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Tuning Parameter Tolerance Design: Foundations, Methods, and Measures

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Abstract. *In this paper, a novel technique is presented to solve tolerance design problems. To achieve the desired performance tolerance, the technique uses a subtle, but significant, change in the design: the addition of a tuning parameter in place of an increase in component precision. Statistical models are used to develop a framework for the tuning parameter design method. Also developed is a new, dimensionless design metric which ranks candidate tuning parameters. A step-by-step method is developed for the application of tuning parameters using this metric. The step-by-step tuning parameter design method is applied to a heavy-duty manual stapler as a clarifying example.*

Keywords: Design metric; Design principles; Dimensionless design parameter; Formal design methods; Tolerance design; Tuning parameter design

1. Introduction

As system or component tolerances become tighter, manufacturing costs generally increase (Drake 1997; Chase and Greenwood 1987; Abdel-Malek and Asadathorn 1994; Speckhart 1972). To create high, or in some cases, sufficient precision systems, engineers often rely on 'good' design to avoid the need for precision components. These techniques of 'good tolerance' design can be classified according to tolerance design principles (McAdams and Wood 1999). In most cases, the application of these principles is informal, and based on experience and trial and error. From robust design come methods more sophisticated than trial and error for application of the tolerance design principles of moving the nominal set points to improve a signal-to-noise ratio (Creveling 1997). The goal of this paper is to provide a theoretical foundation, methodology and a design metric for formalizing the application of a different tolerance design principle. The tolerance design principle of interest here is the use of a tuning parameter to provide the desired system performance.

An insightful example of a tuning parameter is the adjustment parameters on the front wheel of an automobile. To drive and steer correctly (straight) requires precise positioning of the front wheels. As suspension, frame, and steering components are manufactured and assembled, the stack-up error in camber, castor, and toe-in cause a car to steer poorly. Manufacturing an automobile with enough precision such that these tolerance stack-ups are small enough to allow satisfactory steering is impractical. To allow accurate and straight steering, adjustment parameters for castor, camber, and toe-in that allow the alignment to be tuned are designed into the steering and wheel mounting systems.

Though perhaps not always recognized as such, the use of a tuning parameter to solve tolerance design problems is a valuable and common design technique. The usefulness and power of a tuning parameter as a general design technique is developed by Otto and Antonsson (1993). To move the application of tolerance tuning parameter design from informal to formal, there is a need for a solid theoretical foundation and design method. In this paper, these first steps toward a complete theory of tuning parameter design are presented.

The objective here is to develop a formal methodology for tuning parameter design and base it firmly in practical engineering techniques. Such an objective raises several key questions. How does a designer know when a tuning parameter is needed? What is a useful model of a tuning parameter? What are the performance requirements of the tuning parameter? And, which of the existing design parameters serves as a best candidate for the addition of tunability? These questions are answered in this paper through a step-by-step method which provides the framework for a tuning parameter design method. A simple example is also presented to clarify the procedure.

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2. Step 1: Determining Need

The first step in designing a tuning parameter for tolerance design is to determine if one is needed. There is no need to add additional complexity, manufacturing, or assembly steps if a product performance is within the desired tolerance. The simple question is, given a performance parameter, a performance parameter tolerance requirement, a design solution, a chosen manufacturing process, and the associated manufacturing tolerances, does the system perform within the desired tolerance specification?

Here, the question is answered formally and generally using a statistical model of the variational relationship between the performance and the design parameters.

2.1. An Engineering Model for Tuning Parameter Tolerance Design

Here, a model is presented for tolerance design and analysis. The development and presentation of this model is important for several reasons. One key goal of this development is the provision of a solid analytical foundation for tuning parameter design. Also, this model development is important to show the integration of tuning parameter design as a general tolerance design tool.

The discussion in this section and in the remainder of this paper assumes that the performance metric of interest can be represented parametrically. In other words, the product has some performance parameter p, which may be a length, a mass, a time, an energy, or any other metric that affects customer needs. This performance parameter can be expressed as a function of design parameters in the form

$$p = f(d_1, d_2, \ldots, d_n) \tag{1}$$

An assumption is made here that Eq. (1) describes the system completely. The term *completely* is interpreted as follows: a design change occurs in the value of p if, and only if, there is some change in at least one design parameter d_i.

Throughout this paper, the terms *variation, tolerance, natural tolerance, variance*, and *standard deviation* are used with specific and distinct meanings. *Variation* means some change or error in a parameter from an ideal or nominal value. *Tolerance* is the specified maximum acceptable variation from a target value. *Natural tolerance* is used to describe the actual variation of a design or performance parameter. In other words, if the tolerance is greater than the natural tolerance, the performance or part is within the design specification. In the literature, *natural tolerance* more often refers to the variation of a single component feature that results from some specific manufacturing or fabrication technique (Bowker and Lieberman 1959). Here the definition of natural tolerance is extended to include the variation of an entire system as it occurs with some specified configuration and manufacturing process. The term *variance* is used with its common statistical meaning: the second moment about the mean of a randomly occurring value. *Standard deviation* is used with the common statistical meaning.

2.2. Developing the Mathematical Model

Two equations are presented here. The first is the system response of Eq. (1) based on the assumption that each of the design variables d_i is random and normally distributed. A common and reasonable assumption in tolerance design and analysis is to assume that the design variables in Eq. (1) occur as normally distributed random variables (Bowker and Lieberman 1959; Creveling 1997; Chase and Greenwood 1987). In fact, leaders in manufacturing often require suppliers to provide statistical verification to design specification conformance (Wright 1990).

