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Trade-Off of Fuzzy Set-Based Design Parameters

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Abstract

During the early stages of design processes engineers have to deal with imprecision in design and performance parameters due to lack of precise knowledge about them and their final configuration (concept). Imprecision can be represented in terms of intervals, but these representations are not immediately compatible with most engineering research in decision making like classical fuzzy theory based on Zadeh's extension principle. Usually engineers must evaluate multiple interactive design parameters in order to move towards the more desirable designs. The problem here is to find an economic set of values for design parameter that satisfy performance parameters. It is necessary to perform trade-off decisions for fuzzy set-based design parameters of non equal cost.

This research presents a procedure to determine fuzzy sets for cost of performance parameters and the back propagation of a performance parameter into design parameter values, and the use of weighted compensated design strategy to automate the procedure of finding a suitable cost solution for design parameters.

Keywords
Lean design, design theory, fuzzy sets, fuzzy numbers, cost compensation

1. Introduction
The decision making in engineering usually requires the consideration of multiple performance parameters (objectives, goals). The basic procedures to perform decision-making in a fuzzy environment are provided in Bellman and Zadeh [1]. Tools for decision-making in fuzzy environments considering multiple objectives of unequal importance are found in literature like Ammar [2], Baas and Kwakenraak [3], Dias [4], Diaz [5], Dong and Wong [6], Kahne [7], Yager [8]. These methods are based on the combination of performance criteria in a fuzzy "AND" operation [18], which ranks the solution based on the parameter with the worst performance. The fuzzy set theory is not appropriate for decision-making considering economic trade-off, which requires an additive operation for compensation among performance parameters [9]. Instead, Utility Theory is usually applied to perform economic trade-off under uncertainty in choice in engineering design [9, 10].

Another type of decision-making of interest in engineering design is the choice of values for interactive design parameters of unequal cost. This decision-making is even necessary when considering only one performance parameter. The problem here is to find the most economic set of design parameter values that satisfy a performance parameter value. The objective of this paper is to develop the capabilities within the Labeled Fuzzy Sets Method [11, 18] to perform trade-off decisions for fuzzy design parameters of unequal cost.

2. Propagation of Fuzzy Set on Performance Parameter
Propagation of fuzzy sets is based on Zadeh's extension principle. For example considering the fuzzy design parameters $\tilde{d}_1, \tilde{d}_2, \ldots, \tilde{d}_n$ let $d_i \in \tilde{d}_i$ ($i \in [1, n]$). Given a performance parameter expression, represented by the mapping:

$$p=f(d_1, d_2, \ldots, d_n) \;\; \forall\, d_i \in \tilde{d}_i \tag{1}$$

Let $\tilde{P}$ be the fuzzy performance parameter resulting from the mapping. The extension principle then implies that the induced membership function for $\tilde{P}$ is:

$$\mu_p = \sup_{\substack{d_1,\ldots,d_n \\ p \;\; f(d_1,\ldots,d_n)}} \{\min(\mu(d_1), \ldots, \mu(d_n))\} \tag{2}$$

Where $\mu(d_i)$ is the degree of preference for the design parameter value d_i.

Computation of the extension principle for the uncertain parameters in engineering design is simplified by applying the labeled α-cut algorithm developed by Hernandez [11]. An example of the propagation of preference using the labeled α-cut algorithm is presented next.

2.1. Example

The diameter of a planet gear [d_p] is a function of the number of teeth of planet gear [N_p], and diametral pitch [P]. The performance parameter expression relating these parameters is:

$$d_p = \frac{N_p}{P} \tag{3}$$

Particular fuzzy preference sets are assigned for the two design parameters, as shown in Table 1, and figures 1 and 2.

Table 1: Preference on Design Parameters for the Planet Gear

α=0	**α=1**
[R only N_p 12 18]	[R only N_p 18 18]
[P only P 12 20]	[P only P 12 12]

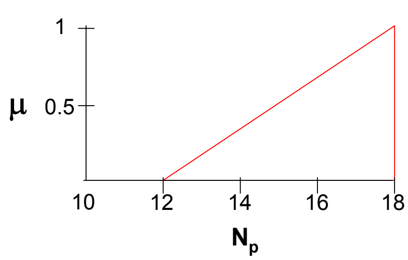

Figure 1: Planet Teeth [N_p]

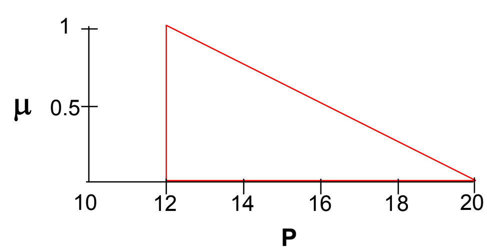

Figure 2: Diametral Pitch [P]

These fuzzy preferences sets are hypothetically defined, just for illustration purpose. The induced labeled fuzzy preference set for performance parameter d_p is obtained by applying the labeled α-cut algorithm, and shown in figure 3.

