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ERROR MEASURES FOR FUNCTIONAL PRODUCT TESTING

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ABSTRACT

During product development, testing of models and prototypes offers significant advantages over direct product testing, including easier, cheaper, and faster fabrication. However, two issues prevent effective functional testing with prototypes: prediction accuracy and confidence in scale testing results.

The traditional similarity method, which is based on dimensional analysis, is commonly applied to perform scale testing. However, the method may not provide accurate scale testing results, especially when available model materials are different from the final product materials. The authors have developed a new empirical similarity method, wherein specimen pairs and partial knowledge of systems are systematically utilized, to improve the prediction accuracy. In this paper we describe the construction of error measures to utilize scale testing results with confidence.

In practice, scale testing results are validated based on experiences with previous testing results. This approach to predicting accuracy is difficult to formalize. We develop and simulate a systematic two-level error estimation procedure. Realistic numerical examples demonstrate the feasibility of the approach.

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NOMENCLATURE

E_e Estimated scale testing error of the empirical similarity method
E_t Estimated scale testing error of the traditional similarity method
d_i System parameter
$\mathbf{G}$ A full set of geometric system parameters
$\mathbf{M}$ A full set of non-geometric (material and loads) system parameters
R_e Reliability of the estimated ESM error E_e
R_t Reliability of the estimated TSM error E_t
X_i State at an i-th spatio-temporal point
$\mathbf{x}$ $n \times 1$ state vector composed of the n states
π_i Dimensionless parameter that does not include the product state X
π_X Dimensionless parameter that includes the product state X
(Subscripts)
m Scaled model
ms Scaled model specimen
p Product or full-scale target system
ps Product specimen

1 INTRODUCTION

Product testing is a task that significantly impacts overall product development cost and time. Effective product testing gives a company a competitive edge in the market

(Holmes, 1984; Liley, 1993). However, industries currently prefer full-scale product testing to scaled model[1] testing, even though scale testing may save significant fabrication and/or testing effort, especially in the later design stages. This preference is mainly due to the lack of scale testing methods that reliably predict the behavior of the products. The accuracy of scale testing results is just not dependable.

Worse yet, testing results cannot be used with confidence for verification or improvement of product performance as there is no systematic approach for *estimating* the prediction accuracy. Not only are the results inaccurate, but the degree of inaccuracy cannot be quantified *a priori*. This paper focuses on the later issue, providing error measures for confident scale testing.

Production vs. Prototyping

Most mass-production manufacturing processes are inappropriate for fabricating test-products. This is due to the required fabrication cost and time. In order to reduce this cost and cycle time, various prototyping processes may be utilized instead. Computer numerically controlled machining (CNC), silicon rubber molding, and rapid prototyping are examples of effective prototyping processes (Wall et al., 1991).

Rapid prototyping usually requires the least cost and time for the fabrication of a single part, but rapid prototyping materials are currently too limited for many functional tests. Several factors differentiate the functional behavior of prototypes and products, including limited base materials, distinct material structures (e.g., isotropic vs. orthotropic), and limited part size. Among these, limited material choice is critical for scale testing, as it restricts the loading zones and conditions that can be applied to design scaled models. To address the problems in performing reliable functional testing with rapid prototypes, we have introduced a new similarity method to perform reliable scale testing with rapid prototypes (Cho and Wood, 1997; Cho et al., 1998(a)(b)). The range of problems that this new method can address extends beyond the problem domain of traditional similarity methods.

Correlation of Product and Prototype Behaviors

Scale testing has a potential to save significant product development time and cost, if one can correlate the scaled model to actual product behavior. Especially during the later product development stages, highly accurate and reliable testing results are required, as the decisions made at these stages directly affect the quality of the final products.

[1] Scaled models, model, and prototypes are interchangeably used in this paper.

The traditional similarity method (TSM), based on dimensional analysis, is commonly used to perform scale testing. However, industrial applications of TSM are limited, especially when highly accurate and confident testing results are required. The problems with the TSM stem from its reliance on very limited information, i.e., the dimensional matrix. Our empirical similarity method derives a mapping between the product and scaled model states by testing a simple specimen pair and using partial knowledge of the systems.

The TSM has been frequently criticized due to the lack of a method to validate the testing results (Baker et al., 1992; Szucs, 1980; Kline, 1965). Currently, engineers rely on experiences of similar test cases, and it is hard to find a systematic approach. With this significant problem in mind, we provide error measures with which one can interpret scale testing accuracy systematically.

The structure of this paper is as follows. In Section 2, both the traditional and our empirical similarity methods (ESM) are briefly introduced with illustrative examples, highlighting possible sources of scale testing errors. In Sections 3 and 4, a formal approach to define error measures for both the TSM and the ESM is described, and realistic examples are provided in Section 5. As a formal approach, we first estimate the scale testing error under certain assumptions. The estimated error bound may deviate from the actual error, if the imposed assumptions are not valid. We thus extend the measure to check the validity of the imposed assumptions, in the second stage. Through the two measures, scale testing accuracy is estimated. Realistic numerical examples demonstrate the feasibility of our approach. Expected impacts and future research areas conclude this paper.

2 SIMILARITY METHODS FOR SCALE TESTING

Traditionally, dimensional analysis provides a systematic approach for designing scaled models, and for correlating the behavior of the (target) product and the scale model. However, the application of the TSM for product development is confined to certain problem domains, mainly due to material issues. The ESM systematically utilizes a geometrically simple specimen pair to overcome these limitations. The fundamentals of the two methods are introduced below, and the sources of prediction errors are clarified.

2.1 Buckingham Π theorem and TSM

The Buckingham Π theorem is a powerful mathematical tool that is widely used for both effective empirical modeling and scale testing (Bridgman, 1937; Langhaar, 1951; Baker, 1992). The theorem can be stated in terms of product de-

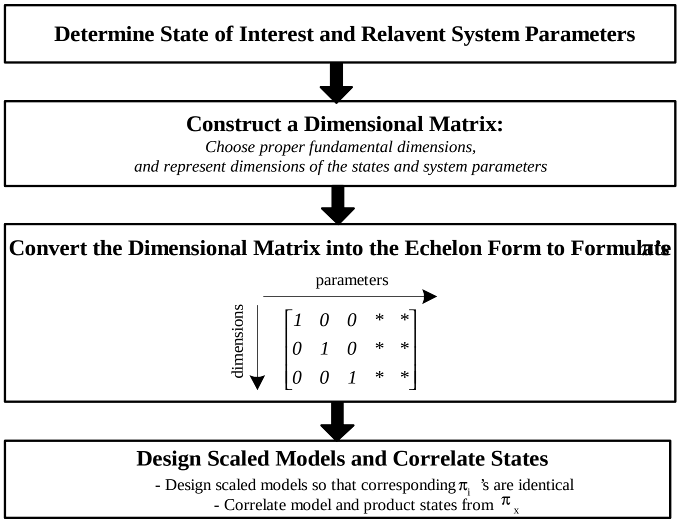

Figure 1. Overall Procedure of Traditional Similarity Methods

sign as follows:

If a product state X can be represented by a functional relationship $X = f(d_1, d_2, \ldots, d_n)$ where d_i is a relevant system parameter, then the dimensionless parameter π_X that includes the product state can be equivalently expressed as the function $\pi_X = F(\pi_1, \pi_2, \ldots, \pi_N)$.

