On Complexity in Metal Cutting and Fractality of Machined Surfaces *

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Abstract

Metal cutting is modeled as a *complex* process, resulting from the interaction of several predictable and irregular mechanisms. Emerging from this complexity, the principal mechanism for surface generation is based in a new model for fracture in metal cutting. In addition, secondary and tertiary mechanisms are explored. *Fractal* parameters are proposed to describe the structure of machining errors, in concert with linear and periodic trends. Experimental results are also presented, wherein the predictable and irregular components are computed for a milling process. The fidelity of the method is tested by synthesizing the profiles, and comparing them to the originals. Finally, applications and future directions to the research are presented.

1 Precision Manufacturing

Precision engineering has emerged as an important facet of manufacturing [10, 24], especially within the last decade. Increasingly stringent demands have been placed on the construction and performance of machine tools used to manufacture high precision components. On the counterbalance, all processes have

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inherent variability, emanating from the physical mechanisms governing surface generation. Metal cutting models have been used [17, 14] to describe and understand the geometric and physical characteristics of the generated surface. However, the *exact* mechanisms involved in surface generation are largely unknown [4, 9]. This is due to the high degree of complexity involved in the metal cutting process. In addition to the need for better models to understand metal cutting, there is also a need for developing techniques based on sound scientific bases to address certain areas in precision manufacturing [10]. One such area is the characterization of precision engineering surfaces. In this contribution, we address these two issues: we examine metal cutting from a fresh scientific perspective, namely, *complex processes*, and qualitatively derive the mathematical tools needed for describing the machined surface.

1.1 Components of the Research

We begin by surveying various types of errors in a simple machining process. Against this topical background, the issue of *complex structures* in manufacturing errors is discussed at a qualitative level. We then digress to present an introduction to fractal dimensions as viable descriptors of these complex structures. Then we revert to further study the mechanisms that lead to error structures in machined surfaces. These include a Laplacian model for fracture, energy cascades, and elastic deformation and recovery in cutting. A superposition model is used to represent the composite nature of the errors, and one of the components includes *fractal* parameters. We then use the profiles from a milling process to verify that the model can fully capture the error structure. The paper concludes with applications in design and thoughts on future research.

2 Surface Generation

In metal-cutting processes, the cutting tool is constrained to move relative to the workpiece and remove unwanted material in the form of chips. During this process, the metal is removed through plastic deformation. The separation takes place by a concentrated *shear* along a distinct shear plane [17]. The plastic deformation also implies that most of the energy used is converted into heat. Frictional forces also affect the process as the chip slides over the tool face. These surface generation mechanisms are influenced by a number of error sources, described in more detail in the following section.

2.1 Machining Errors - Origin and Classification

In any material removal process, three main components are involved in generating the surface: the machine tool structure, the cutting tool, and the workpiece. The machine tool provides the kinematic means for relative positioning between the cutting tool and the workpiece, and the energy required for metal removal. However, the instantaneous position between the tool and the workpiece is affected by a number of factors, causing errors in the form and size of the machined surface. Based on their origin, these errors can be classified as "internal" and "external" to the manufacturing system.

Internal errors can be subclassified as *setup, quasistatic,* and *dynamic* errors [6]. Setup errors are geometric errors caused by incorrect fixturing/gaging. Quasistatic errors are kinematic/geometric errors of the machine tool and its components, which are transferred to the workpiece during machining, e.g., guideway or bearing errors, thermal strains, etc. Dynamic errors include vibrations [10], controller errors, inertial deflections of the workpiece and/or the tool, random excitations due to non-homogeneous workpiece material properties, e.g., hardness [28], and so on. External errors are due to environmental changes, e.g., transmission of vibrations from adjacent machine tools.

In the following section, we discuss the formation of structure in machining errors from a philosophical viewpoint. For this purpose, we observe that the various errors mentioned above can be regarded as either deterministic or random errors. The final structure is a culmination of interactions between these errors, and this issue is addressed at length below.