The second equation of interest is the standard deviation or variance of p based on the same assumption. In the derivations of each of these equations, it is assumed that the design variables are independent. A discussion of cases in which the design variables are dependent is beyond the scope of this article, and is left for future work. The complete derivations are included in the appendix.

2.2.1. The System Response

Based on the derivation in the appendix, the system response is

$$\bar{p} = f(\overline{d_1}, \overline{d_2}, \ \ldots, \ \overline{d_n}) \ + \ \frac{1}{2}\sum_{i=1}^{n} \frac{\partial^2 f}{\partial d_i^2} Var(d_i) \tag{2}$$

where

$$\bar{p} = E(p) \tag{3}$$

and

$$\overline{d_i} = E(d_i) \tag{4}$$

Though not a true equality (it is an approximation), hereafter the system response will be written as one.

2.2.2. The System Variance
Based on the derivation in the appendix, the variance on the system performance is determined as

$$Var(p) = Var[f(d_1, d_2, \ldots, d_n)] = \sum_{i=1}^{n} \left(\frac{\partial^2 f}{\partial d_i}\right)^2 Var(d_i) \quad (5)$$

It is more common to describe tolerances in terms of the standard deviation, rather than the variance. To express Eq. (5) in terms of the standard deviation, the square root of both sides gives

$$\sigma_p = \sqrt{\sum_{i=1}^{n} \left(\frac{\partial f}{\partial d_i}\right)^2 \sigma_i{}^2} \quad (6)$$

where σ_i is the standard deviation of the design parameter d_i.

2.2.3. Determining Need
It is common practice in engineering to interpret the tolerance specification as $N\sigma$ (Creveling 1997). For example, if the weight target value and tolerance for a breakfast cereal box is listed as $24 \pm 1oz$, the standard deviation of the weight of the box is $(1/N)oz$. The analogous relation holds between the natural tolerance and standard deviation. Generally, N is taken to be 3. For $N = 3$ (Creveling 1997), the probability of the performance being within $p \pm 3\sigma_p$ is 99.7% (Hahn and Shapiro 1994). This convention is used in the remainder of this paper. In other words, *tolerance* implies a system performance requirement on the standard deviation of $\sigma_p = tol/N$ with $N = 3$. Similarly, the natural tolerance of the system is interpreted as $ntol = N\sigma_p = 3\sigma_p$.

Using this convention and the equations developed above, a straightforward calculation is used to determine whether a tuning parameter is needed. Using Eq. (6), if the condition

$$tol/3 < ntol/3 = \sigma_p = \sqrt{\sum_{i=1}^{n} \left(\frac{\partial f}{\partial d_i}\right)^2 \sigma_i{}^2} \quad (7)$$

is violated, a tuning parameter is needed, i.e. if the natural tolerance exceeds the tolerance, the addition of tunability is required.

3. Step 2: Performance Requirements for Tuning Parameter Candidates

The next step in the tuning parameter design process is to determine the performance requirements for candidate tuning parameters. The goal of the method presented in this paper is to use an engineered, rather than intuitive, approach to determine which design parameters are superior candidates for the addition of tunability. To achieve this goal, the tuning parameter performance requirements for multiple design parameters are found and compared in Step 3. The performance requirements for a tuning parameter arealso used as design constraints in Step 4 as tunability is designed into the system. Therefore, the performance requirements are discussed and developed in Step 2.

There are two key performance requirements for a tuning parameter. The first critical requirement of this parameter is that, through some range in its adjustment, an out-of-specification performance be tuned to the target value. The second important performance parameter is that the tuning parameter provide the required performance tolerance for the system. This section includes a discussion of the tuning parameter range and tolerance requirements.

3.1. Determining the Tuning Parameter Range Requirements

The performance parameter distribution that is implied by some performance specification $p \pm tol$ is shown in Fig. 1. The dotted line represents the performance parameter distribution of the actual design. As shown, the natural tolerance is larger than the specified tolerance. Thus, as discussed in the previous section, a tuning parameter needs to be added to the system.

Consider a single product taken from the distribution, with performance p^*, as shown in Fig. 1. The requirement of the tuning parameter is that when this value is some $p^* \neq \bar{p}$, then the value of d_k is changed to some new value d_k^t such that $p* = \bar{p}$. Here, the parameter to which tunability is added is denoted by d_k. This condition can be stated more explicitly as follows: if the particular value of a performance metric p is

$$p^* = f(d_1^*, d_2^*, \ldots, d_k, \ldots d_n^*) \neq \bar{p} \quad (8)$$

where the superscript * indicates the specific values that the design parameters have taken to cause p^*, then d_k must be changed to d_k^t such that

$$p^t = f(d_1^*, d_2^*, \ldots, d_k^t, \ldots d_n^*) = \bar{p} \quad (9)$$

where p^t is the value achieved by tuning the system. The different values to which d_k^t must be adjusted are

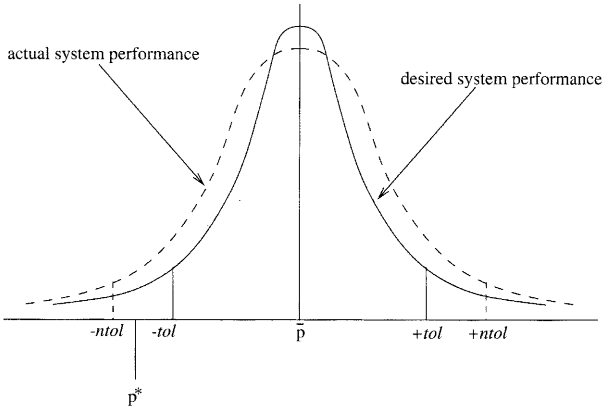

Fig. 1. The response of a performance parameter. The solid line represents the desired response. The dotted line represents the actual response.

