Surfaces The profile data are generated by simulation or experimentation (Tumer et al., 1995). Considering an ensemble of M profiles, each error profile is represented by a vector $\vec{X}_m = [x_m^1, x_m^2, ..., x_m^i, ..., x_m^N]^T$, where, $i = 1, 2, ..., N$ indicates the number of simulated/discretized points on the profile, and $m = 1, 2, ..., M$ is the identification number of the profile. The total number of data points (N) is called the dimensionality of the profile. The departure, $\vec{y}_m$, or deviation of the individual error data from the mean ($\vec{\bar{X}}$) is first computed. Using these departures, the covariance matrix for the deviation ensemble is calculated as:

$$\vec{C} = \frac{1}{M} \sum_{m=1}^{M} \vec{y}_m \, [\vec{y}_m]^T \tag{1}$$

The covariance matrix is symmetric and non-negative and contains the intrinsic variational properties of the data (Sirovich and Kirby, 1987). For a given $\vec{C}$, the KL basis functions are the eigenvectors $\vec{u}_i$ of $\vec{C}$ (Sirovich and Kirby, 1987):

$$\vec{C} \; \vec{u}_i = \lambda_i \; \vec{u}_i \qquad\qquad 1 \leq i \leq N. \tag{2}$$

Since $\vec{C}$ is an $N \times N$ matrix, there exist N eigenvalues and eigenvectors. The eigenvalues and eigenvectors can thus be easily computed to obtain principal components for the data. Each of these principal components can be used to characterize individual fault components that result in errors on part surfaces. These principal components might, for example, correspond to basis vectors for profile height variations, straightness errors, or circularity errors.

This KL basis is optimal in the sense that a subset of these N eigenvectors (say n, $n < N$) are used to generate a truncated representation, such that the mean square error induced by the truncation is a minimum; i.e., the truncated representation closely resembles the original. This subset of eigenvectors corresponds to the n largest eigenvalues of $\vec{C}$ (Sirovich and Kirby, 1987). The mean square error in KL is determined by the sum of the eigenvalues for $i > n$; i.e., $MSE = \sum_{i=n+1}^{N} \lambda_i$ (Fukunaga, 1990). Therefore, deleting the ith eigenvalue from the set of KL eigenvalues decreases the mean square error by λ_i (Fukunaga, 1990). As a result, an important step in the optimal representation of the profile data is computing the eigenvalues of $\vec{C}$, ranking them in an ascending order, and selecting the $n < N$ dominant eigenvalues that make up the majority of the energy ($90 - 95\%$ is generally sufficient) (Ball et al., 1991; Fukunaga, 1990).

Reconstruction of Profiles Any particular profile in the ensemble can be reconstructed from the significant eigen-

vectors (Tumer et al., 1995). The complete reconstruction of a profile is given by:

$$\vec{X} = \overline{\vec{X}} + \sum_{i=1}^{n} a_i \vec{u}_i. \tag{3}$$

where $\vec{y} = \sum_{i=1}^{N} a_i \vec{u}_i$ is the profile deviation, a_i is the decomposition coefficient or component corresponding to the deviation $\vec{y}$ and the eigenvector $\vec{u}_i$, and is calculated using $a_i = [\vec{y}]^T \vec{u}_i$. As the eigenvalues are calculated, the decomposition coefficients are also computed. While the basis vectors provide the general structure of the spatial modes on manufactured parts, the decomposition coefficients indicate the magnitude of the errors. Then the profile is reconstructed from a truncated version of Equation 3. The optimality of the basis requires only the n significant eigenvectors to be considered in the summation, instead of all N terms. As a result, the dimensionality of the problem is reduced from N to n, so that most of the significant profile information is contained in the first n eigenvectors. This low-dimensionality is an expected result for dynamical systems (Srinivasan et al., 1995).

Synthesis of error profiles using the KL method is currently being investigated. One conceivable approach is to statistically generate the amplitudes of the decomposition coefficients a_i in order to synthesize and predict error surfaces given the deterministic basis functions. This aspect will be investigated further.

The Fractal-Wavelet Representation

A fractal representation of irregular objects was first introduced by Mandelbrot (Mandelbrot, 1983). The concept is quantified by means of a fractal dimension (Grosser et al., 1992). The fractal representation of irregular surfaces is based on the idea of intermediate dimensions, i.e., the fractal dimension, which describes irregular geometries that deviate from the ideal dimensions of 1 for a line, and 2 for a surface, and so on.

In order to enable the forward and inverse mapping of fractal-based error information, the use of Wavelet transforms are introduced as a suitable model for the analysis and synthesis of profile errors (Srinivasan et al., 1995). Because fractals and wavelets share two properties of primary interest in machining processes, namely non-stationarity and statistical self-similarity, the mathematical theory of wavelets is shown to be directly applicable to entities possessing fractal structure. The link between fractals and wavelets have been researched before (Mallat, 1989; Wornell, 1991). However, the first novel application of the Wavelet transforms in conjunction with fractals to the analysis and synthesis of surfaces from manufacturing processes was presented by the authors in (Srinivasan et al., 1995; Tumer et al., 1995). As a result, the mathematics of this novel approach are presented in more detail in the following sections.