In this statement, π_i denotes a dimensionless parameter that is a power function of the subset of design parameters (e.g., $\pi_i = d_1^{r_1} d_2^{r_2} d_3^{r_3}$ where r_i is a rational number), and the number of π_i, N, is less than $n - m$, where m is the number of fundamental dimensions. By comparing non-dimensionalized functions F of two systems, one can design a scaled model of a product and correlate its behavior. It should be noted that this approach does not require specific knowledge of the governing function f, as only dimensional information is utilized.

We present TSM by application to the problem of pulling a ship (Bridgman, 1937). The theoretical details of the method can be found in historical texts on dimensional analysis (Bridgman, 1937; Langhaar 1951), and one may refer to (Bluman, 1989) for mathematical verification and advanced approaches.

2.1.1 Fundamentals of TSM: Consider two systems that can be described with an identical function f, which relates the behaviors of interest (states), X and X^*, and relevant system parameter sets $\{d_1, d_2, \ldots, d_n\}$ and $\{d_1^*, d_2^*, \ldots, d_n^*\}$ that represent system geometry, material properties, applied loads, boundary conditions, etc. Then

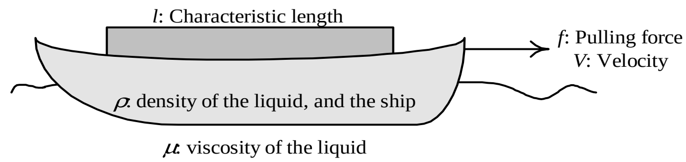

Figure 2. Ship Pulling Problem.

the states of the two systems can be represented as

$$\begin{aligned} X &= f(d_1, d_2, \ldots, d_n), \\ X^* &= f(d_1^*, d_2^*, \ldots, d_n^*). \end{aligned} \tag{1}$$

From the Buckingham Π theorem, these system equations can be equivalently represented as

$$\begin{aligned} \pi_X &= F(\pi_1, \pi_2, \ldots, \pi_N), \\ \pi_X^* &= F(\pi_1^*, \pi_2^*, \ldots, \pi_N^*). \end{aligned} \tag{2}$$

From Equation (2), one can conclude that

$$\pi_X = \pi_X^*, \tag{3}$$

if

$$\pi_i = \pi_i^*, \forall i = 1, 2, \ldots, n. \tag{4}$$

Equations (3) and (4) are the fundamental basis of the TSM, where the former is called the *prediction equation*, and the latter are referred as *design equations* or *scaling laws*. One should design scaled models such that no design equations are violated, in order to predict the product states through the prediction equation. The overall procedure of the TSM is shown in Figure 1.

2.1.2 Ship Pulling Example: Consider the ship pulling problem illustrated in Figure 2, where l is the characteristic length of the ship geometry, v the velocity of the ship, f is the required pulling (or drag) force, g is the gravitational constant, and μ is the viscosity of the surrounding liquid. The required pulling force can be predicted through a scaled model experiment by deriving dimensionless parameters and comparing them.

One can systematically derive dimensionless parameters from a dimensional matrix, which is composed of dimension vectors of the states and all relevant system parameters (Barr 1984; Bluman and Kumei, 1989). The dimension vector implies a $m \times 1$ column vector (m is the number of fundamental dimensions) that is composed of powers of the fundamental dimensions. For example, the dimension vector of the viscosity μ becomes (-2 1 1)^T, as the dimensions of μ are $L^{-2}T^1F^1$ in terms of fundamental dimensions L(length), T(time), and F(force). By combining the dimension vectors of system parameters, a dimensional matrix is constructed to derive following dimensionless parameters:

$$\pi_f = \frac{f}{l^2v^2\rho}, \pi_1 = \frac{lg}{v^2}, \pi_2 = \frac{\mu}{lv\rho}. \quad (5)$$

One should prepare the scaled ship experiment such that none of the following similarity conditions is violated:

$$\begin{aligned} \frac{l_m g_m}{v_m^2} &= \frac{l_p g_p}{v_p^2}, \\ \frac{\mu_m}{l_m v_m \rho_m} &= \frac{\mu_p}{l_p v_p \rho_p}, \end{aligned} \quad (6)$$

where subscripts m and p denote the model and the product. If we want to perform scale testing with a 1/10 size model ship ($l_p = 10l_m$), and 1/10 ship velocity ($v_p = 10v_m$), the following equations govern the design of the model:

$$\begin{aligned} \frac{l_m g}{v_m^2} &= \frac{10 l_m g_m}{(10v_m)^2}, \\ \frac{\mu_m}{l_m v_m \rho_m} &= \frac{\mu_p}{100 l_m v_m \rho_p}. \end{aligned} \quad (7)$$

As the first design equation in (7) cannot be satisfied without changing the gravitational constant g, we cannot perform scale testing unless we can change the gravitational field. Like this case, if any of the design equations is violated, then the product and model are said to be distorted or dissimilar. If not, we say the model is well scaled.

As illustrated in the ship pulling example, there are situations when one cannot construct a well-scaled model, particularly if some of the model parameters are difficult to control (e.g., the gravitational constant in the ship pulling example). This is a typical problem with the TSM, and restrictions on model parameters (e.g., limitations in prototyping materials) make the construction of a well scaled model even more difficult. Likewise, one cannot perform reliable scale testing if the unknown governing equations f of the model and the product ship are not identical (e.g., the deflection correlation of springs constructed of linear and nonlinear materials), as TSM presumes the identity of the governing equations. Moreover, one should identify unknown system parameters, as one should choose the model parameters relative to the product system parameters. Thus there is a need to develop a new method that can overcome such problems in TSM, if we want to perform reliable scale testing in broad problem domains.