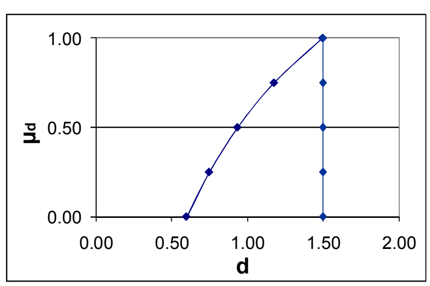

Figure 3: Labeled Fuzzy Preference Function for d_p

The extension of this example to propagate cost instead of preference is shown in the next section.

3. Induced Fuzzy Set for Cost of Performance Parameters

The fuzzy preference functions on the Method of Imprecision (MI) [13, 19] measure the degree to which a designer prefers a value of a design parameter. The degree of preference μ in the fuzzy set for preference of the method of imprecision is analogous to the degree of membership μ of the classical fuzzy set theory. The preference μ is a normalized ranking of the designer's belief or desire that the value will be used in the design [12]. A value of $\mu=1$ represents the maximum belief or desire that the value will be used. A value of $\mu=0$ indicates that the value will not be used. The feasible preference μ in the Labeled Fuzzy Sets represents the preference for using a feasible value of the labeled fuzzy preference function. A value of $\mu=1$ has the same interpretation of maximum preference of the MI. However, a value of $\mu=0$ represents the minimum preference to use the feasible value; that is, the value might be the last preferred to use, but it can be used because it is feasible.

A typical problem in engineering design is to select the most economical set of design parameters to satisfy a functional requirement. This problem requires the propagation of cost of design parameters into the performance parameters. First, the costs for values of design parameters $C_{d1}, \ldots, C_{dn}$ are normalized into the degree of preference $\mu(d_1, \ldots, d_n)$. Then, $\mu(d_1, \ldots, d_n)$ are propagated into the degree of preference of performance parameters $\mu(P_1, \ldots, P_n)$ applying the Labeled α-Cut Algorithm. Finally, an inverse map of $\mu(P_1, \ldots, P_n)$ into parameter costs $C_{P1}, \ldots, C_{Pn}$ is determined. The following example illustrates these steps.

3.1. Example

For the previous example, let us assume the following cost functions C_N and C_P (see figures 4 and 5) for the design parameters N and P.

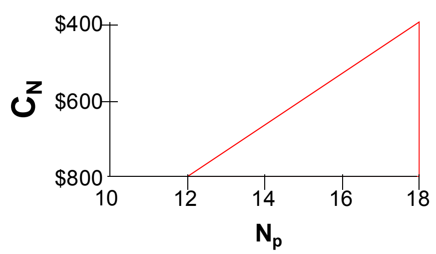

Figure 4: Cost Function for N_p

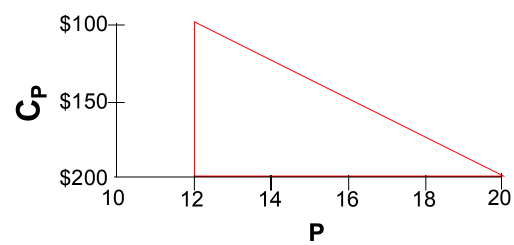

Figure 5: Cost Function for P

Assuming a proportional relation between cost and the normalized preference function for N_p and P (Figures 6 and 7), we obtain the following mapping equations:

$$\mu_N = -\frac{C_N}{400} + 2 \quad ; \quad \mu_P = -\frac{C_P}{100} + 2 \tag{4}$$

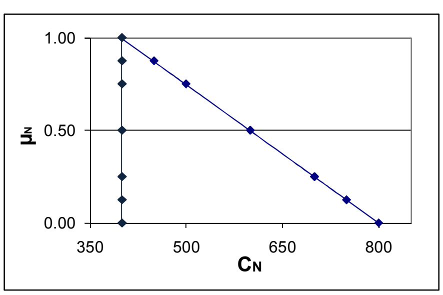

Figure 6: Preference Function for N_p Cost

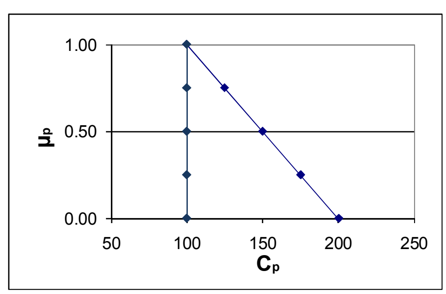

Figure 7: Preference Function for P Cost

Then, the vertical axis of the cost functions for N and P can be mapped to the degree of preference for their corresponding fuzzy preference functions, shown in figures 1 y 2. The propagation of these fuzzy preference functions applying the Labeled α-Cut Algorithm results in the fuzzy preference function μ_p for the performance parameter d in figure 3.