Background Based on the use of wavelet theory for multiresolution signal decomposition (Mallat, 1989), the approximation and detail spaces of surface profile signals are developed. The necessary mathematical operations for this task are based on Hilbert spaces. The approximation space is the minimum information needed to represent a signal, whereas the detail space is the additional information needed to exactly reconstruct a signal. Multiresolution analysis implies the study of a physical profile structure at different resolutions, thus enabling the study of different scales of errors. The central idea is to examine a given signal (error profile) as successive approximations with an increased degree of smoothing. Successive approximations correspond to different resolutions; the difference between two approximations is called the detail. As a final step, the power spectra of fractal surfaces are used to derive a relationship between wavelets and fractals. From this relationship, two measures of precision, namely the fractal dimension and the magnitude factor, are derived. From this fractal-wavelet relationship, profiles of manufactured surfaces can be directly synthesized.

Approximation The discrete approximation is interpreted in terms of filtering the signal with a low-pass filter (approximation). Consider a discrete filter H with the following impulse response:

$$h_k = (\phi_{-1}(u), \phi(u-k)), \qquad k \in \mathbf{Z} \tag{4}$$

The mirror filter (Mallat, 1989) $\tilde{H}$ is defined as having the impulse response $\tilde{h}_k = h_{-k}$. Using this notation, the discrete approximation at the resolution 2^m can be written as follows:

$$A_m^k f = (f(u), \phi_m(u - 2^{-m}k)) = \sum_{n=-\infty}^{\infty} \tilde{h}_{2k-n} A_{m+1}^n f \tag{5}$$

This implies that the approximation at resolution 2^m is obtained by filtering the approximation at resolution 2^{m+1} with the filter $\tilde{H}$ and retaining every other data point (Srinivasan et al., 1995).

Detail The discrete detail at resolution 2^m can be written with the filter interpretation as follows: let g_k be the impulse response of the discrete filter G (Srinivasan et al., 1995):

$$g_k = (\psi_{-1}(u), \phi(u-k)), \qquad k \in \mathbf{Z} \tag{6}$$

Defining a corresponding mirror filter $\tilde{G}$ with impulse response $\tilde{g}_k = g_{-k}$, the discrete detail is given by:

$$D_m^k f = (f(u), \psi_m(u - 2^{-m}k)) = \sum_{n=-\infty}^{\infty} \tilde{g}_{2k-n} A_{m+1}^n f \quad (7)$$

Derivation of Fractal Measures Fractal profiles and surfaces are characterized by power spectra $S(\xi)$ of the form $S(\xi) \propto \xi^{-\beta(D_f)}$, where ξ is the frequency and D_f is the fractal dimension, and β is the spectral exponent. Consider the discrete details of a fractal profile. As the power spectrum is related to the autocorrelation function, and hence the variance (Bendat and Piersol, 1986), Wornell (Wornell, 1991) presents a scaling argument in terms of the variance of the discrete detail signals $(D_m^k f)_d$:

$$\sigma^2[(D_m^k f)_d] = V_0\ 2^{\beta m}, \quad (8)$$

where V_0 is a constant, called the magnitude factor, which determines the magnitude of variation. With this equation, the fractal dimension D_f is calculated from the slope (i.e., spectral exponent β, with $D_f = \frac{3-\beta}{2}$) of the log-log plot of the variance versus the scale 2^m; the magnitude factor is the y-intercept (offset) of the same plot (Srinivasan et al., 1995).

Reconstruction and Synthesis of Profiles Following the implementation of the approximation and detail operators for surface profiles, a method is needed for reconstructing the profile signal. Since the approximations and details at the coarser resolutions are known, the reconstruction yields the requisite approximation at finer resolutions. The reconstruction formula is written in terms of the filtering operation by simplification and by using the filter definitions in Equations 4 and 6:

$$A_{m+1}^k f = 2 \sum_{n=-\infty}^{\infty} h_{k-2n} A_m^n f + 2 \sum_{n=-\infty}^{\infty} g_{k-2n} D_m^n f \quad (9)$$

In the synthesis of profiles, details $D_m^n f$ are derived based on the power spectrum relationship between the fractal measures and the details (Equation 8) (Srinivasan et al., 1995). The details are constructed as a random process with a set of mutually uncorrelated, zero-mean random variables, with the structure determined by the fractal measures (Wornell, 1991). As a result, details can be synthesized given sets of fractal measures. The predicted profiles can then be synthesized using Equation 9.

A MACHINING APPLICATION: GRINDING

The previous section presents the fundamentals of the methods under investigation. The following sections present an investigation of the suitability of the four methods to characterize actual surfaces from a precision grinding process.