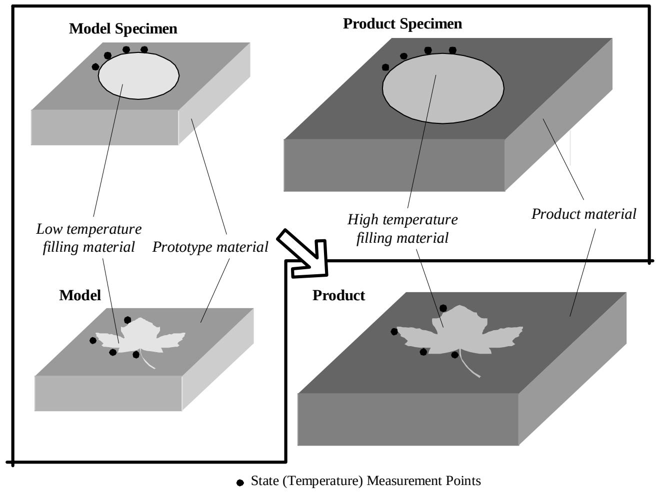

Figure 3. Fundamental Terms and Concept of the ESM

2.2 Empirical Similarity Method

The empirical similarity method (ESM) aims to perform reliable functional product testing with time and cost effective prototypes (e.g., rapid prototypes) in wide problem domains. As noted in the previous section, constructing well-scaled models becomes harder when available base materials are limited. As a consequence, scale testing results with rapid prototypes through the TSM may not provide meaningful state predictions, considering the limitations in choosing prototyping materials. In contrast, the ESM utilizes a geometrically simple specimen pair to correlate the model and product states empirically. This approach enables scale testing with relaxed restrictions on model materials.

2.2.1 Fundamental Concept:

Figure 3 illustrates the fundamental concept and terms of the ESM through a mold die example. As shown in the figure, the model and product specimens are geometrically simplified versions of the model and product. The model and product specimens are fabricated with the prototyping and production processes, respectively. Likewise, the model is fabricated with the prototyping process. With ESM, in contrast to the TSM, the states of the specimen pair and the model are measured to empirically abstract the state changes under pure geometrical (between the model specimen and model) and non-geometrical (material and loading) variations (between the model specimen and product specimen). By integrating the abstracted state transformations with partial knowledge of the governing phenomena, the product states are predicted.

In comparison to the TSM, wherein the product states are predicted through the model states and dimensional information, the ESM requires additional fabrication and testing effort for the specimen pair. However, the additional effort is still very small compared to direct product testing, especially when the actual product geometry is complex.

2.2.2 Empirical Similarity State Transformation: Let $\mathbf{x}_{ms}, \mathbf{x}_{ps}$, and $\mathbf{x}_m$, be $n \times 1$ state vectors that are composed of the measured states of the specimens and the scaled model, and $\mathbf{x}_p$ represent the product states to be predicted. Assuming that the product state $\mathbf{x}_p$ is a function of the specimen pair and the model states, one can represent $\mathbf{x}_p$ as

$$\mathbf{x}_p = g(\mathbf{x}_{ms}, \mathbf{x}_{ps}, \mathbf{x}_m). \tag{8}$$

In general, the state vectors can be represented as

$$\begin{aligned} \mathbf{x}_{ms} &= r^*(\mathbf{G}^*, \mathbf{M}^*) \\ \mathbf{x}_{ps} &= r(\mathbf{G}^*, \mathbf{M}) \\ \mathbf{x}_m &= r^*(\mathbf{G}, \mathbf{M}^*) \\ \mathbf{x}_p &= r(\mathbf{G}, \mathbf{M}), \end{aligned} \tag{9}$$

where r and r^* are unknown governing functions, $\mathbf{G}$ is the set of geometric parameters, and $\mathbf{M}$ is the set of system parameters that are related to the material and applied loads. On the basis of the equations and boundary element expressions of the system, one can construct the state transformation for several classes of problems.

Boundary element methods (Gipson, 1987; El-Zafrany, 1993; Ingham, 1994) are useful numerical techniques for constructing the prediction equation (8), based on the unique capability to represent partial states in terms of boundary conditions using a Green's function. In comparison to boundary element methods, other numerical techniques (e.g., finite element or finite element methods) require full information about all states within a bounded domain. A detailed procedure for constructing the prediction equation through the boundary element method is described in (Cho et al., 1998(b)). Two exemplary state predictions through the ESM are provided below.

(Case 1: Linear ESM, Fundamental State Transformation) If two systems are distorted with respect to the boundary conditions only, there exists a state transformation matrix $\mathbf{T}_m$ that simultaneously satisfies

$$\mathbf{x}_p = \mathbf{T}_m \mathbf{x}_m, \tag{10}$$

and

$$\mathbf{x}_{ps} = \mathbf{T}_m \mathbf{x}_{ms}. \tag{11}$$

A simple way to determine the similarity transformation matrix $\mathbf{T}_m$ is to pseudo-inverse $\mathbf{x}_{ms}$ to estimate the matrix from the second specimen equation (Strang, 1988). However, the pseudo-inversion matrix approach may not provide reliable transformation, as the number of unknowns (matrix elements) is larger than the number of known equations. Another way to derive $\mathbf{T}_m$ is to simultaneously consider the state transformation matrix $\mathbf{T}_g$ that represents state changes under the pure geometric variations. By considering relationship between the four state vectors, $\mathbf{x}_{ms}$, $\mathbf{x}_{ps}$, $\mathbf{x}_m$, and $\mathbf{x}_p$, the following additional relationship that aids to determine the transformation matrices can be derived

$$\mathbf{T}_m \mathbf{T}_g = \mathbf{T}_g \mathbf{T}_m. \tag{12}$$

Due to the space limitations, the details to derive the transformation matrices $\mathbf{T}_g$ and $\mathbf{T}_m$ that satisfy Equations 10,11, and 12 are not described in this paper. The linear ESM is particularly useful when it is difficult to impose well-scaled loads, or when the boundary conditions are improperly known.

(Case 2: Polynomial ESM) If a polynomial functional relationship exists between the model and product states, then one can predict the product states from

$$P(\mathbf{x}_p) = P(\mathbf{x}_m) P(\mathbf{x}_{ms})^+ P(\mathbf{x}_{ps}), \tag{13}$$

where the polynomial function P of a vector $\mathbf{x}$ is defined as

$$\begin{aligned} P(\mathbf{x}) &= P([X_1 X_2 \cdots X_n]^T) \\ &= \begin{bmatrix} X_1^1 & X_2^1 & \cdots & X_n^1 \\ X_1^2 & X_2^2 & \cdots & X_n^2 \\ \vdots & \vdots & \ddots & \vdots \\ X_1^z & X_2^z & \cdots & X_n^z \end{bmatrix}. \end{aligned} \tag{14}$$

In equation (14), the subscripts denote measuring points, the superscripts are exponents (powers of the states), and

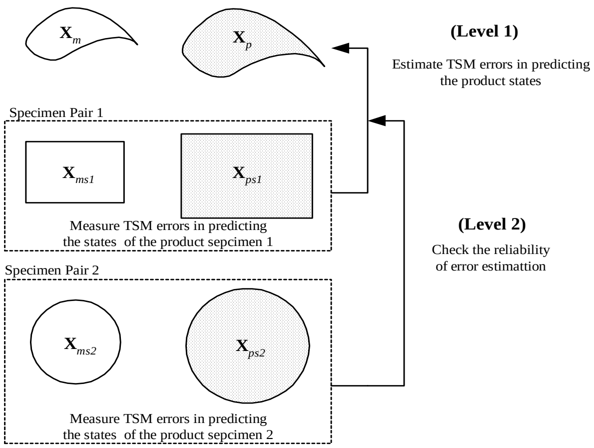

Figure 4. Overview of the Two-Level TSM Error Estimation

z is the order of the polynomial approximation. Problems that correlate the states of two systems with linear-nonlinear or nonlinear-nonlinear material properties fall into this category.