a key performance requirement for the tuning parameter. Inverting Eq. (9) and solving for the tuning parameter gives

$$d_k^t = f^{-1}(\bar{p}, d_1^*, d_2^*, \ldots\ d_{k-1}^*, d_{k+1}^*, \ldots, d_n^*) \quad (10)$$

Thus, the particular value of d_k^t needed to tune can be determined from the original model. In cases where Eq. (10) does not exist in closed form, numerical techniques need to be used in this equation and in the following optimization.

To determine the adjustability requirement on d_k, a statistical worst-case condition is considered for the design variables $d_i\ \forall_i \neq k$. The worst possible value that occurs for each is $d_i^{max} = \overline{d_i} + ntol_{di}$ or $d_i^{min} = \overline{d_i} + ntol_{di}$. The adjustable range requirement for d_k is expressed as the solution to two constrained optimization problems:

$$d_k^{low} = min[f^{-1}(\bar{p}, d_1, d_2, \ldots\ d_{k-1}, d_{k+1}, \ldots, d_n)] \quad (11)$$

subject to

$$d_i^{min} \leq d_i \leq d_i^{max}, \quad i = 1, 2, \ldots, k-1, k+1, \ldots, n \quad (12)$$

and

$$d_k^{high} = max[f^{-1}(\bar{p}, d_1, d_2, \ldots\ d_{k-1}, d_{k+1}, \ldots, d_n)] \quad (13)$$

subject to

$$d_i^{min} \leq d_i \leq d_i^{max}, \quad i = 1, 2, \ldots, k-1, k+1, \ldots, n \quad (14)$$

Simply put, the requirement on d_k is that it have some range of adjustability from d_k^{low} to d_k^{high} such that it can always tune p to $\bar{p}$ for any value d_i that occurs. Though the solution to $2 \times n$ optimization problems is potentially computationally time-consuming, the worst-case range on the design variables d_i is generally small thus, reducing the optimal search space on the constraint requirement. In practice, due to this small search space, the extrema very often lie on the search space boundaries.

3.2. Tuning Parameter Tolerance Requirements

The addition of a tuning parameter essentially removes the random effects of the design parameters that are not tuned. The value of d_k^t is set in *response* to the values the design parameters have taken for a single case. Thus, from the perspective of output performance, the untuned design parameters are no longer stochastic variables but fixed constants. From the perspective of the output performance, however, the tuning parameter remains a stochastic variable. The addition of a tuning parameter adds a new set of design parameters to the system. Also, there is some tuning procedure, such as aligning the front of the steering system as discussed in Section 1, that results in a random variation of the performance. Using this model, each particular value of d_k^t is actually some expected value $E(d_k^t)$ with some standard deviation $\sigma_{d_k^t}$.

Proceeding with a derivation similar to that presented in Step 1, the standard deviation of the system can be represented as

$$\sigma_p = \left| \left(\frac{\partial f}{\partial d_k} \right) \right| \sigma_{d_k} \quad (15)$$

because there is now only one random variable: d_k. Using Eq. (15), the tolerance requirement for the tuning parameter is expressed in terms of the system tolerance requirement as

$$tol_{d_k} = 3\sigma_{d_k} = \frac{1}{\left| \left(\frac{\partial f}{\partial d_k} \right) \right|} 3\sigma_p = \frac{1}{\left| \left(\frac{\partial f}{\partial d_k} \right) \right|} tol_p \quad (16)$$

By substituting the system tolerance requirement into Eq. (16), the tolerance requirement for each candidate is determined.

3.3. Summarizing the Results

Each design parameter, if used as the tuning parameter, has different individual performance requirements. These parameters are used in the next step to determine which design parameter is the most suited for the addition of tunability. Also, once the desired tuning parameter is chosen, it must be designed to meet the performance requirements.

4. Step 3: Selecting the Preferred Tuning Parameter

The next step in the process is to select the most appropriate tuning parameter. An 'ideal' tuning parameter requires a small range of adjustability to control the output, while at the same time the stochastic variability in the tuning parameter causes little variability in the output. On these merits the different candidates tuning parameters are compared.

The first measure of tuning parameter superiority is the range that it requires to control the system, as discussed in Step 3. The range is defined as

$$R_k = d_k^{high} - d_k^{low} \tag{17}$$

The smaller this value, the less the design parameter d_k, and thus the design, has to be adjusted and altered. The second measure of tuning parameter superiority is its sensitivity coefficient

$$S_k = \left|\frac{\partial f}{\partial d_k}\right|. \tag{18}$$

Simply put, the smaller the value of S the more error allowed in setting d_k^t during adjustment.

To make an accurate comparison between different tuning parameters, these two measures can be combined. For both measures, smaller is better, so they can be multiplied or added to give a single measure. The product of S and R are in the same units as p; thus, multiplying gives superior clarity for comparing a single measure among the tuning parameters. The result of dividing the system performance tolerance by this product yields a dimensionless number that allows all the candidate tuning parameters to be compared. This number is a metric that indicates the relative goodness of adding tunability to a design parameter. The equation for this metric is

$$\tau_k = \frac{tol_p}{S_k R_k} \tag{19}$$

Table 1 shows an abstract example using Eqs (17), (18) and (19) to rank candidate tuning parameters. In Table 1, the design parameter that is reviewed first for the addition of tunability is d_p. Based on the mathematical behavior of the system, this parameter will be the most advantageous to tune.