Precision Grinding

Grinding is the most accurate of the common manufacturing processes and one of the most frequently used in manufacturing precision machine components (Slocum, 1992). Accuracy of the grinding process is a function of a number of factors, such as accuracy of the machine, dressing of the wheel, temperature control of the coolant, fixturing method, and feed rate. Desired precision accuracies can be reached when these factors are properly controlled. Consequently, it becomes crucial to monitor the process, and to predict the error profile given the machine and process parameters. In this work, the focus is on surface grinding, which is designed to produce high tolerance, low surface roughness, and flat planar surfaces. The horizontal-spindle reciprocating-table surface grinder is noted for its precision. The amplitude of vibration of a grinding system does not have to be very large to affect process performance.

In this section, experimental data from a surface grinding process are analyzed using the four methods discussed above. The experimental data contain profiles with high and low values of the grinding wheel speed and workpiece speed. These factors result in surface variations, which must be detected and monitored by means of measures, as well as predicted given the machine and process parameters. For high surface precision, the workpiece speed is the most important parameter. In addition, first order interactions with the workpiece speed and the combined effect of workpiece speed with other parameters are also significant. High surface precision in surface grinding requires a low workpiece speed, a high wheel speed, and a low infeed rate (Lewis and Schleicher, 1976).

The questions that must be answered are as follows: Can these methods detect the changes in the surface precision caused by varying the two parameters (wheel and workpiece speeds)? Can the methods be used as a tool for analysis and synthesis of error profiles? Can the methods provide measures that satisfy the set of criteria established previously?

Experimental Setup

The experiments are run on a surface grinder, with a medium grit size and hardness grinding wheel. The wheel is dressed before each experiment. The radial depth of cut (infeed) on the workpiece surface is $0.0127\ mm$ $(0.0005\ in.)$. The parameters that are controlled in the experiments are the grinding wheel speed and the workpiece speed. Two

runs (I and II) and two measurements for each run (a and b) are performed using the same combination of wheel and workpiece high and low speeds. These combinations are specified with the letters A to D. The high and low wheel speed values are 31.8 m/s (6259.0 ft/min) and 17.17 m/s (3500 ft/min). The high and low workpiece speed values are 0.27 m/s (53.6 ft/min) and 0.03 m/s (6.1 ft/min).

Autocorrelation Function Method

To compare surface precision trends using the autocorrelation function (ACF) technique, simulations are run to compute and graph the ACF of each of the grinding profiles. A sample ACF plot is shown in Figure 1. The ACF plots show that the experimental profiles have a dominant stochastic component, and no significant periodicities. In addition, the experimental profiles have a trend, shown as a slow decay exhibiting a slope.

The autocorrelation method has been used quite frequently in the literature (Whitehouse, 1994). From the examples found in literature, a successful application of the ACF method is when changes in machine and process parameters are monitored on a qualitative basis. The method is helpful for a broad classification of the surface profiles and the estimation of the presence and absence of features (Whitehouse, 1994). However, when applied to random grinding data, the method cannot provide precise quantitative measures. There are some inherent problems with the method. Most data from machining processes have time-varying statistics, and are therefore nonstationary. Nonstationary effects may be due to statistically unpredictable freak marks on the surface due to inhomogeneity of the material, chip damage, a deep scratch, and so on. Such effects can cause corrosion and potential failure and should be detected and quantified as they occur. Accounting for nonstationarities in the profile is difficult with ACF, since they tend to integrate out from position to position (Whitehouse, 1994), giving an average representation of the surfaces. Although several parameters can be extracted from the ACF plots (Whitehouse, 1994), none of them gives a proper mathematical description of the surfaces. Due to the high stochastic component shown on the plots such as Figure 1, when plots for different data sets are compared, a distinguishing characteristic is not detectable. This difficulty in extracting characteristic measures makes an accurate synthesis of profiles from the ACF method alone impossible. Therefore, the autocorrelation method alone is not an effective tool for monitoring and predicting surface precision for grinding.

Fourier Spectrum Method

Figure 1 also shows a sample spectrum plot of a ground surface profile. Note that the spectrum method has been used widely in the literature (Hingle, 1987; Whitehouse, 1994). The Fourier spectrum of a surface is an efficient