Through the prediction equations, the ESM has the capability to predict product behaviors more precisely. However, the ESM currently has several limitations. First of all, there exist problem domains for which one cannot construct an ideal state transformation using the measured states of the specimen pair. In this case, one may try other types of state transformations (e.g., bilinear transformation), and choose the best transformation. However, this approach will introduce prediction error depending on the trial transformations, and one need a guidance to decide proper transformations. Secondly, the ESM assumes that the boundary conditions imposed on the model and model specimen, and the product and product specimen are topologically identical. This assumption may be violated in some situations. Due to such limitations, we need a method to estimate prediction errors of the ESM.

3 TSM ERROR MEASURES

The scale testing error e_i is defined as the difference between the unknown true product state at a spatio-temporal point and the state estimated through a specific similarity method. In the TSM, one predicts product states by multiplying the model state by a constant state scale factor λ_X, and the TSM error can be represented as

$$e_i = X_{p,i} - \lambda_X X_{m,i}. \tag{15}$$

In the equation $X_{p,i}$ is the unknown true product state, and $X_{m,i}$ is the measured state of a scaled model at a spatio-temporal point p_i. In the equation, the state scale factor λ_X is determined from Equation (3) on the basis of assumed system parameters.

In general, e_i depends on the spatio-temporal point and system parameters (e.g., geometry of the target system). As the dependency of e_i on system parameters is not known, it is a difficult task to estimate e_i from the previous testing results on similar problems. As a systematization of the current practice that validates TSM results through previous experiences on similar problems, a two-level error estimation approach depicted in Figure 4 is proposed. As shown in the figure, the TSM error in predicting the product state is estimated through a specimen test under a certain assumption (Level 1), and another specimen testing is performed to check the validity of the assumption (Level 2). The following two subsections detail the two-level error estimation procedure, and provide two error measures for confident scale testing with the TSM.

3.1 Estimation of the TSM Prediction Error

As an attempt to represent the TSM error in terms of the specimen testing error, we assume that the states of the scaled model, the product and the specimen pair can be represented as

$$\begin{aligned} X_{m,i} &= g(\mathbf{G})h(\mathbf{M}_m), \quad \text{Scale Model} \\ X_{p,i} &= g(\mathbf{G})h(\mathbf{M}_p), \quad \text{Target Product} \\ X_{ms,i} &= g(\mathbf{G}_s)h(\mathbf{M}_m), \quad \text{Model Specimen} \\ X_{ps,i} &= g(\mathbf{G}_s)h(\mathbf{M}_p), \quad \text{Product Specimen} \end{aligned} \tag{16}$$

where $\mathbf{G}$ and $\mathbf{M}$ are respectively sets of geometric and non-geometric parameters, and g and h are unknown functions. If we denote the functions $g(\mathbf{G})$ and $h(\mathbf{M}_p)$ as G and H, and $g(\mathbf{G}_s)$ and $h(\mathbf{M}_m)$ as G^* and H^*, one can derive the following condition that relates the states of the four objects,

$$\frac{X_{m,i}}{X_{ms,i}} = \frac{X_{p,i}}{X_{ps,i}} = \frac{G}{G^*}. \tag{17}$$

From the definition of the prediction error, errors in predicting product and product specimen states at a spatio-temporal point i can be represented as

$$\begin{aligned} e_i &= \lambda_X X_{m,i} - X_{p,i}, \\ e_{s,i} &= \lambda_X X_{ms,i} - X_{ps,i}, \end{aligned} \tag{18}$$

where λ_X is the theoretical state scale factor. By combining Equations (17) and (18), one can represent the TSM errors as

$$e_i = \gamma_i(\lambda_X X_{ms,i} - X_{ps,i}), \tag{19}$$

where $\gamma_i = \dfrac{G}{G^*}$ is a form factor that represents the state change under the pure geometric variation. As a consequence, one can derive the following TSM error equation,

$$e_i = \gamma_i e_{s,i}. \tag{20}$$

Using this equation, one can estimate the product testing error e_i at a point, as the form factor γ_i can be determined by measuring the states of the model and the model specimen, and the specimen error $e_{s,i}$ through a specimen pair. The error estimation equation implies that the estimated error is proportional to the form factor that reflects the state changes under the geometrical variation. As we are interested in estimating the maximum error, we define the Level 1 TSM error measure as

$$E_t = max(|e_i|), \tag{21}$$

where $i = 1, 2, \ldots, N$, and N is the number of measurement points.

The reliability of the defined error measure depends on the correctness of the imposed assumption (Equation 16), and the next subsection quantifies the reliability of the Level 1 TSM error measure by employing another specimen pair.

3.2 Accuracy of TSM Error Estimation

To quantify the correctness of estimated TSM error e_i or Level 1 error measure E_t, we employ another specimen pair as shown in Figure 4. We define a measure that represents the accuracy of the error measure E_t by considering the inter-relationship of the states of two specimen pairs shown in Figure 5. In the figure, the circles that represent the states at a point have the following meanings:

> The black circles denote the measured specimen states.
> The white circles represent the states of product specimens predicted through a theoretical state scale factor λ_X.
> The gray circle is the state of the additional product specimen when the assumption imposed to estimate E_t (Equation 16) is true.

Thus, the length L represent the estimated TSM error e_i, and ΔL represents the error in our assumption. For example, the estimated TSM error e_i is perfect when ΔL vanishes; and the ratio of ΔL to L becomes large, if the assumption becomes incorrect. We define a measure r_i that reflects reliability of e_i as

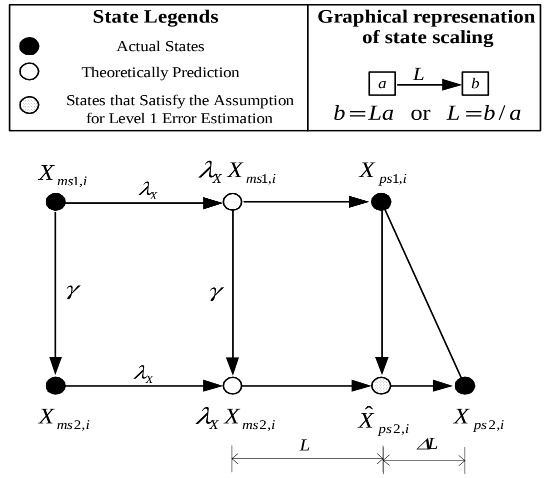

Figure 5. View of State Scale and Form Factors

$$\begin{aligned} r_i &= \frac{Actual\,Error - Estimated\,Error}{Actual\,Error} \\ &= \frac{X_{ps2,i} - \hat{X}_{ps2,i}}{X_{ps2,i} - \lambda X_{ms2,i}}. \end{aligned} \tag{22}$$