3 Manufacturing: Deterministic or Random?

Consider a machining process for generating flat surfaces. If there are no errors, any surface element can be conceived as a perfectly straight line, represented by a constant value with respect to an arbitrary datum.

As error sources affect the process, deviations from this constant value occur. For example, if we consider vibration to be present as the sole error source, the dynamics can be described by the following deterministic model:

$$M\ddot{y}(t) + C\dot{y}(t) + Ky(t) = F_i, \tag{1}$$

where $y(t)$ is the vibration of the tool, assumed normal to the machined surface, $\dot{y}(t)$ indicates differentiation with respect to time, M, C, and K are the lumped mass, damping factor, and stiffness of the toolpost structure respectively, and F_i is the instantaneous cutting force calculated as:

$$F_i = U[d - y(t)]f, \tag{2}$$

where U is the unit cutting force, d is the depth of cut, and f is the feed. If the profile is modeled using Equation 1, the solution can be expressed as a function of t, with a limited number of parameters, e.g., a decaying sinusoidal function; computing the profile remains a straightforward exercise.

Surface generation is also influenced by properties of the work material [10], e. g., hardness variations. However, adding the heterogeneous hardness of the work material introduces unpredictability in the cutting force, which is modified as below.

$$F_r = U\, d\, f\, \left[\left(\frac{p_h(t)}{\overline{p_h}} \right)^m - 1 \right], \tag{3}$$

where $p_h(t)$ is the instantaneous hardness in the workpiece material, $\overline{p_h}$ is the mean of the hardness distribution, and m is the Meyer exponent which gives a nonlinear relationship between the random cutting force and the hardness ratio $(p_h(t)/\overline{p_h})$. The hardness variation is assumed to follow a Gaussian distribution [28]. Inclusion of the random cutting force modifies the system equation (Equation 1) as follows:

$$M\ddot{y}(t) + C\dot{y}(t) + Ky(t) = F_i + F_r. \tag{4}$$

If a deterministic solution approach is adopted for the above scenario, it requires knowledge of the hardness values at each and every point in the work material. It is possible to obtain this information, at least in principle. This would make the profile more predictable, compared to the probability distribution approach adopted in Equation 3. The next step however is to know the same information for all the workpieces in each batch; the quantity of information grows at astronomical rates! This is an example of a case when we resort to probablistic distributions due to lack of adequate information about the problem.

Deferring to the fact that there are random elements in the manufacturing process, a hypothetical model for manufacturing processes is presented as below:

$$\mathcal{M}(\frac{d}{dt}; y; \alpha; f_n) = 0, \tag{5}$$

where, $\mathcal{M}$ is a model operator [11], $\frac{d}{dt}$ is a differentiation operator, y is the modeled profile, α represents the deterministic effects (e.g., process parameters), and f_n embodies random factors, noise, etc. It can be seen that this is the most general form of Equations 1 and 4. All error sources, inputs, and process parameters can be identified with some factor in Equation 5. It is to be noted that the random effects and noise can result from random inputs to the process, or from lack of adequate information about the process. As more information about f_n becomes available, it is subsumed in y. The heterogeneous hardness is an example of this case, where the knowledge of the variation, albeit probabilistic, is used to augment the deterministic model, resulting in Equation 4.

An alternative interpretation of deterministic and random systems concerns the dimensionality, or the number of parameters required to completely specify the system. In the case of the ideal profile, a constant value specifies the system completely. If the profile is purely periodic, e.g., sinusoidal, amplitude, frequency, and phase are required for the specification. However, once the random factors are considered (e.g., hardness), the number of parameters required to specify the system increases. In fact, without recourse to the probabilistic description, the parameters can even tend to infinity!

Equation 4 combines the deterministic and random effects by superposition. While this is one conceivable method of combining two effects in the machining process, there is a plethora of other effects (f_n), which are not accounted for in the equation. How do all these deterministic and random effects interact in a real process? The answer leads to the notion of a *complex system, defined as a system consisting of several interacting components* [2]. The features of the interactions are explored at a conceptual level in the following section.