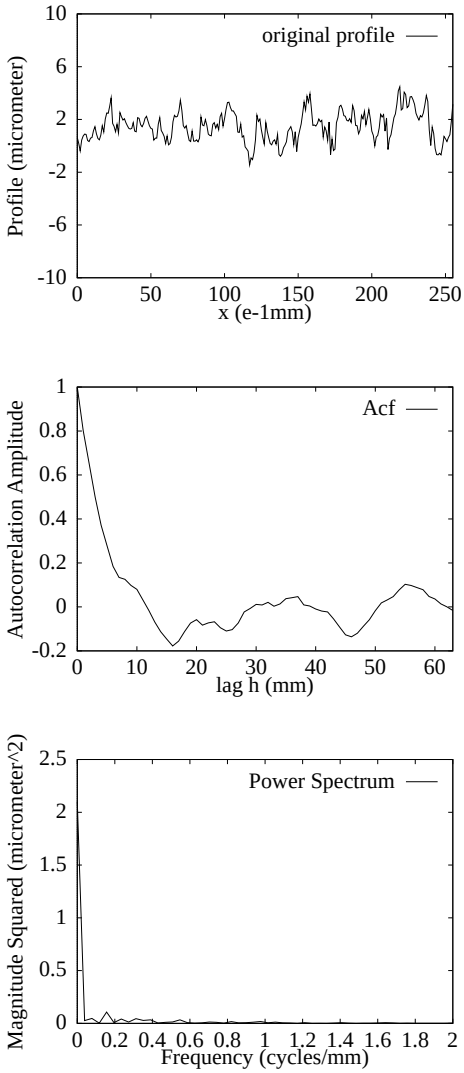

Figure 1. Grinding Profile, Autocorrelation Function, and Power Spectrum.

tool in observing trends and changes in a process, as well as identifying the modes and dominant frequencies of surfaces. For example, to reveal tool wear, the relative change in the magnitude of the harmonics of a spectrum is compared to the fundamental frequency component (e.g., feed). As more wear scars appear on the tool, more harmonics are added to the clean spectrum, hence increasing the relative harmonic to fundamental ratio. So, changes in the system conditions can be detected by monitoring for the emergence of new harmonics, and the change in their amplitudes (Hingle, 1987; Whitehouse, 1994).

However, this method also has some inherent problems associated with it. The frequencies are not meaningful when

studying highly random surfaces like grinding. As is shown in the example (Figure 1), the method does not provide a proper measure of the randomness characteristic of surfaces from grinding. In addition, there is some information loss due to the property of averaging over space, making detection of smaller features on the surface difficult. Furthermore, nonstationary profiles cause problems with this method as well (Whitehouse, 1994). Any difference in the nature of the data function is smoothed out because the total signal is encompassed under the integral sign (Whitehouse, 1994). When comparing ground surface profiles, such as the one shown in Figure 1, it is very difficult to distinguish between different grinding profiles, due to masking effect and information loss described above. As a result, the method cannot be used to monitor the grinding process quantitatively, or to synthesize random surfaces effectively.

Karhunen-Loève Method

The KL expansion technique is applied to the ensemble of error profiles from surface grinding. Eight profiles from each measurement, with 256 sample points each, are assembled into a matrix, which is then fed into an algorithm to compute the covariance matrix $\vec{C}$ and its 256 eigenvalues and eigenvectors (Ball et al., 1991; Sirovich and Kirby, 1987). $M = 8$ profiles are used in this study as a result of the 2^2 factorial experimental design. Recall that the purpose is to detect the changes caused by varying the levels of two parameters. Two different levels are used, and the experiments are replicated once to assure repeatability. $N = 256$ data points are sampled on each profile to assure computational efficiency. This sample size is obtained after decimating the original data captured at a rate of 799 $samples/mm$; decimation is deemed sufficient as long as the shape of the waveform does not change (Bendat and Piersol, 1986).

Analysis of Grinding Profiles The KL expansion results in the computation of 256 eigenvalues; only seven have significant values. These seven eigenvalues represent the majority of the energy present in the signal, compared to the total set of 256 eigenvalues. The seven eigenvalues, shown in Table 1, correspond to the $n = 7$ principal components (modes) of the error profiles. Note that the first 4 eigenvalues constitute 95% of the total energy and hence should adequately represent the original data (Ball et al., 1991).

The main four eigenvectors for Replicate I are shown in Figure 2. The first mode exhibits a linear slope; the remaining modes appear to exhibit random characteristics. The eigenvectors from Replicate II show similar trends. The physical meaning of these characteristics will be investigated further. To assure statistical accuracy in estimating the eigenvalues and eigenvectors, a large number M of sample profiles must be used (Fukunaga, 1990; Sirovich and Kirby, 1987). As a result, the shape of the eigenvectors in Figure 2 will be verified by using a greater number of samples M. Additional profile measurements from the eight experimental surfaces will be performed to investigate the characteristics of these modes further.

Table 1. Principal KL Eigenvalues for Grinding.

Principal Component	Eigenvalue (Rep I)	Eigenvalue (Rep II)
1	2869.44	1812.1
2	179.37	126.29
3	125.53	88.22
4	102.98	65.92
5	86.1	56.56
6	61.64	40.48
7	27.5	31.19

The eigenvalues from the KL expansion indicate the effectiveness of each error feature (Fukunaga, 1990). Note that the eigenvalues are always positive and real-valued, and when normalized by the total energy of the system, belong to a finite range between 0 and 1. As a result, the main criteria for mathematical measures are satisfied. In addition, the method can efficiently identify the dominant error modes, which correspond to the highest eigenvalues in the solution set of eigenvalues of the covariance matrix. Note that the method has not yet been investigated to verify whether changes in parameters are reflected in the minimum set of eigenvalues. In theory, the decomposition coefficients a_i corresponding to each deviation $\vec{y}$ and eigenvector $\vec{u_i}$ should be different for each set of profiles, thus allowing a comparison between different surfaces. However, the main question is whether these coefficients will correspond to actual physical factors. Further research remains to be performed on this aspect of the technique.