From Figure 5, the unknown state $\hat{X}_{ps2,i}$ can be represented as

$$\begin{aligned} \hat{X}_{ps2,i} &= \gamma X_{ps1,i} \\ &= \frac{X_{ms2,i}}{X_{ms1,i}} X_{ps1,i}. \end{aligned} \tag{23}$$

By substituting the equation into Equation 22, we can express r_i as

$$\begin{aligned} r_i &= \frac{X_{ps2,i} - \dfrac{X_{ms2,i}}{X_{ms1,i}} X_{ps1,i}}{X_{ps2,i} - \lambda X_{ms2,i}} \\ &= \frac{X_{ms1,i} X_{ps2,i} - X_{ms2,i} X_{ps1,i}}{X_{ms1,i} X_{ps2,i} - \lambda_X X_{ms1,i} X_{ms2,i}}. \end{aligned} \tag{24}$$

Consider the following two extreme cases for proper interpretation of r_i. If the imposed assumption is perfect, $X_{ps2,i} = \hat{X}_{ps2,i}$, and r_i becomes 0. In contrast, r_i approaches to 1 if the imposed assumption becomes invalid. As a consequence, one can conclude that

1. The estimated error is identical to actual prediction error, if $r_i = 0$.
2. The estimated error does not represent actual prediction error at all, if $r_i = 1$.

As we should be aware of the worst case (lowest confidence), we define the Level 2 TSM error measure that represents confidence on E_t as

$$R_t = max(r_i), \tag{25}$$

where $i = 1, 2, \ldots, N$, and N is the number of measurement points.

Through the two TSM error measures, E_t and R_t, we can judge TSM results as follows.

If E_t is not within tolerable error range reject TSM results.
If E_t is within tolerable error range and R_t is small, accept TSM results.

A numerical example for the TSM error measures is provided in Section 5, in order to demonstrate the validity of our approach and the error measures defined in Equations (20) and (24).

4 ESM ERROR MEASURES

The ESM estimates the product states assuming that *the product state is a function of the states of a specimen pair and a scaled model.* Assuming that this assumption is true, an error measure E_e that represents state prediction accuracy of the ESM is defined. Similar to the TSM error estimation procedure, we also define R_e to check the validity of the imposed assumption, by employing another specimen pair. In this approach, two additional specimen pairs are necessary (one to define E_e, and another to define R_e). The mathematical formulation of the error measures is described in the following subsections.

4.1 Estimation of the ESM Prediction Error

Consider the model, the product and the two specimen pairs shown in Figure 6. Assume that there exists an ideal function h that satisfies

$$\begin{aligned}\mathbf{x}_p &= h(\mathbf{x}_{ms1}, \mathbf{x}_{ps1})\mathbf{x}_m \\ &= h(\mathbf{x}_{ms2}, \mathbf{x}_{ps2})\mathbf{x}_m.\end{aligned} \tag{26}$$

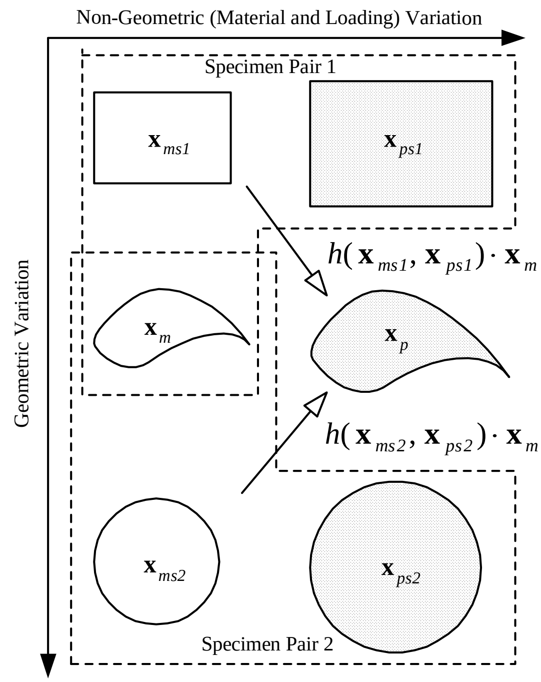

Figure 6. Product State Predictions via Two Specimen Pairs.

Then the ESM errors using the specimen pairs 1 or 2, $\mathbf{e_1}$ and $\mathbf{e_2}$, can be represented as

$$\begin{aligned}\mathbf{e}_1 &= \mathbf{x}_p - g(\mathbf{x}_{ms1}, \mathbf{x}_{ps1})\mathbf{x}_m \\ &= h(\mathbf{x}_{ms1}, \mathbf{x}_{ps1})\mathbf{x}_m - g(\mathbf{x}_{ms1}, \mathbf{x}_{ps1})\mathbf{x}_m \\ \mathbf{e}_2 &= \mathbf{x}_p - g(\mathbf{x}_{ms2}, \mathbf{x}_{ps2})\mathbf{x}_m \\ &= h(\mathbf{x}_{ms2}, \mathbf{x}_{ps2})\mathbf{x}_m - g(\mathbf{x}_{ms2}, \mathbf{x}_{ps2})\mathbf{x}_m,\end{aligned} \tag{27}$$

where the function g is the state transformation constructed to predict the product states. It should be noticed that the prediction errors $\mathbf{e}_1$ and $\mathbf{e}_2$ are not identical in general, as the functional values of $g(\mathbf{x}_{ms1}, \mathbf{x}_{ps1})$ and $g(\mathbf{x}_{ms2}, \mathbf{x}_{ps2})$ are generally distinct. Combining the two equations, we define the mean ESM error $\mathbf{e}$ as

$$\begin{aligned}\mathbf{e} =& \frac{\mathbf{e}_1 + \mathbf{e}_2}{2} \\ =& \mathbf{x}_p - \frac{1}{2}\Big[g(\mathbf{x}_{ms1}, \mathbf{x}_{ps1}) - g(\mathbf{x}_{ms2}, \mathbf{x}_{ps2})\Big]\mathbf{x}_m \\ =& \frac{1}{2}\Big[h(\mathbf{x}_{ms1}, \mathbf{x}_{ps1}) + h(\mathbf{x}_{ms2}, \mathbf{x}_{ps2}) \\ & - g(\mathbf{x}_{ms1}, \mathbf{x}_{ps1}) - g(\mathbf{x}_{ms2}, \mathbf{x}_{ps2})\Big]\mathbf{x}_m.\end{aligned} \tag{28}$$

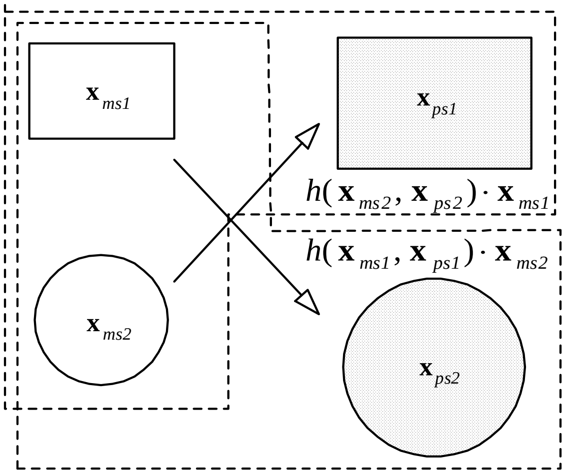

Figure 7. Specimen State Predictions through Another Specimen Pair.