3.1 A Pictorial Illustration

With reference to Figure 1, assume that the dynamics of a given manufacturing process can be represented as a line AB. Such a representation can be rigorously established in the phase space of the process [3]. With the progression of time, the process evolves or assumes a new dynamic state under the influence of various effects, mentioned in Equation 5. Deterministic effects tend to have a low dimensionality, and hence induce a convergence in the dynamics; on the contrary, random effects are high dimensional and promote divergence. The divergence is shown in Figure 1 as a stretching operation, and the convergence is shown as a folding operation[1]. The stretching and folding operations are repeated several times during the machining process; from this intricate balance of opposing effects, a definite structure emerges [3]. In other words, unpredictability due to the random effects is bounded. This is intermediate between the low dimensional deterministic effects (e.g., Newtonian mechanics), and the high dimensional random effects.

The pattern or structure which emerges from the interaction of various errors in the machining process is modeled at a high-level by Equation 5. It follows that this structure of the dynamics is reflected in the geometry of the machined surface. We now need a mathematical tool to link the geometry and dynamics involved in machining. This leads us to the next topic of fractals and fractal dimensions.

3.2 Structure of Machining Errors

The manifestation and interaction of the various errors described above leads to errors in the geometric characteristics of the workpiece. The conventional representation of these errors is by means of a tolerance

[1] The association of deterministic effects with convergence and random effects with divergence is made for purposes of illustration only; deterministic effects can also induce a divergence, and this is the intrinsic property of chaotic systems [3], a class of complex systems.

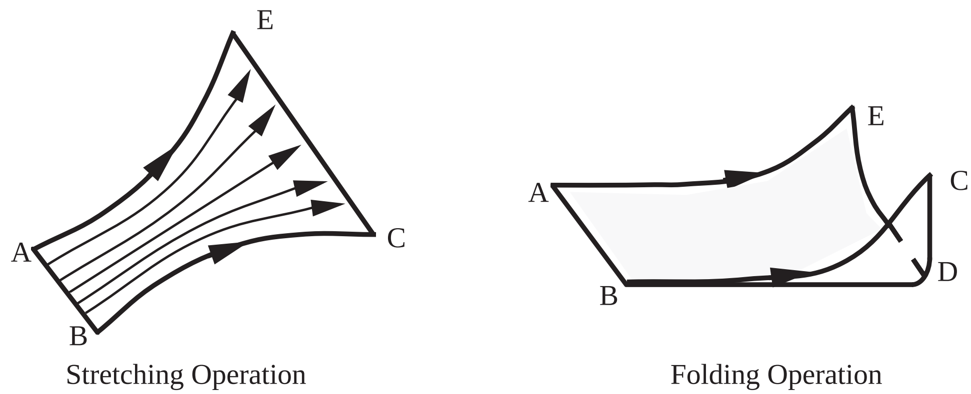


Figure 1: The Emergence of Structure in a Complex System.

zone [1]. However, it is hypothesized that fractal parameters are an alternative and potentially more complete descriptors of the geometric deviations in machining. The following section presents the central idea of fractals and and its extension to represent error structure.

3.3 A Brief Introduction to Fractals

Fractals [13] are defined as sets which contain nested structures across many scales. The most useful index for describing a fractal is the so called *fractal dimension*, which possesses the distinctive characteristic of being fractional [13]. This idea can be extended to represent form errors in machined surfaces. For example, a perfectly straight element of a surface has a topological dimension of unity. However, due to the various errors, the real element is more complex and irregular. This irregular element can be characterized by a fractal dimension, where the fractional part of the dimension serves as a measure of the deviation from ideal form. Fractal dimensions have also been explored for their utility in surface finish characterization and tribology [12].

Next, we reinforce the use of fractal dimensions to describe the complexity of machined surfaces by examining the mechanics of surface generation more closely.