Reconstruction and Discussion The reconstruction procedure for KL is tested using the grinding profile with low wheel speed and low work speed. The seven principal eigenvectors are used in Equation 3, and the reconstructed profile, along with the actual experimental profile, is shown in Figure 3. The reconstruction is nearly perfect, and achieved by using just seven eigenvectors instead of 256, thanks to the optimal representation properties of the KL approach.

A paramount advantage in the use of the Karhunen-Loève method is the ability it extends to isolate the primary design and manufacturing components contributing to the errors, even without prior knowledge of their identity. Each significant eigenvector or mode is interpreted as a factor influencing the profile generation process. The severity of these factors is determined by the amplitude coefficients

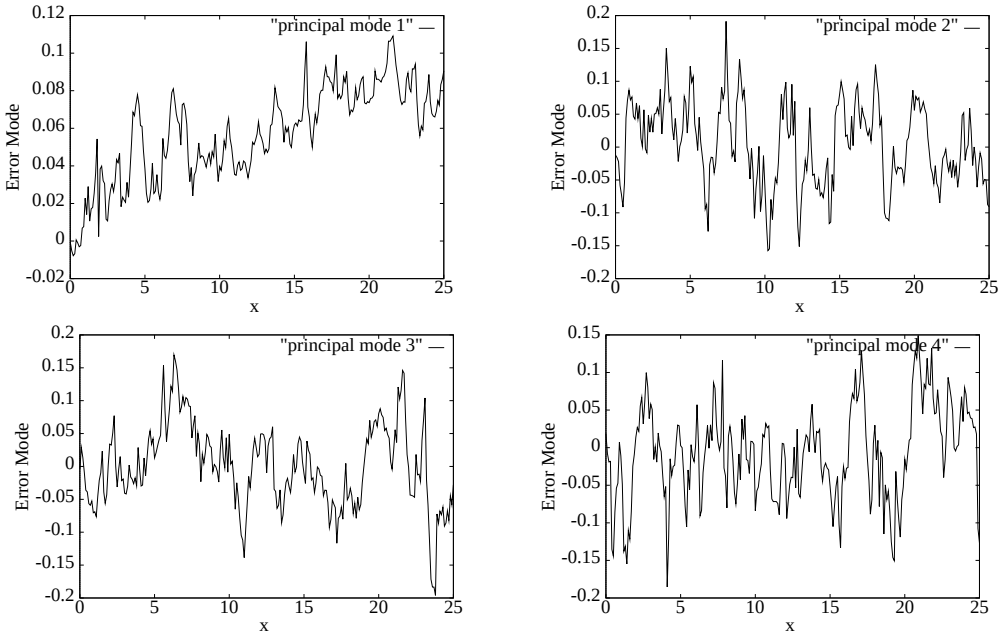

Figure 2. Main Four KL Modes for Grinding, Replicate I.

a_i. However these are abstract factors, and are usually not directly interpretable in terms of physical factors. *Target transformations* are prescribed to transform the abstract factors to real, physical factors (Malinowski, 1991). However, this is a non-trivial problem. A joint inference from ANOVA and Karhunen-Loève decomposition is potentially valuable information to determine these transformations. In addition, the transformation is computationally burdensome. For instance, in the grinding experiments, the size of the covariance matrix is 256×256.

Fractal-Wavelet Method

The fractal-wavelet technique computes the fractal dimension and the magnitude factor of the machined profiles. When this method is applied to profiles from grinding, changes in parameters are reflected in the fractal measures. The fractal dimension reflects the structure of the error profile, and the magnitude factor captures the size of the variations on the surface. Note that the fractal dimension and the magnitude factor are real-valued and belong to a finite range. Furthermore, a change in the fractal dimension and the magnitude factor occurs when the wheel speed and the workpiece speed vary. Since it is possible to distinguish between different profiles using this technique, and since the wavelet transform allows mapping to the time domain, the fractal-wavelet method can be used for analysis *and* synthesis of error profiles. However, some inconsistencies are encountered when the measures are compared to the expected trends (Srinivasan et al., 1995; Tumer et al., 1995). These

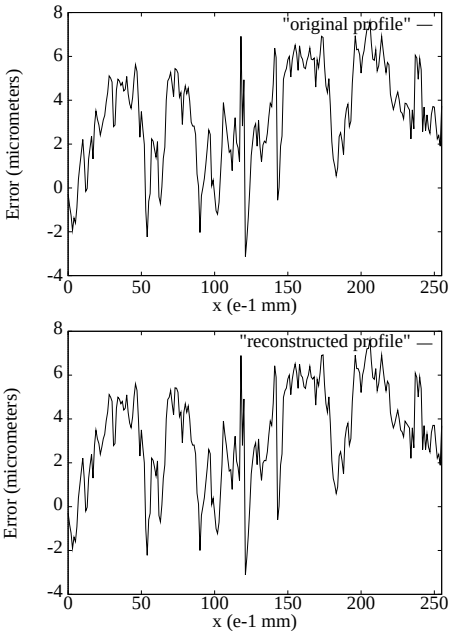

Figure 3. Experimental and Reconstructed Profiles using the KL Method.

inconsistencies are attributed to the presence of deterministic components on the surface profiles, which biases the values of fractal measures (Srinivasan et al., 1995; Tumer et al., 1995). Specifically, the fractal measures are valid to

Table 2. Trend Parameters for Grinding.