As the ideal function h is not known, we consider the state transformations between two specimen pairs, as shown in Figure 7. The error in predicting the state of the first product specimen, $\mathbf{x}_{ps1}$, through $\{\ \mathbf{x}_{ms2}, \mathbf{x}_{ps2}, \mathbf{x}_{ms1}\ \}$, can be represented as

$$\mathbf{e}_{21} = \Big[h(\mathbf{x}_{ms2}, \mathbf{x}_{ps2}) - g(\mathbf{x}_{ms2}, \mathbf{x}_{ps2}) \Big] \mathbf{x}_{ms1}. \tag{29}$$

Similarly, the prediction error of the second product specimen's states, $\mathbf{x}_{ps2}$, through $\{\ \mathbf{x}_{ms1}, \mathbf{x}_{ps1}, \mathbf{x}_{ms2}\ \}$, becomes

$$\mathbf{e}_{12} = \Big[h(\mathbf{x}_{ms1}, \mathbf{x}_{ps1}) - g(\mathbf{x}_{ms1}, \mathbf{x}_{ps1}) \Big] \mathbf{x}_{ms2}. \tag{30}$$

From Equations (29) and (30), one can approximate the deviation of the ideal and constructed functions h and g at the specimen states as

$$\begin{aligned} h(\mathbf{x}_{ms2}, \mathbf{x}_{ps2}) - g(\mathbf{x}_{ms2}, \mathbf{x}_{ps2}) &\approx \mathbf{e}_{21}\mathbf{x}^{+}_{ms1} \\ h(\mathbf{x}_{ms1}, \mathbf{x}_{ps1}) - g(\mathbf{x}_{ms1}, \mathbf{x}_{ps1}) &\approx \mathbf{e}_{12}\mathbf{x}^{+}_{ms2}. \end{aligned} \tag{31}$$

By substituting these equations into Equation (28), we can estimate the mean prediction error $\mathbf{e}$ as

$$\mathbf{e} \approx \frac{1}{2} \left\{ \mathbf{e}_{21}\mathbf{x}^{+}_{ms1} + \mathbf{e}_{12}\mathbf{x}^{+}_{ms2} \right\} \mathbf{x}_{m}. \tag{32}$$

Since the specimen prediction errors $\mathbf{e}_{21}$ and $\mathbf{e}_{21}$ are known from the specimen testing shown in Figure 7, we can estimate the mean prediction error. Similar to the TSM error measures, we define the Level 1 ESM error measure as

$$E_e = max(e_i) \tag{33}$$

where e_i is the element of the vector $\mathbf{e}$. One needs two specimen pairs in order to estimate the prediction error. As the prediction error is estimated assuming that the ideal state transformation can be expressed by Equation (26), the reliability of the error estimation is dependent on the correctness of the assumption. Thus, we define another measure that estimates the reliability of the error estimation in Equation (32).

4.2 Accuracy of ESM Error Estimation

To utilize the scale testing results with confidence, we need to know the prediction error bound. We define an error measure that estimates the prediction error assuming that Equation (26) is valid. If the assumption is valid, the residual of Equation (32),

$$\mathbf{R} = \mathbf{e} - \frac{1}{2} \left\{ \mathbf{e}_{21}\mathbf{x}^{+}_{ms1} + \mathbf{e}_{12}\mathbf{x}^{+}_{ms2} \right\} \mathbf{x}_{m\cdot}, \tag{34}$$

should be negligible, and $\mathbf{R}$ quantifies the reliability on the estimated error $\mathbf{e}$. As we do not know true $\mathbf{e}$, another specimen pair is fabricated as a replacement of the scaled model and the product pair. Considering the three specimen pairs, $\mathbf{R}$ can be calculated from

$$\begin{aligned} \mathbf{R} = &\left[\mathbf{x}_{ps3} - \frac{1}{2} \Big\{ g(\mathbf{x}_{ms1}, \mathbf{x}_{ps1}) - g(\mathbf{x}_{ms2}, \mathbf{x}_{ps2}) \Big\} \mathbf{x}_{ms3} \right] \\ &- \left\{ \mathbf{e}_{21}\mathbf{x}^{+}_{ms1} + \mathbf{e}_{12}\mathbf{x}^{+}_{ms2} \right\} \mathbf{x}_{m} \end{aligned} \tag{35}$$

$\mathbf{R}$ can be interpreted as the difference between normalized errors, and the measure $\mathbf{e}$ for the error estimation is meaningful if the maximum value of $\mathbf{R}$ is small. Similar to the TSM, we define the Level 2 ESM error measure as

$$R_e = max(|r_i|), \tag{36}$$

where r_i is the element of $\mathbf{R}$.

5 NUMERICAL EXAMPLES: THERMAL BEHAVIOR OF TURBINE BLADES

We illustrate the error estimation procedure by considering the problem to predict the steady-state temperature of the turbine blade turbine blade shown in Figure

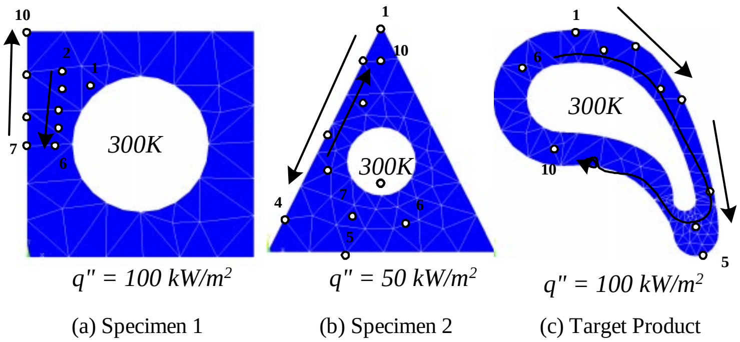

Figure 8. Geometry and 10 Sampling Points of Specimen and Product Turbine Blades

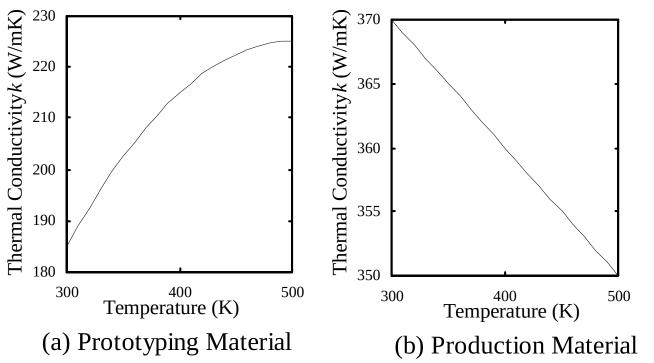

Figure 9. Thermal Conductivity of Prototyping and Production Materials

8(c) through scale testing. The turbine blade is a geometrically complex part, and thermal modeling to determine thermal deformation and stresses is integral to its design and manufacturing. In this example, finite element analysis with ANSYS^{TM} is performed to simulate temperatures, as a preliminary study for more realistic experimental investigations under the preparation. We are preparing experimental studies, in which metal and polymer blades will be fabricated from a traditional machining process and the selective laser sintering process, respectively.