The next step is to find the inverse mapping of the membership of preference μ_d to the cost C_d for the performance parameter d.

$$C_d=f(\mu_d) \tag{5}$$

The value of μ_d was obtained applying the Zadeh's extension principle.

$$\mu_d = \sup_{\substack{N,\,P \\ d\ f(N,\,P)}} \{\min(\mu_N, \mu_P)\} \tag{6}$$

Therefore, the relation of μ_d with μ_N and μ_P is an "AND" [11, 19], which is not additive. In fact, from the concept of α-cuts we know that the application of the extension principle can be simplified by calculating the operations when μ_d, μ_N and μ_P have the same α-cut value. Therefore, for a performance parameter calculated using the extension principle we get:

$$\mu_d(\alpha\text{-cut}) = \mu_N(\alpha\text{-cut}) = \mu_P(\alpha\text{-cut}) \tag{7}$$

However, cost is additive and we need to calculate it to perform the required trade-off. The cost of parameters N and P as a function of their corresponding degree of preference μ_N and μ_P can be calculated as:

$$C_N = -400(\mu_N - 2) \quad ; \quad C_P = -100(\mu_P - 2) \tag{8}$$

Therefore, the cost C_d of the performance parameter d can be calculated assuming an additive metric:

$$C_d(\mu_d) = C_N(\mu_N) + C_P(\mu_P) \tag{9}$$

Substituting (8) in (9) the inverse mapping for cost of the design parameters, we get:

$$C_d = -400(\mu_N-2) - 100(\mu_P-2) \tag{10}$$

Using the fact that for each α-cut of μ_d: $\mu_d = \mu_N = \mu_P$ (11)

Therefore, substituting (11) in (10): $C_d = -400(\mu_d-2) - 100(\mu_d-2)$ (12)

Arranging terms, we can simplify this inverse mapping to:

$$C_d = -500(\mu_d-2) \tag{13}$$

Equation (13) can be used to map the membership axis of the labeled fuzzy set of figure 3 into the cost function of figure 8.

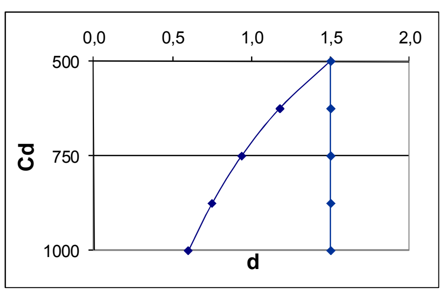

Figure 8: Labeled Fuzzy Set for Cost of d

4. Back-Propagation of Performance Parameter Into Design Parameter

The previous section presented the procedure used to determine fuzzy sets for cost of performance parameters, by the forward calculation of the induced preference of design parameters. This section will now analyze the back propagation of a performance parameter value into design parameter values.

One important ramification of fuzzy set theory is that once a forward calculation of an induced fuzzy set for a performance parameter is made, then backward propagation can be obtained with no further computation [13]. For example, take the case of the fuzzy preference function for the planet gear diameter shown in figure 3. The maximum preference of planet diameter is 1.5 in. This value of performance parameter is obtained when the maximum preferred values for design parameters N=18 and P=12 are used. Let assume that the planet diameter is constrained to be equal to 1 in. This constraint is represented as a dash line in figure 9.

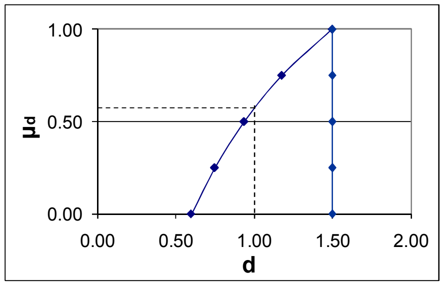

Figure 9: Best Preference Point

The intersection point of the constraint with the fuzzy preference function is indicated. This point represents the best preference that can be obtained for the value of 1 in, which corresponds to μ=0.57. From the Labeled α-Cut Algorithm, we know that this value of performance parameter with this preference was obtained using a number of teeth N=15.43 and a diametral pitch P=15.43. These values are shown in figures 10 and 11.