Run	Slope	Offset (μm)	Run	Slope	Offset (μm)
I.Aa	0.0121	1.7526	I.Ca	-0.0110	-0.8157
I.Ab	0.0193	1.7482	I.Cb	-0.0229	-2.5229
I.Ba	0.0208	-0.8857	I.Da	-0.3854	1.4967
I.Bb	0.0119	0.7139	I.Db	0.0117	0.5889

represent random high frequency components only; as a result, a method is proposed by the authors to systematically detect and quantify the deterministic components of the surface (e.g., periodicities and linear trends), and subtract them from the original profile (Srinivasan et al., 1995). An improved approach, using the fractal-wavelet method along with the ACF, Fourier spectrum, and regression analysis (Srinivasan et al., 1995) is presented next.

Subcomponents of Surface Profiles Any surface from a machining process will have deterministic, as well as stochastic characteristics. The deterministic components include trends and periodicities in a profile. The stochastic component is best represented by the fractal model of the irregular surface. The fractal-wavelet representation handles nonstationary effects caused by the randomness of the grinding process, whereas the previous mappings such as ACF and FT handle the deterministic trends and periodicities. In the following sections, the different components of a surface are identified first, and then combined using a superposition model.

Analysis of Profiles In analyzing a given experimental surface profile from a precision grinding process, first a linear regression analysis is carried out on the profile data to obtain the trend information. In this case, all grinding profiles have an intercept and a slope, as can be inferred from the profile plots, and more definitively from the power spectra and the ACF plots. The ACF plot indicates the presence of a slope, by exhibiting slow decay as the lag increases (Bendat and Piersol, 1986; Frieden, 1991). The power spectrum of a profile with an offset shows a peak at zero frequency. The trend $y_t(x)$ can be approximated by a straight line, and isolated by using linear regression techniques (Sen and Srivastava, 1990). This procedure compresses the trend information into two parameters, i.e., an intercept y_{t0}, and a slope s_t, using $y_t(x) = y_{t0} + s_t \cdot x$, where $0 \leq x \leq L$. The linear regression analysis is carried out for the grinding data and is shown in Table 2 (Srinivasan et al., 1995).

The periodic component is estimated from the surface profile by using a nonlinear regression procedure, providing three additional measures, namely frequency, amplitude, and offset (Sen and Srivastava, 1990). For the case of precision grinding, since neither the grinding profiles nor the ACF and Fourier spectrum plots indicate the presence of a periodic component, the nonlinear regression analysis is not carried out (Srinivasan et al., 1995).

Table 3. Fractal Parameters for Grinding.

Run	Wheel Speed	Work Speed	Spect. Exp.	Fractal D_f^{Δ}	Magn. (μm^2)
I.Aa	low	low	0.4561	1.2719	0.1588
I.Ab			0.5080	1.2459	0.1881
I.Ba	low	high	0.6402	1.1798	0.1034
I.Bb			0.4255	1.2872	0.1202
I.Ca	high	low	0.8234	1.0882	0.1364
I.Cb			0.6964	1.1517	0.1102
I.Da	high	high	0.1857	1.4071	0.0627
I.Db			0.1627	1.4186	0.0657

After detrending the grinding profiles, the fractal-wavelet method is used to extract the fractal dimension and magnitude factor. The fractal dimension effectively describes the structure resulting from complex processes (Mandelbrot, 1983). As a result, the fractal component takes into account effects that cannot be represented with the other two components. The results from this technique, shown in Table 3, follow expected trends (Srinivasan et al., 1995). First, the parameter changes are reflected by the fractal measures, and hence the profiles are shown to be fractal. Second, the fractal measures have smaller values, hence corresponding to expected precision levels from a grinding process. Finally, in Replicate I, as expected, the profile with the high wheel speed and low workpiece speed has the smallest fractal measure. Recall that the fractal-wavelet method had some problems associated with it. These problems are handled by first removing the deterministic trends, such as linear trends and periodicities.

Synthesis of Precision-Ground Surfaces Once the measures are known, the wavelet reconstruction procedure is used to synthesize the fractal component of the surface, based on the idea that the details from the wavelet analysis are related to the fractal information (Srinivasan et al., 1995; Wornell, 1991). This reconstruction step is followed by invoking respective models to synthesize the deterministic components of the surface profiles. As a final step, a superposition model is used to reconstruct the entire surface (Srinivasan et al., 1995). Assuming independence of the three components identified above, a superposition model combines the trend, periodic, and fractal components.