Table 1. Calculating and ranking the candidate tuning parameters

d	S	R	τ	rank
d_1	S_1	R_1	τ_1	5
d_2	S_2	R_2	τ_2	3
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
d_p	S_p	R_p	τ_p	1
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
d_n	S_n	R_n	τ_n	2

5. Step 4: Implementing the Tuning Parameter Concepts

The last step in tuning parameter design is the generation of a concept that changes some design parameter from a value of $\overline{d_k}$ to one that can be adjusted from d_k^{low} to d_k^{high} in response to performance requirements, as discussed in Step 2. The solution chosen depends upon the constraints and physics of the system and the tuning parameter.

For example, consider the performance requirement of smooth rotation, or balance, for a rotating system. The tuning parameter chosen is the location of a center of mass. The tuning parameter design solution may be either to remove or add mass at a specific point. Whether one of these solutions is superior depends on the physics and constraints of the design that may not be represented in Eq. (1). For example, when balancing the wheels of an automobile, mass is added using small lead weights. In this case, adding mass is the superior solution due to the clear negatives of removing mass from the rim or tire. On the other hand, when balancing electric motor rotors in hand held power drills, mass is generally removed (by drilling or grinding away part of the rotor core) because spatial constraints make the addition of material problematic.

6. A Clarifying Design Example

In this section, a simple design example is presented to clarify the concepts and show the implementation of the methods presented in the above sections. The example is the tolerance design of a spring that provides the function of storing energy. The spring comes from a heavy-duty construction stapler, as shown in Fig. 2.

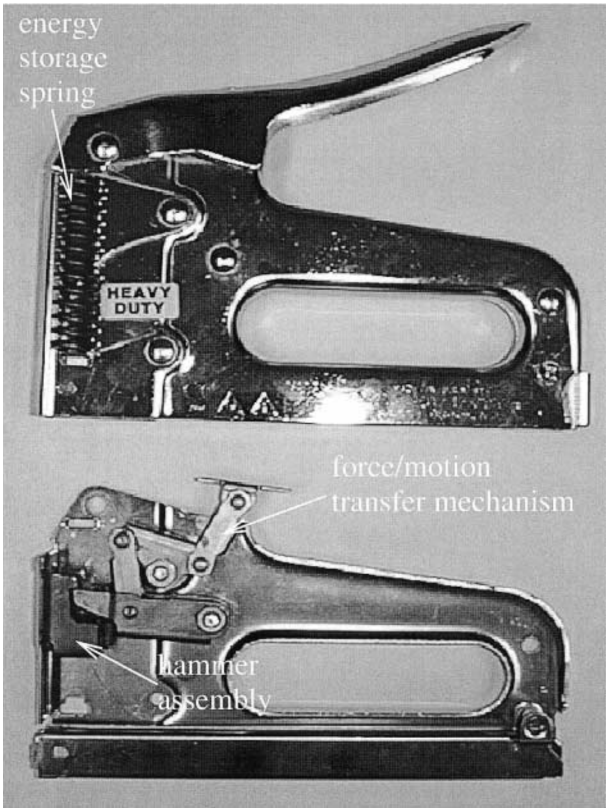

Fig. 2. The heavy-duty construction stapler.

Table 2 contains design and performance information for the spring system parameters. As a result of minimum energy requirements to sink a staple and maximum energy requirements to limit user fatigue, the performance requirement for the energy storage is $E = 1.8 \pm 0.05\ J$. The variations listed in Table 2 for the spring constant and the initial and final spring lengths are natural tolerances. The design task is the selection of the most preferred design parameter to which tunability is added, thus allowing the tolerance to be achieved. To complete the tuning parameter design, a concept is developed to add this tunability.

Table 2. Tolerance design data for the spring

Parameter	Description	Value
E	energy stored in spring	$1.8 \pm 0.05\ J$
K	spring constant	$12{,}000 \pm 600 N/m$
x_f	final compression of spring	$2 \pm 0.02 cm$
x_i	initial compression of spring	$1 \pm 0.02 cm$

6.1. Step 1: Determining Need

The key performance metric for the spring is the energy it stores. Thus, the system performance model for the spring is

$$E = \frac{1}{2} K(x_f^2 - x_i^2) \tag{20}$$

Using Eq. 6 and the values from Table 2 gives the performance as

$$E = 1.8 \pm 0.10 \tag{21}$$

where 0.10 is the natural tolerance and is defined as $3\sigma_E$. Comparing the natural tolerance of the spring system to the tolerance requirement in Table 2 shows that $tol = 0.05 \not> 0.10 = ntol$; thus, the system needs to be tuned.

7. Step 2: Performance Requirements for Tuning Parameter Candidates

Continuing the tuning parameter design process, the performance requirements for the candidate tuning parameters are determined.