The geometry and boundary conditions of the specimen blades, for the prediction of the temperature of the blade through the TSM or the ESM, are shown in Figures 8(a) and (b). For clear comparisons of the model and product behaviors, the size and loads of the models and products are kept identical.

The model and product blades are distorted due to nonlinear thermal conductivity, and the distinct model and product material behaviors are shown in Figure 9. Finite element analysis to simulate the temperature of the turbine blades is performed, and the steady-state temperature

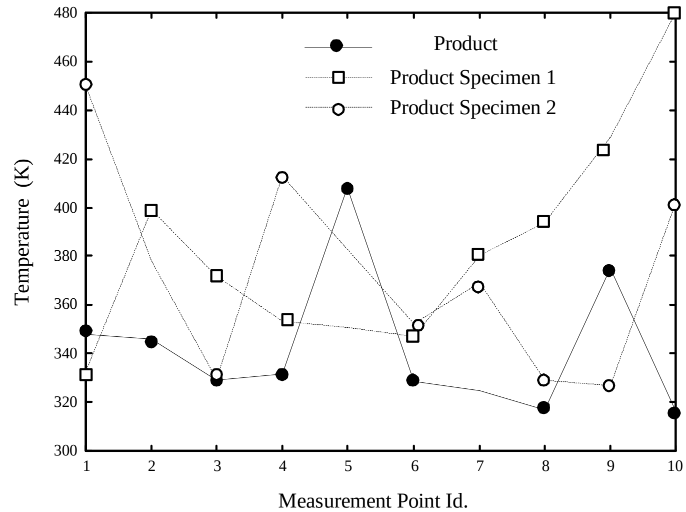

Figure 10. Temperatures of the Product and the Product Specimens at 10 Sampling Points

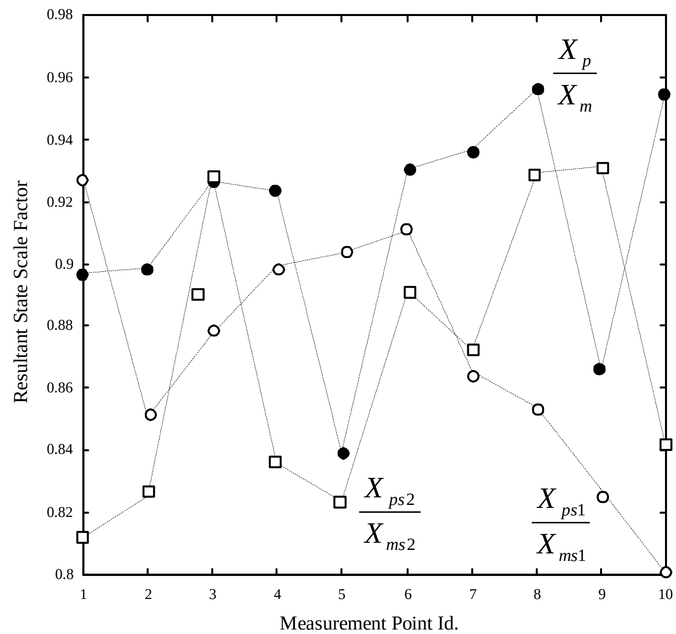

Figure 11. State Scale Factors of the Specimens and the Product

of the product and product specimens at selected measurement points are plotted in Figure 10. The state scale factors, the ratios of the corresponding product and model states are plotted in Figure 11, and they approximately range from 0.8 to 1.0. Recalling that the factor should be independent on the location of measurement points and system geometry in case of well-scaled problems, one can notice the difficulty in predicting accurate temperature of

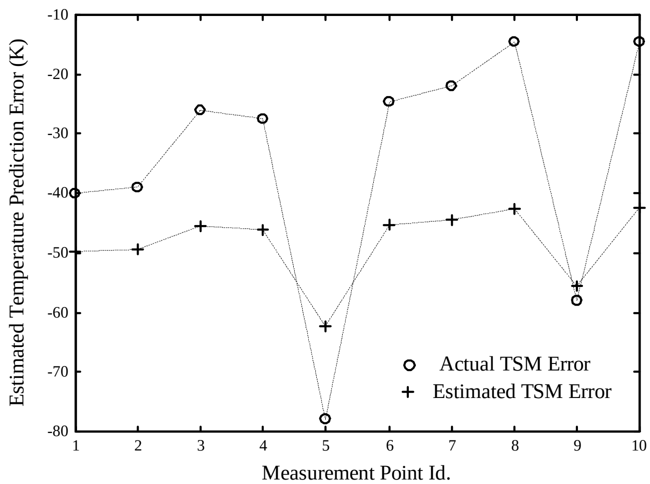

Figure 12. Estimated vs Actual TSM Prediction Error

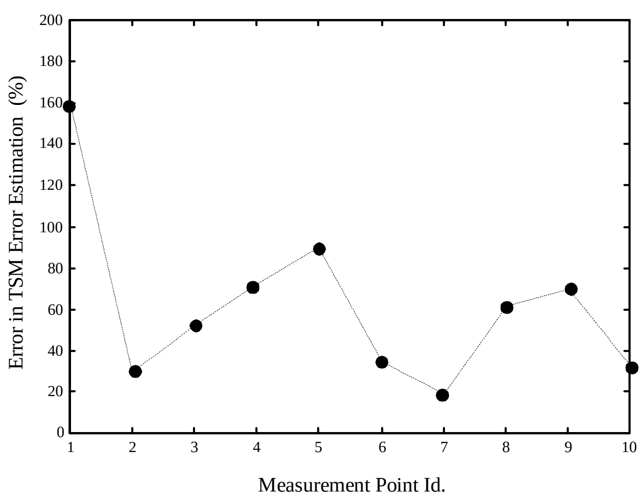

Figure 13. Reliability of TSM Error Estimation

the product blade through the TSM. In the following subsections, the temperature of the product blade is predicted though the TSM and ESM, and then the prediction error is estimated through the Level 1 error measures. Finally, the error estimation results are interpreted through the defined Level 2 error measures.