An example of an actual grinding profile, and the cor-

responding synthesized version is shown in Figure 4. A preliminary visual comparison proves how well the two plots conform in overall shape and magnitude characteristics. Here, the fractal measures are obtained using the analysis method described above, and fed into the wavelet reconstruction algorithm to synthesize an error profile (Srinivasan et al., 1995). In actuality, the fractal measures could, for example, come from the specifications of the part. The error profile can then be synthesized prior to machining, and used by the engineer to make *a priori* design decisions about the part.

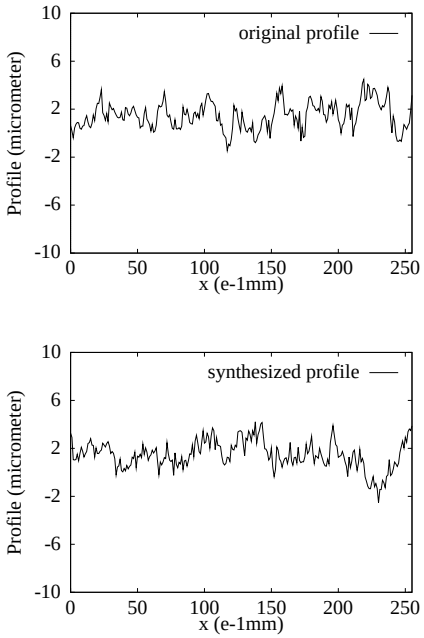

Figure 4. Experimental and Synthesized Profiles using the Fractal-Wavelet Method.

Precision Measures from the Fractal-Wavelet Method
Using the fractal-wavelet approach, a minimum set of measures for any surface profile from a manufacturing process includes the following parameters: fractal dimension and magnitude factor from the fractal-wavelet method; intercept and slope from the linear regression analysis; and frequency, amplitude, and offset from the nonlinear regression analysis. In the case of precision grinding, no periodic component is detected. As a result, the set of measures includes the first four measures only, listed above. All four of these parameters satisfy the mathematical criteria established for measures. The fractal dimension and magnitude factor for the grinding data are shown in Table 3. They are both positive and real values. The fractal dimension has a base point of 1 and belongs to the finite range between 1 and 2. The magnitude factor has a base point of 0 and can be normalized to be reduced to a finite range. The intercept and slope for grinding are shown in Table 2. They are both positive and real values. The fractal dimension has a base point of 1 and belongs to the finite range between 1 and 2. The magnitude factor has a base point of 0 and can be normalized to be reduced to a finite range. The intercept and slope for grinding are shown in Table 2. They are both real-valued measures with a base point of 0. The values deviate from the base point in both the negative and positive directions, and can be reduced to belong to a finite range when normalized. Recall that this minimum set of measures can now be used to effectively characterize the precision of surfaces from grinding processes. These measures can potentially be used to monitor surface errors produced during manufacturing processes, as well as to predict surface errors prior to machining.

FRACTAL-WAVELET METHOD IN DESIGN

In order to show the utility in design, the fractal-wavelet method discussed above is used in a design example, specifically, gears. The design example involves the computation of a functional parameter based on original error surfaces and synthesized error surfaces. If error prediction and control is to be implemented using this method, both types of surfaces must show comparable functional performance characteristics. An effective way to judge the validity of the methods is to compare functional parameters from experimental profiles with functional parameters from profiles synthesized using the methodology described above.

Gear Teeth Precision and Transmission Error

For good gear performance, the toothed wheel has its teeth arranged so that, when they are meshing with the teeth on another toothed wheel, motion is transmitted. This transmission can, however, be affected when the teeth are not machined accurately to the required precision levels. Many high-precision, fine-pitch gears have their teeth ground from the blank, either by form-grinding or by generating grinding (Parmley, 1985). Consequently, the degree of precision of ground gear teeth becomes a very desirable piece of information.

In precision gear applications, the transmission of motion from shaft to shaft must have a high degree of linearity. Theoretically, involute gears will function perfectly. In reality, however, there are deviations from ideal motion transmission as a result of involute profile variations and machining errors (Parmley, 1985), described by an expression of transmission error (Mark, 1982). Transmission error is defined as the deviation in the position of the driven gear for any position of the driving gear, relative to the position the driving gear would occupy if both gears were geometri-

cally perfect and undeformed.

To facilitate the assessment of machining errors on performance, defined by the transmission error, the problem is presented as follows. The transmission error, $TE(x)$, is expressed as a function of the local composite error, $\epsilon_i(x)$, and the local stiffness, $k_i(x)$, of the ith tooth pair, and the normal transmitting force in the transverse section of gears (W):

$$TE(x) = \frac{W + \sum_{i=1}^{n} \epsilon_i(x) k_i(x)}{\sum_i^n k_i(x)}, \qquad 0 \le x \le L \tag{10}$$

where n is the total number of tooth pairs in the zone of contact.