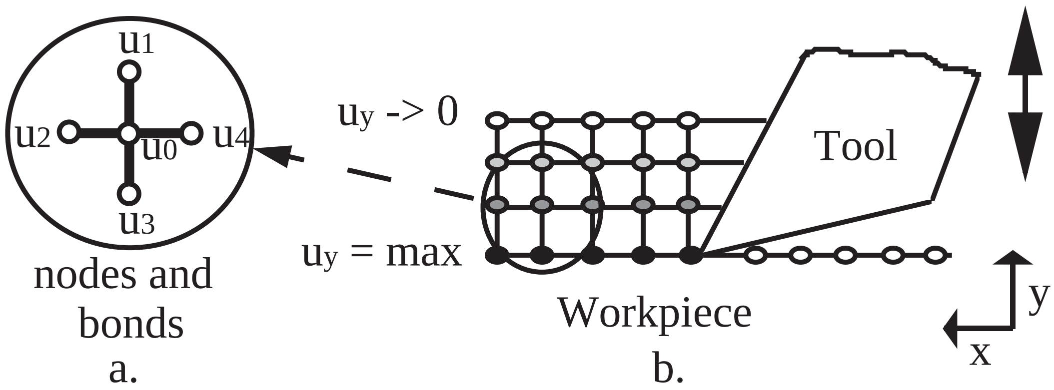


Figure 2: Idealized Cutting Process for Laplacian Model Development.

4 Causal Mechanisms for Error Structures

4.1 A Laplacian Model for Fracture

Fracture is a predominant feature of machining; a new surface is generated by physically removing a layer of material from the workpiece. However, the *exact* mechanisms involved in surface generation during machining are mostly unascertained [4, 9]. A new proposition for the fracture mechanism, based on Laplace's equation can explain the physics of fractal structure generation.

Laplace's equation is used to model diverse physical processes like diffusion limited aggregation (DLA) and dielectric breakdown (DBM) [15]. Some researchers have extended the use of Laplace's equation to study fracture mechanics [21, 25]. The following model extends these ideas to describe fracture in metal cutting.

Fracture in machining is the outcome of nucleation and propagation of cracks [5]. Consider a magnification of the cutting zone, with the workpiece material discretized in terms of cells as shown in Figure 2a and 2b. Each cell can be visualized as a rectangular entity, with nodes at the corners, and bonds connecting adjoining nodes. The material in the vicinity of the tool tip is subjected to a tensile stress field, and deforms plastically. This leads to crack formation and consequent chip separation [5]. A *displacement* $u_y(x, y)$ is defined for each node, and is a function of x and y, as shown in Figure 2.

It can be shown that the mechanism of fracture can be described by Laplace's equation [18, 22]:

$$\frac{\partial^2 u_y}{\partial x^2} + \frac{\partial^2 u_y}{\partial y^2} = 0. \tag{6}$$

In an error-free, ideal cutting process, the displacement is equal and maximum along the line corresponding to the theoretical depth of cut. However, in reality, the various error sources described in Section 2.1 change the ideal distribution of displacements. For example, the vibration in the machine frame propagates to the cutting region, causing a change in the nodal displacements and a varying depth of cut. In addition, the varying hardness of the workpiece induces a reordering in the displacement magnitudes. Solution of similar Laplacian models in the fields of dielectric breakdown, diffusion-limited aggregation, etc., have given rise to fractal models for the corresponding phenomena [15].

The emergence of structure can be traced as follows: the cell matrix is defined along with boundary conditions for the displacement; i.e., nodes collinear with the current position of the tool tip have maximum displacement, and nodes lying on the outer boundary of the unmachined workpiece are assigned the least displacement($\rightarrow 0$). The displacement at all nodes is calculated by solving Laplace's equation. With the displacements of all nodes, we can calculate the strains on each bond i, indicated by $\Delta(u_y)_i$. In a deterministic scenario, the first bond to break would be the one subjected to the maximum strain. However, in order to account for the various error mechanisms, a *fracture probability* is defined for each bond, with the probability proportional to $|\Delta(u_y)_i|$. One possible definition is [15]:

$$p_i = \frac{|\Delta(u_y)_i|^\eta}{\sum_i |\Delta(u_y)_i|^\eta}, \tag{7}$$

where η is a parameter characterizing the random effects. This parameter embodies the random effects denoted by f_n in Equation 5, and modulates the deterministic effects according to Equation 7. As η is assigned different values, a whole gallery of structures can be generated. The above probabilities are used to define a probability distribution. This probability distribution is then used to choose the bond that breaks in this step. This process is then repeated until the fracture is complete.