5.1 TSM Error Estimation

The temperature of the product blade is predicted through the TSM, assuming that the thermal conductivity of the model and product materials is constant. If this assumption is true, the model and product blades should show identical temperature distributions, as the governing equation of the steady-state thermal problem is independent of thermal conductivity.

The temperature prediction error of the blade through the scaled model is the difference between the states of two systems, as the ideal state scale factor λ_X is 1.0. The actual prediction errors at 10 selected spatial points are plotted and compared to the error estimated from Equation (20), using the specimen pair 1. In Figure 12, the actual prediction error and error estimated through the specimen pair 1 are compared. As shown in the figure, the actual and estimated errors show similar trends, but the local error estimation is not reliable.

In order to check the correctness of the imposed assumption, r_i is calculated from Equation (24) on the basis of the states of specimens 1 and 2, and r_i is plotted in Figure 13 (a). As r_i is defined as the ratio of the error in error estimation with respect to the actual error, approximately 150% error in error estimation is expected from the plot.

In summary, we have estimated the error in the TSM, by utilizing a scaled model and the specimen 1. From the error estimation, the maximum local prediction error is expected to be $60K$ (from the estimated TSM error at $5th$ measurement point in Figure 12). As a consequence, we may need to reject the TSM results, and perform scale testing through the ESM or full-scale testing, if more accurate testing results are required. By considering another specimen pair, we have estimated the reliability of the estimated error that is comparable to a correlation coefficient in linear regression analysis. Overall, the reliability measure R_t, i.e. the maximum of r_i, is 160%, and the estimated maximum TSM error is expected to be around $60 \times 1.6 \approx 100K$. In general, $100K$ temperature error is not acceptable, and we predict the temperature using the ESM in the next subsection.

5.2 ESM Error Estimation

As discussed earlier, we can obtain a better prediction of the temperature of the product using the scaled model and a specimen pair through a polynomial transformation. In this subsection, scale testing with the ESM is simulated on the basis of Equation (13), with the first and third order polynomials.

The simulated prediction errors are compared to the estimated errors calculated from Equation (32), and a comparison of the estimated and actual prediction error is plotted in Figure 14. In both cases, we can notice the improved prediction accuracy, and good estimation of the ESM error. Especially, the ESM with the 3rd order polynomial provides remarkable prediction accuracy that is comparable to full-scale product tests.

To check the reliability of the estimated error, $\mathbf{r}_i$ is calculated using an additional specimen pair with the geometry

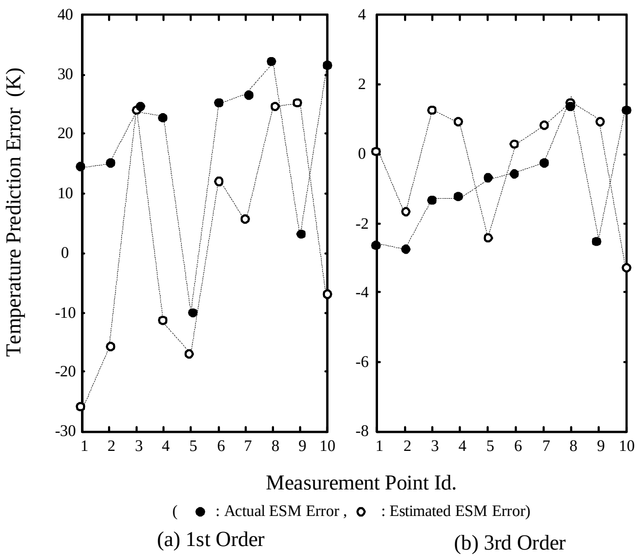

Figure 14. Estimated vs Actual ESM Prediction Error

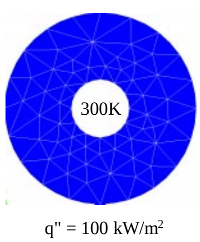

Figure 15. Additional Specimen (Specimen 3) for Relaibility Testing

shown in Figure 15, and the results are plotted in Figure 16.

In case of the ESM with the first order polynomial, the ESM errors range from up to 30 K (Figure 14(a)). The prediction accuracy is dramatically improved through the third order polynomial, and the maximum ESM error is within 3 K (Figure 14(b)). The maximum ESM errors estimated from Equation 32 are $30K$ and $3K$ in the first and third order polynomial cases (Figure 16), respectively. In both cases, one can notice the reliability of the error estimation (Level 1 error measure). Currently, the physical meaning of the ESM reliability measure is not yet clear, and we need to clarify the meaning of the measure or define a new meaning measure. However, through the two error measures, one can at least decide desirable order of the polynomials. From the plots in Figures 14 and 16, one can conclude that we can best estimate the product temperature through the 3rd order transformation, as both the Level 1 error measure E_t and the level 2 error measure R_t are small.

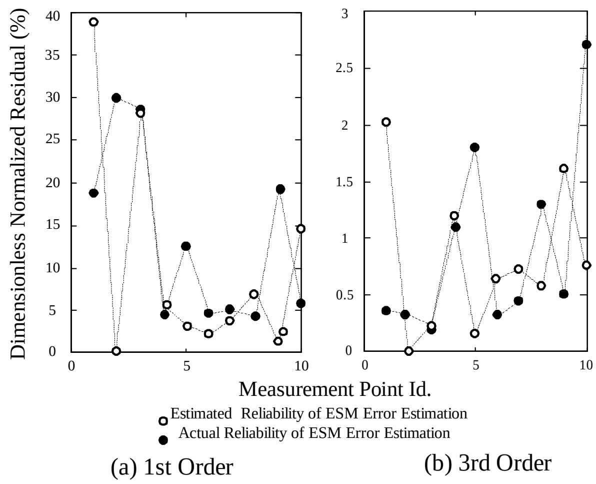

Figure 16. Reliability of ESM Error Estimation: Normalized Residual, $\mathbf{R}$.

6 EXPECTED IMPACT AND FUTURE WORK

In this paper, we have addressed the problem of error estimation in scale testing. We describe two error measures, one to estimate the prediction error, and the other to check the reliability of the measure of error estimation. These error measures can be utilized not only to validate scale testing results, but also to guide the construction of proper state transformations for accurate prediction of product states. As a consequence, one can provide accurate testing results with confidence. Direct product tests, which are currently performed due to the lack of confidence in scale testing results, can be potentially replaced with the effective scale testing. This strategy is expected to lead to significant cost and time savings, especially at the later stages of the product development process (Wall et. al, 1991; Ulrich and Eppinger, 1995).

Although the feasibility of the error measures has been demonstrated through numerical simulations, more diverse simulations and accompanying experiments in several problem domains must be performed to concrete the measures. In parallel to studying error measures, the authors are continuing research to establish the general theoretical framework of the empirical similarity method, including construction of similarity transformations using approximate lumped models (Cho et al., 1999).

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