Functional Performance from Profiles

The parameters for the gear pair used in this study are given in (Houser, 1985; Tumer et al., 1995). The length of contact L is calculated as (Mark, 1982):

$$L = \sqrt{R_{tg}^2 - R_{bg}^2} + \sqrt{R_{tp}^2 - R_{bp}^2} - \tan\phi(R_{bg} + R_{bp}) \tag{11}$$

where R_{bg} is the radius of the base circle for the gear, and R_{tp} is the outside circle radius of the pinion, and so on. As the contact ratio is unity, the expression for transmission error becomes:

$$T.E.(x) = \frac{W + \epsilon(x) k(x)}{k(x)}, \qquad 0 \le x \le L. \tag{12}$$

The stiffness values $k(x)$ are calculated by assuming $k(x) = k_0 \cdot x$, where k_0 is a constant stiffness.

One of the grinding profiles is used to represent the error profile on a pair of meshing gear teeth. Equation 10 is used to calculate "experimental transmission error", and then the calculation is repeated with the corresponding synthesized errors, to obtain "synthesized transmission error" (Tumer et al., 1995). The plots of these errors for one mesh cycle are shown in Figure 5. Note that the overall pattern of the error profiles and the maximum and minimum values compare well. From the definition of the transmission error, (Equation 12), the component due to the compliance, i.e., $W/k_i(x)$, decreases with the progression of meshing. The component due to the profile errors is called the transmission error of "unloaded gears" (Houser, 1985). For a perfect gear pair, this component is zero. However, the total transmission error for both the experimental and synthesized profiles is increasing in a mesh cycle; the magnitude of the error, $10\mu m$ could be acceptable, for example in a concrete mixer. For more critical applications, like machine tool gears, this error is unacceptable, indicating the need for more precise gear finishing methods.

This example establishes the applicability of the proposed approach in synthesizing realistic part models. Using similar results, the designer can choose the proper process and machine parameter combinations to reach a compromise between performance and time of manufacture. For example, the designer can vary process parameters and compare system performance. Is the performance acceptable if higher speeds (required for higher precision) are used, thus reducing manufacturing time?

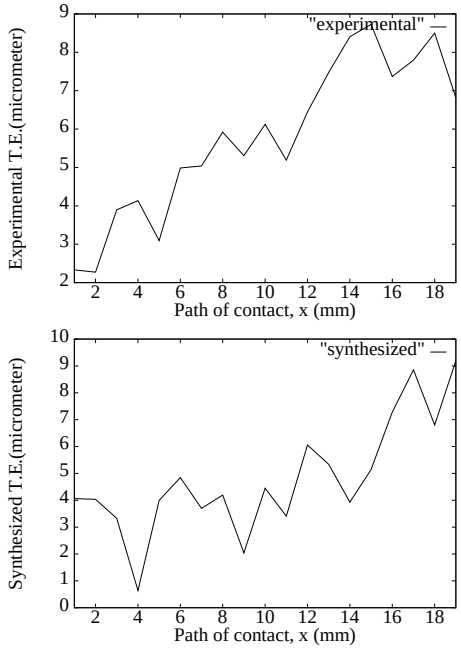

Figure 5. Transmission Error: Experimental & Synthesized Grinding Profiles.

CONCLUSIONS AND FUTURE WORK

In this paper, we investigate the feasibility of finding characteristic measures of precision for precision-ground surfaces. Four methods are investigated based on the objectives and criteria established to allow for the monitoring and prediction of surface errors. The methods are applied to machined surfaces from a surface grinding experiment. The results are used to identify the shortcomings of each method. A minimum set of characteristic measures is derived from a combination of the fractal-wavelet method with qualitative results from the Fourier spectrum and the autocorrelation function method. This improved fractal-wavelet approach is then applied to a design problem to verify that functional performance parameters from experimental surfaces and from synthesized surfaces compare effectively.

In addition, the Karhunen-Loève technique is presented as an alternative to the fractal-wavelet approach. Using this

method, any surface characteristics can be detected and isolated. These individual error components (e.g., stochastic, deterministic, or nonstationary) can then be analyzed and quantified separately. However, there are currently limitations in determining the physical significance of the measures from the KL method, as well as difficulties due to the computational complexities. Future work will explore the extensions to this alternative.

The ultimate goal in this work is to develop a unified approach to detect and quantify errors from manufacturing machines. The authors are conducting an investigation of other processes, including surfaces from Selective Laser Sintering, as well as other types of signals, such as deviations in the laser positional accuracy. In addition, an array of signal processing methods are being investigated to determine their suitability to detect and quantify any type of error signal from manufacturing processes, including higher-order spectral methods, and time-frequency distributions. The results of this work will provide the design and manufacturing fields with unified tools to determine and quantify the precision from manufacturing machines, helping in the integration and automation of the two fields.

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