The model outlined above does not describe the finer details, e.g., how the displacements at a given node change due to varying depth of cut. However, it serves as a causal model, explaining at a qualitative level the interplay of deterministic and random ingredients in the evolution of fractal structures.

4.2 Energy Cascade in Fracture

Another possible source for the presence of structure in machining errors is suggested by the role of energy in fracture [8]. For purposes of discussion, we restrict the study to fracture in brittle materials. In conventional analysis of machining, the energy expended for fracture is ignored [23], as it forms an insignificant proportion of the energy required for plastic deformation. However, the fracture energy can play a role in the formation of surface structure.

Consider the workpiece material as a hierarchy of scale structures. The ideal brittle fracture is characterized by Griffith's law [7], stated as: the growth of a crack results in a reduction in the system potential energy, and this is balanced by the work of separation that forms the new surface. This ideal law leads to surfaces that are theoretically smooth. However, in the presence of a scale hierarchy, the fracture energy forms a cascade, released from larger scales to smaller ones, and finally to microscopic scales, where the new crack is formed. This leads to the more realistic, irregular cracks. Gol'dshtein and Moslov [8] have described the energy cascade using a power law. This is another possible manifestation of how the behavior at one scale can influence the behavior at another scale.

4.3 Premature Deformation in the Tool Path

The tool-workpiece interface is characterized by elasto-plastic deformation [9], a phenomenon that can induce long-term correlations in the following manner. During the process of cutting, very high stresses are generated at the tool tip, leading to fracture of the work material. However, in addition, secondary compressive stresses are generated in the material just ahead of the tool tip. Depending on the magnitude of the compressive stress, the material in the tool path will be deformed, elastically or plastically. The portions deforming elastically recover, but those deforming plastically do not, i.e., there is differential recovery along the tool path. Consequently, there exists a varying deformation pattern in the workpiece ahead of the cutting zone. This can conceivably affect the cutting characteristics (e.g., depth of cut), when the relative motion brings the tool to those points. This is yet another possible cause for the existence of long-range correlations in the surface profiles.

5 Mathematical Implications: Fractal Models

The above subsections have explored the various possibilities for the manifestations of fractal structures in surface errors. These structures are characterized by special long range correlations between various scales, determined by the cutting mechanism at one location influencing the structure at neighboring locations. The conventional models for surface errors postulate either a white noise model for the stochastic component, or a statistical model such as ARMA [16]. These assumptions typically are not based on physical reasoning, but the fractal-based approach is an elegant model for the dependence structure [13]. The above background provides the physical motivations to adopt fractal-based models.

5.1 Complete Surface Descriptions

While fractals are quite useful to describe machined surfaces, there are some features of machined surfaces, that are best described by more conventional techniques [26]; e.g., periodic feed marks, and slope error generated by an inclined machine table, etc. Hence we present the following superposition model to provide a complete description of machined surfaces:

$$y(x) = y_t(x) + y_p(x) + y_f(x) + y_o(x), \tag{8}$$

where x is the direction of tool travel (one-to-one correspondence with time t, assuming constant feed rate), $y_t(x)$ is the linear trend component of the error, $y_p(x)$ is the periodic component, $y_f(x)$ is the fractal component. The last component, $y_o(x)$ is called the outlier component, which accounts for the *sporadic* data

Component	Parameters	Estimation
linear trend	y_{t0} = y-intercept s_t = slope	linear regression
periodic	y_{p0} = offset, d_a=amplitude f_r = frequency	nonlinear regression
fractal	D_f = fractal dimension V_0 = magnitude factor	wavelet decomposition

Table 1: Profile Parameters and Estimation Methods.