Beginning with the design parameter of the spring constant K, the range requirements and tolerance are determined. Solving Eq. (20) for K gives

$$K = \frac{2E}{x_f^2 - x_i^2} \tag{22}$$

The optimization problem presented in Eqs (11)–(14) is formed for Eq. (22):

$$K^{low} = min\left[\frac{2\overline{E}}{(x_f)^2 - (x_i)^2}\right] \tag{23}$$

and

$$K^{high} = max\left[\frac{2\overline{E}}{(x_f)^2 - (x_i)^2}\right] \tag{24}$$

constrained to

$$x_f - ntol_{x_f} \le x_f \le x_f + ntol_{x_f}$$
$$x_i - ntol_{x_i} \le x_i \le x_i + ntol_{x_i} \tag{25}$$

In this constrained space, Eq. (22) has no extrema; thus, k^{high} and k^{low} are found on the borders of the constraint space from Eq. (25). To solve for the lower tunability requirement on K, Eq. (22) is written as

$$K^{low} = \frac{2\overline{E}}{(x_f + ntol_{x_f})^2 - (x_i - ntol_{x_i})^2} \tag{26}$$

Equation (26) is the lower constraint equation. Substituting the values from Table 2 gives

$$K^{low} = \frac{2 \times 1.8}{(2.0+0.02)^2 - (1.0-0.02)^2} \frac{J}{cm^2} = 11,538 \ \ N/m \tag{27}$$

Similarly, to solve for the K^{high},

$$K^{high} = \frac{2\overline{E}}{(x_f - ntol_{x_f})^2 - (x_i + ntol_{x_i})^2} \tag{28}$$

Substituting in the values from Table 2 gives

$$K^{high} = \frac{2 \times 1.8}{(2.0-0.02)^2 - (1.0+0.02)^2} \frac{J}{cm^2} = 12,500 \ \ N/m \tag{29}$$

To solve for the tolerance, Eq. (16) is used. For the spring performance model in Eq. (20), the result is

$$tol_K = 3\sigma_K = 3\frac{1}{\left|\frac{\partial E}{\partial K}\right|}\sigma_E = 3\frac{1}{\frac{1}{2}(x_f^2 - x_i^2)}\sigma_E \tag{30}$$

where K is the candidate tuning parameter, d_k. Substituting in the appropriate values from Table 2 gives

$$tol_K = 333 \ \ N/m \tag{31}$$

Repeating this procedure for each of the design parameters, the performance requirements for the other candidate tuning parameters are determined and listed in Table 3.

Table 3. Candidate tuning parameters performance requirements for the spring design

Parameter	d_k^{low}	d_k^{high}	tol
K	11,538 N/m	12,500 N/m	333 N/m
x_i	0.87 cm	1.11 cm	0.04 cm
x_f	2.05 cm	1.95 cm	0.02 cm

8. Step 3: Selecting the Appropriate Tuning Parameter

To rank the tuning parameters, the tuning range requirement, the tuning parameter sensitivity, and the tunability metric τ are calculated for each design parameter.

Beginning again with the spring constant, the range R_K is

$$R_K = K^{high} - K^{low} = 12,500 - 11,538 = 961.5 \ \ N/m \tag{32}$$

Solving for the sensitivity $\mathcal{S}_K$ and evaluating at the nominal values gives

$$\mathcal{S}_K = \left|\frac{\partial E}{\partial K}\right| = \frac{1}{2}(x_f^2 - x_i^2) = 1.5 \times 10^{-4} \ \ m^2 \tag{33}$$

Writing Eq. (19) for the spring gives

$$\tau_K = \frac{tol_E}{\mathcal{S}_K R_K} = \frac{(0.05)}{(1.5 \times 10^{-4})(961.5)} = 0.35 \tag{34}$$

Proceeding in a similar fashion, the rest of the data are determined and recorded in Table 4.

Table 4. Calculating and ranking the candidate tuning parameters for the spring design

d	S	R	τ	rank
K	$1.5 \times 10^{-4} m^2$	961.5 N/m^2	0.35	1
x_i	120N	0.24 cm	0.16	3
x_f	240N	0.10 cm	0.20	2

The tuning parameter ranks in Table 4 show the spring stiffness as the superior tuning parameter candidate. Because of the difficulty of changing the stiffness of a compression coil spring after it has been manufactured, however, the spring is not considered further as a tuning parameter candidate.

The next most desirable candidate for a tuning parameter is the final length of the spring. The performance requirements for this design parameter are that it be adjustable from $x_f^{low} = 1.95 \ cm$ to $x_f^{high} = 2.05 \ cm$ and the embodied tuning solution has a tolerance of $\pm 0.02 \ cm$. A simple concept for adding tunability to x_f is presented next.

9. Step 4: Implementing the Tuning Parameter Concepts

The final length of the spring is determined by a combination of stop tabs on the stapler housing and tabs on the stapler hammer assembly, as shown in Fig. 3. The positions of these tabs in the sheet metal stamps are such that at x_f, the assembly can move no further. The six-bar linkage, shown in Fig. 2 and referred to as the transfer mechanism, slides off the

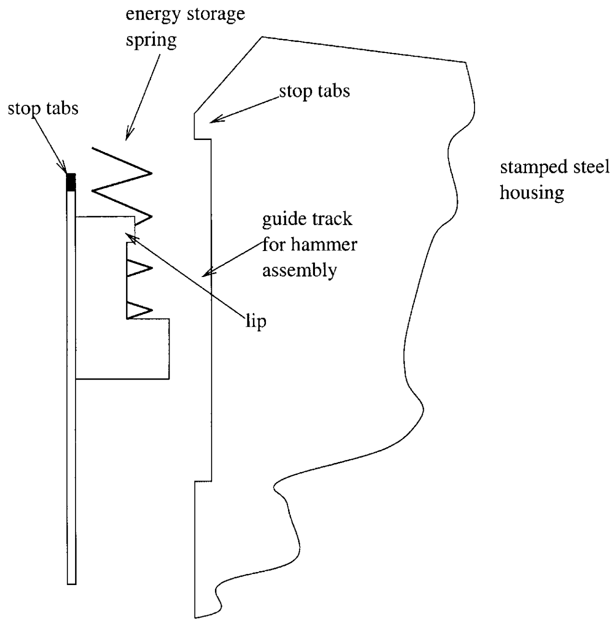

Fig. 3. The initial solution used to control the final length of the spring in the stapler.

lip as shown in Fig. 3 and releases the energy stored in the spring. The result is a transformation of the energy stored in the spring to kinetic energy in the hammer assembly. The staple hammer pushes the staple into the object to be stapled.