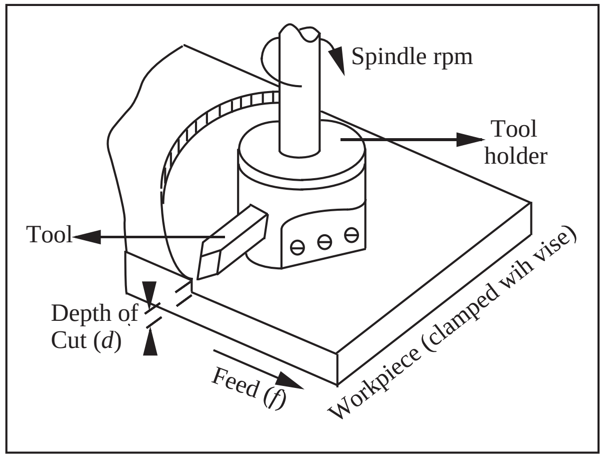
points that are not statistically accounted for in the others, and is included for completeness.

The parameters that are required to describe each of the above components, and the mathematical techniques used to compute these parameters, are summarized in Table 1. It is noteworthy that the fractal component has two parameters: a fractal dimension to quantify the structure of this component, and a magnitude factor to capture the magnitude or range of the error component. The associated mathematical and computational details are found in [18, 19].

In the above discussion, the physics of surface generation is qualitatively examined, treating it as a complex process. This examination naturally leads to the identification of some parameters that can capture the structure of machining errors. Next, the practicability of using the aforementioned parameters as indices of the error is tested by experimentation, as described in the following sections. It is emphasized that the goal of this experiment is to check the fidelity of the surface parameters in *capturing and reproducing* the error structure and *not* to isolate the different sources of error, e.g., vibration, in the machine tool.

6 Experimental Results: Milling

The process selected for study is a face milling operation, where the cutting is carried out by means of a flycutter. A flycutter is a single point tool, mounted in a suitable tool holder, which in turn is mounted in the spindle of a vertical milling machine. It is used to machine light cuts on flat workpieces (Figure 3). There are multiple sources of error in this process: errors from roughing, clamping the workpiece, hardness variations, tool wear, vibrations, and so on. For this experiment, the flycutting operation is performed on



Note: Cutting speed *s* is a function of spindle rpm and cutter diameter D
$s = \pi * D * rpm$

Figure 3: Flycutting on a Milling Machine.

a Bridgeport vertical milling machine using an HSS cutter [23]. The workpiece material is Aluminum 2024; no cutting fluid is used in the experiments.

The geometric error of interest is *straightness*, considered for a single element of the flat milled surface. The height variations are recorded using a digital electronic indicator, withwith an accuracy of 1 microm.ter; The horizontal increment is 0.1 mm, chosen to accommodate the smallest diameter of the probes used for tolerance measurement [27]. The number of data points is $N = 256$.

The profile parameters listed in Table 1 are estimated from this data. The results of the surface parameter estimation are shown in Table 2. We present the results for two combinations of cutting conditions, i. e., speed, feed, and depth of cut.

6.1 Synthesis of Profile

The next step in verifying the validity of this method of surface representation is to recreate the profile from these parameters, using the superposition model (8), and compare with the original experimental profile. In other words, the profile is *synthesized* from the individual components, computed from their corresponding profile parameters. Complete details of the synthesis are presented in [19].

Parameter (Units)	Test 1	Test 2
y_{t0} (mm)	0.039692	-0.006262
s_t	-0.000207	0.000129
y_{p0} (mm)	0.037047	-0.004608
d_a (mm)	-0.014537	0.009259
f_r (Hz)	50.162	49.136
D_f	1.889262	1.788253
V_0 (mm^2)	0.000122	0.000013

Table 2: Profile Parameters for Milling.