A number of solutions may be generated to implement the x_f tuning parameter. A simple and elegant solution to tuning x_f is to remove the positional dependency between the basket and hammer components (Fig. 4). This dependency may be removed by eliminating the locating boss feature on the basket. After elimination, tuning may then be achieved by adjusting the relative position of the basket and the hammer.

To implement this tuning strategy, a three step manufacturing process is possible: (1) measure and set x_f; (2) lock the position of the hammer relative to the basket at this x_f; and (3) drill holes and rivet the hammer to the basket. A gauge block accomplishes

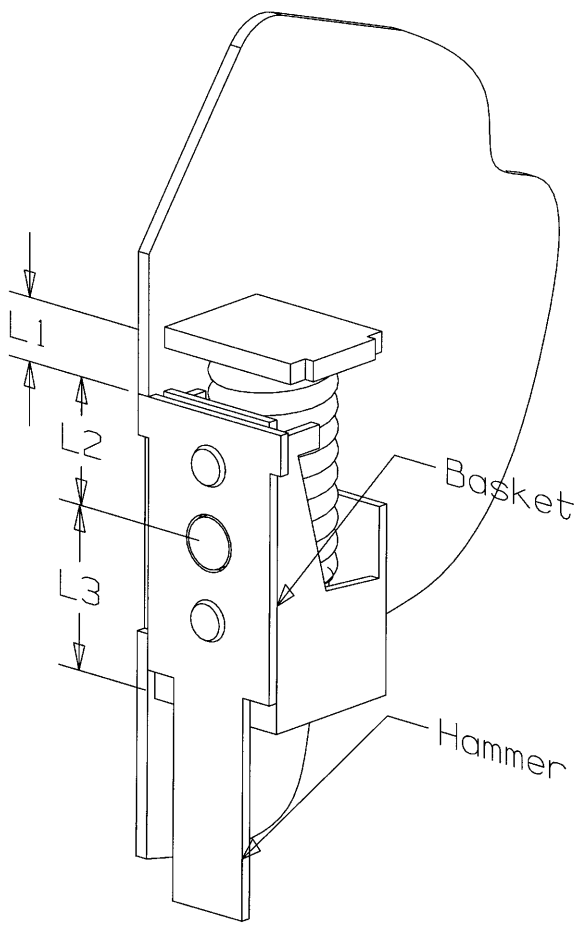

Fig. 4. The primary tolerance chain for the final length of the spring.

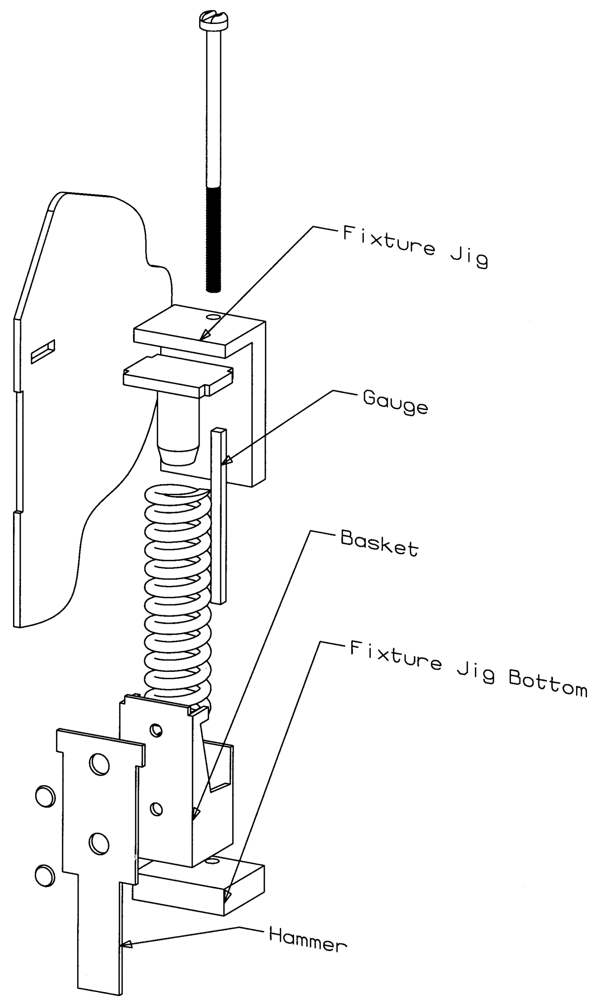

Fig. 5. An exploded view of the spring assembly with design changes to allow for tuning.

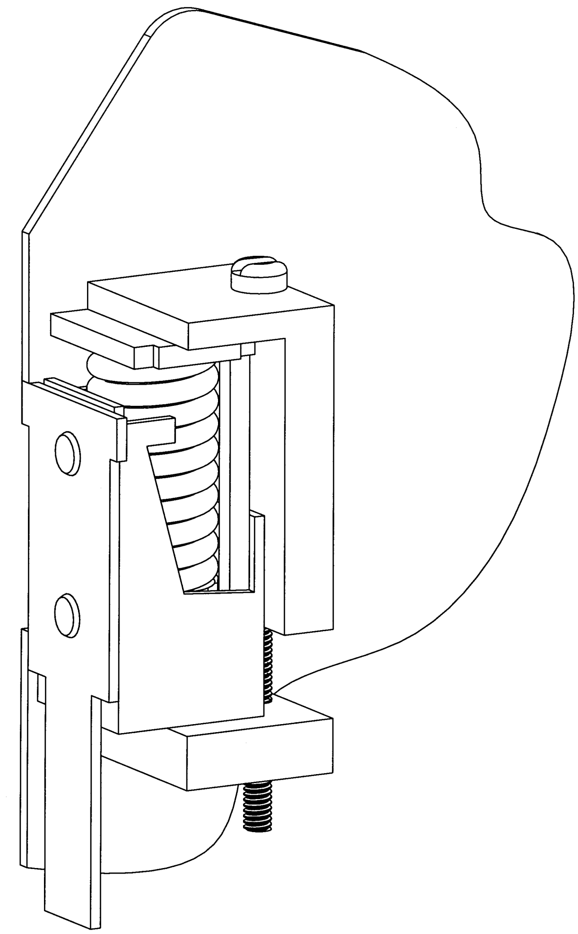

Fig. 6. The spring assembly with design changes to allow for tuning.

the measurement; a fixture jig, in concert with the stapler housing, provides the locking; and the original process is used to complete the riveting. Figure 5 shows an exploded view of the jig, gauge, and assembly (with rivet holes shown). Figure 6 shows the assembly gauge and jig still in place with after the drill and rivet process.

Using the solution, tunability is added to the stored energy, allowing the store energy function of the stapler to operate within the natural tolerances of the manufacturing process for both the original design parameters K and x_i, as well as the implementation of the solution for tunability chosen for x_f.

10. Future Work and Conclusions

The theoretical foundations for tuning parameter design presented here make a significant contribution to tolerance design and design for precision. Through the use of the design methods and analytical measures developed, system performance precision can be increased without increasing component precision. The contribution here is an initial step toward a complete theory of tuning parameter design. Before the theory of tuning parameter design can be considered mature, important research remains as future work. To expand the theory for tuning parameter design, the assumptions made to facilitate the research presented here must be addressed.

The methods presented here are for designing tunability or adding functionality into an existing design parameter. In the above development, an assumption is made about the completeness of the design model in Eq. (46). Because of this assumption, the addition of new tuning parameters is neglected in the sense that *new* changes the character or form of Eq. (46). For tuning parameter additions that change the parametric representation or perhaps the entire physics of the system, further formal methodologies are required.

Other important future work involves tuning more than one design parameter. The method, as presented here, only considers the tuning of a single design parameter. Multiple tuning parameters would allow the range of tunablity for each parameter to be smaller than required for a single tuning parameter.

In the statistical and mathematical development presented, only a single performance parameter is considered. In practice, there are often multiple important performance parameters for a given product or subsystem. Expanding the scalar performance parameter p to a vector $\vec{p}$ and the vector design parameter $\vec{d}$ (explicitly referred to as $d_1, d_2, \ldots, d_n$ in the text) to a matrix D remains a clear research need. Such research will result a significant step toward generalizing tuning parameter design for a broader range of engineering design problems.

The approach presented in this paper uses Eq. (46) for much of the design method development. For some systems, developing these equations is so difficult that experimental analysis is preferred for performance analysis. If experimentation is needed, the approach to tuning parameter design presented here remains valid but requires extension. The key extension required is that the sensitives and tuning parameter ranges must be determined experimentally instead of analytically.

It is worthwhile noting that there are solutions other than the addition of a tuning parameter to solve the problem presented in Eq. (7), such as process improvement and parameter redesign. Which solution is best depends on the specific design problem at hand.

This paper develops a formalized, mathematically based method for tuning parameter design. The key contributions of this paper are the development of a foundation for a complete theory for tuning parameter design. Using the methods and tools developed here, tuning parameter design can be performed in a repeatable, constructive manner without undue dependence on arbitrary and *ad hoc* design decisions. This method develops quantitative requirements for candidate tuning parameters, thus allowing designers clear insight into answering the key design question of which design parameter should be tuned.

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Appendix

Deriving the System Response

Restating Eq. (1)

$$p = f(d_1, d_2, \ldots, d_n) \tag{35}$$

The derivation proceeds by taking a Taylor series expansion of f about the expected values of the design values: $[E(d_1), E(d_2), \ldots, E(d_n)]$. The expansion gives

$$\begin{aligned} f(d_1, d_2, \ldots, d_n) \approx\; & f[E(d_1), E(d_2), \ldots, E(d_n)] \\ & + \sum_{i=1}^{n} \frac{\partial f}{\partial d_i}[d_i - E(d_i)] \\ & + \frac{1}{2}\left\{ \sum_{i,j=1}^{n} \frac{\partial^2 f}{\partial d_i \partial d_j}[d_i - E(d_i)]\left[d_j - E(d_j)\right] \right\} \end{aligned} \tag{36}$$

discarding terms third order and higher. In Equation. (36), all the derivatives are evaluated at the expected values. In other words,

$$\frac{\partial f}{\partial d_i} = \left. \frac{\partial f(d_1, d_2, .., d_n)}{\partial d_i} \right|_{d_l = E(d_l), l=1,2,\ldots,n} \tag{37}$$