The experimental *vs.* synthesized comparisons for the two profiles are shown in Figures 4 and 5.

6.2 Application in Precision Manufacturing

It can be seen that the synthesized profiles reflect the structure and magnitude of errors in the original profiles remarkably well. This leads into a discussion of the utility of this method.

In most precision manufactured components, the tolerances are specified by geometric tolerancing [1]. Typically, a tolerance zone is prescribed to contain all the errors for a geometric characteristic like straightness. The tolerance zone is purely a magnitude-sensitive parameter, and does not account for the structure of the errors within the tolerance zone. For instance, it is possible for two different surfaces to have the same magnitude of error, but with different frequencies. It can be seen that the method outlined here can be used to obtain a more complete description of manufacturing errors, compared with the tolerance zone concept.

7 Summary

Manufacturing processes are beset with errors originating from several sources. Some typical errors occurring in metal removal processes are discussed, and the applicability of fractal parameters as error descriptors is examined from the standpoint of the complex interactions between multiple sources. Mechanisms leading to fractal structures are discussed qualitatively. A simple model to combine linear, periodic, and fractal components is presented. The corresponding profile parameters are calculated for milling profiles; the profiles are resynthesized from the parameters and compared to the experimental profiles.

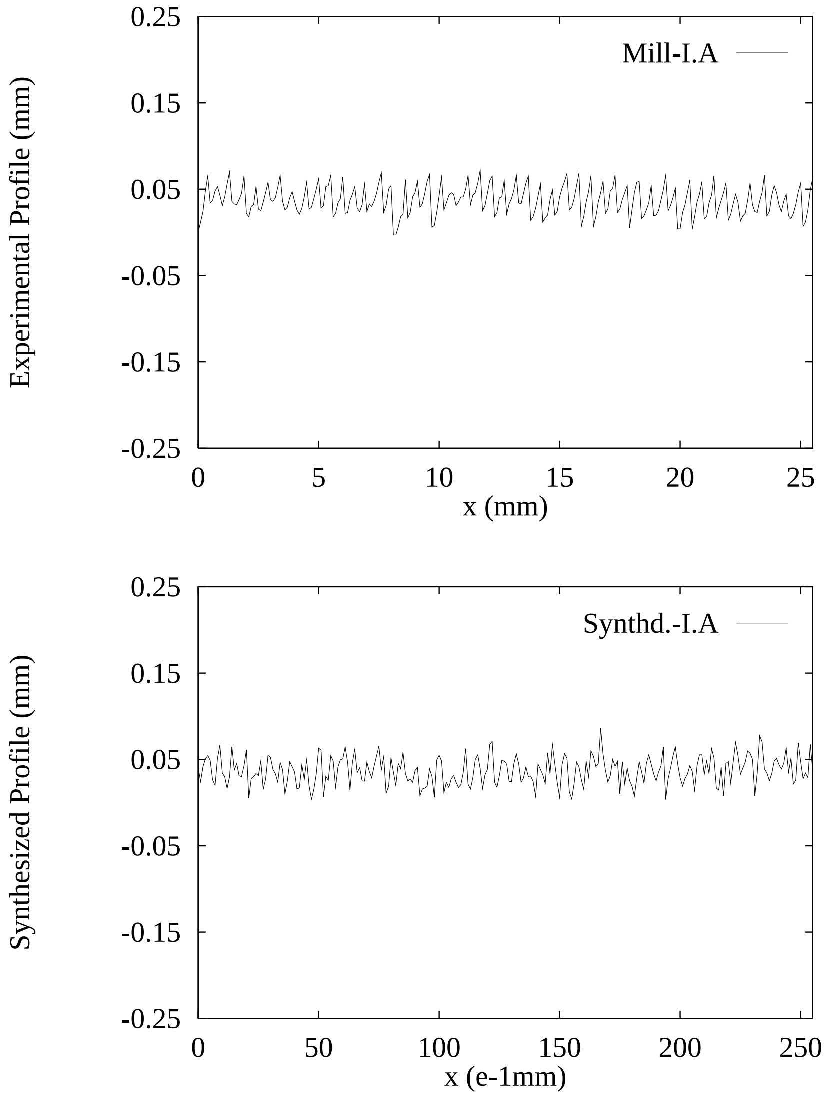


Figure 4: Experimental and Synthesized Profiles: Test 1

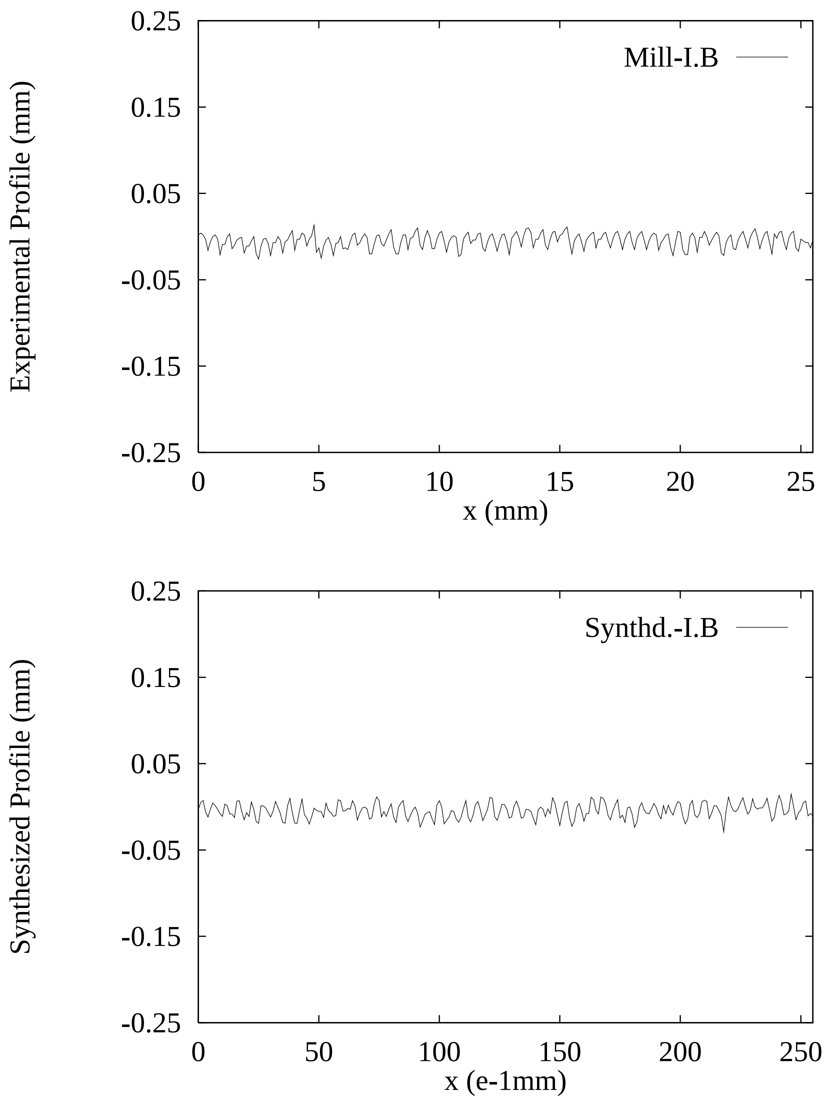


Figure 5: Experimental and Synthesized Profiles: Test 2

In concomitant research, complete factorial experiments for the milling process have been conducted [18], considering speed, feed, and depth of cut as the process parameters. Profile parameters for each case have also been calculated. In addition, to further the application of these parameters in design, a complete methodology for functional form tolerancing has been developed [20]. The future challenges lie in harnessing the full power of fractals and driving toward a more complete understanding of the dynamics of manufacturing processes, wherein the temporal evolution of the structure is described from the design and manufacturing parameters.

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