Taking the expected value of both sides of Eq. (36) gives

$$\begin{aligned} E[f(d_1, d_2, \ldots, d_n)] =\; & E\{f[E(d_1), E(d_2), \ldots, E(d_n)]\} \\ & + E\left\{ \sum_{i=1}^{n} \frac{\partial f}{\partial d_i}[d_i - E(d_i)] \right\} \\ & + E\left\{ \frac{1}{2} \sum_{i,j=1}^{n} \frac{\partial^2 f}{\partial d_i \partial d_j}[d_i - E(d_i)]\left[d_j - E(d_j)\right] \right\} \end{aligned} \tag{38}$$

Recalling the properties of the expected value operator when applied to independent random variables,

$$E\left\{[x_i - E(x_i)]^r [x_j - E(x_j)]^s\right\} = E[x_i - E(x_i)]^r E[x_j - E(x_j)]^s \quad i \neq j, \tag{39}$$

$$E(x_1 + x_2 + \ldots + x_n) = E(x_1) + E(x_2) + \ldots + E(x_n) \tag{40}$$

$$E(c) = c \tag{41}$$

with c a constant, and

$$E(cx) = cE(x) \tag{42}$$

where x is any random variable (Hahn and Shapiro 1994). Applying these to the first term on the right in Eq. (38) gives

$$E\{f[E(d_1), E(d_2), \ldots, E(d_n)]\} = f[E(d_1), E(d_2), \ldots, E(d_n)] \tag{43}$$

Applying the property given by Eq. (41) to the second term in Eq. (38) gives

$$\begin{aligned} E\{\sum_{i=1}^{n} \frac{\partial f}{\partial d_i}[d_i - E(d_i)]\} &= \sum_{i=1}^{n} E\{\frac{\partial f}{\partial d_i}[d_i - E(d_i)]\} \\ &= \sum_{i=1}^{n} \frac{\partial f}{\partial d_i}\{E[d_i - E(d_i)]\} \\ &= 0 \end{aligned} \tag{44}$$

Applying the property given by Eq. (39) to the third term in Eq. (38), and using the definition of the variance, gives

$$E\left\{\frac{1}{2}\sum_{i,j=1}^{n}\frac{\partial^2 f}{\partial d_i \partial d_j}[d_i - E(d_i)]\left[d_j - E(d_j)\right]\right\} =$$

$$\frac{1}{2}\sum_{i=1}^{n}\frac{\partial^2 f}{\partial d_i^2}Var(d_i) + 0 \tag{45}$$

Substituting Eq. (43), (44), and (45) back into Eq. (38) gives the expected value of the system performance as

$$E(p) = E[f(d_1, d_2, \ldots, d_n)] \approx f[E(d_1), E(d_2), \ldots, E(d_n)] + \frac{1}{2}\sum_{i=1}^{n}\frac{\partial^2 f}{\partial d_i^2}Var(d_i) \tag{46}$$

Though not a true equality (it is an approximation), hereafter the system response will be written as

$$\overline{p} = f(\overline{d_1}, \overline{d_2}, ..., \overline{d_n}) + \frac{1}{2}\sum_{i=1}^{n}\frac{\partial^2 f}{\partial d_i^2}Var(d_i) \tag{47}$$

where

$$\overline{p} = E(p) \tag{48}$$

and

$$\overline{d_i} = E(d_i) \tag{49}$$

Deriving the System Variance

Here, the variance on the system performance is determined. The derivation retains terms up to the second order.

The variance of p is

$$Var(p) = E(p^2) - [E(p)]^2 \tag{50}$$

from the definition of the variance operator (Bowker and Lieberman 1959). Substituting Eq. (1) into Eq. (50) gives

$$Var[f(d_1, d_2, \ldots, d_n) = E[f(d_1, d_2, \ldots, d_n)]^2 - \{E[f(d_1, d_2, \ldots, d_n)]\}^2 \tag{51}$$

To obtain an approximation to the first term on the right-hand side of Eq. (51), the square of Eq. (36) is taken, and expected values are taken on a term-by-term basis (employing Eq. (40)). Terms are retained up to second order. This approach leads to

$$E\{[f(d_1, d_2, \ldots, d_n)]^2\} = \{f[E(d_1), E(d_2), \ldots, E(d_n)]\}^2 + \sum_{i=1}^{n}\left(\frac{\partial f}{\partial d_i}\right)^2 E[d_i - E(d_i)]^2 \tag{52}$$

The terms not shown here are all higher than second order. The second term on the right-hand side of Eq. (51) is the square of Eq. (46); thus,

$$\{E[(f(d_1, d_2, \ldots, d_n)]\}^2 = \{f[E(d_1), E(d_2), \ldots, E(d_n)]\}^2 \tag{53}$$

Substituting Eq. (52) and (53) into Eq. (51) gives the equation for the system variance:

$$Var[f(d_1, d_2, \ldots, d_n)] = \sum_{i=1}^{n}\left(\frac{\partial f}{\partial d_i}\right)^2 E[d_i - E(d_i)]^2 \tag{54}$$

Using the definition of the variance on the design variables gives

$$Var(p) = Var[f(d_1, d_2, \ldots, d_n)] = \sum_{i=1}^{n}\left(\frac{\partial f}{\partial d_i}\right)^2 Var(d_i) \tag{55}$$