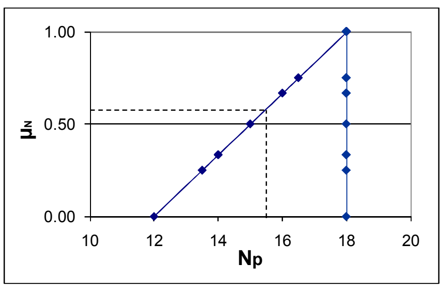

Figure 10: Labeled Fuzzy Set for N

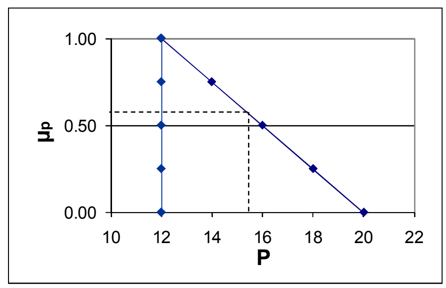

Figure 11: Labeled Fuzzy Set for P

However, these values are not standard integer numbers and cannot be used directly in practice. We have to find a set of standard design parameter values for N and P that results in the desired planet diameter of 1 in. There are several possible and feasible alternative solution sets of N and P that satisfy the requirement.

$$(N, P)=\{(12, 12), (13, 13), (14, 14), (15, 15), (16, 16), (17, 17), (18, 18)\} \qquad (14)$$

Any one of these alternatives will satisfy the planet diameter of 1 inch. However, their corresponding degree of preferences is less that the maximum obtained using the extension principle (6).

Therefore, the vertical dashed line shown in figure 9 represents all the possible alternatives satisfying d_p=1 in, calculated as:

$$\mu_d = \{\min(\mu_N, \mu_P)\} \; ; \; \text{for } d=N/P=1\text{”} \qquad (15)$$

The extension principle will select the set of design parameters resulting in the maximum preference for the performance parameters. However, for design parameters of unequal cost, this selection is not necessarily the most economical. Consider first the inverse mapping of preference to cost calculated in (12), and then that the relation between the degree of preferences μ_N and μ_P and their support values corresponding to the pinion diameter d=1 is:

$$\mu_N = \frac{N}{6} - 2 \; ; \quad \mu_P = -\frac{P}{8} + 2.5 \qquad (17)$$

Substituting these equations into the expression for C_d, we obtain:

$$C_d = 1550 - 400/6\ N + 100/8\ P \qquad (18)$$

The partial derivative of the cost function C_d with respect to N and P are:

$$\frac{\partial C_d}{\partial N} = -\frac{400}{6} \quad ; \quad \frac{\partial C_d}{\partial P} = \frac{100}{8} \tag{19}$$

Therefore, the cost function C_d will increase and decrease monotonically with respect to P and N. On the other hand, N and P are interactive parameters related with d through the governing relation d=N/P. To satisfy a specific value of d, both N and P have to be increased or decreased simultaneously. A trade-off strategy to select values for N and P that minimize C_d has to be performed. An increase in N will result in a more significant reduction of C_d than a similar increase in P. Therefore, N and P should be increased to reduce C_d. The possible intervals of values of N and P are:

$$N=[12\ 18] \;\; ; \;\; P=[12\ 20] \tag{20}$$

Given the monotonicity of C_d with respect to N, we can try to maximize the value of N, where its maximum value is 18. In order to satisfy the requirement of d=1 inch, the value of P should also be 18. Table 2 shows the different design parameter preference and cost for the feasible standard values, and corresponding performance parameter and cost. The use of the values that result in the maximum degree of preference μ_d results in a cost of $715 dollars. However, this is not the best economical selection of design parameters. The most economical solution is given when N=18 and P=18. This alternative results in the minimum cost of $575. The degree of preference for this alternative is μ_d=0.25. This “best” alternative for design cost is indicated in figure 12.

Table 2: Design Cost for a 1 inch Planet Gear

N	μ_N	C_N	P	μ_p	C_p	d_p	μ_d	C_d
12	0.00	800.00	12	1.00	100.00	1	0.00	900.00
13	0.17	733.33	13	0.88	112.50	1	0.17	845.83
14	0.33	666.67	14	0.75	125.00	1	0.33	791.67
15	0.50	600.00	15	0.63	137.50	1	0.50	737.50
15.43	0.57	571.33	15.43	0.57	142.88	1	0.57	714.21
16	0.67	533.33	16	0.50	150.00	1	0.50	683.33
17	0.83	466.67	17	0.38	162.50	1	0.38	629.17
18	1.00	400.00	18	0.25	175.00	1	0.25	575.00

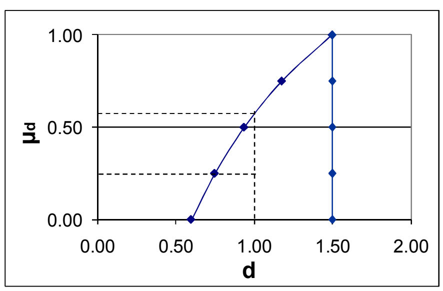

Figure 12: Labeled Fuzzy Set for d

The next section describes a procedure to find this “best cost solution”.

5. Weighted Compensated Design

The previous section describes in detail the determination of a best cost solution, considering the cost of the design parameters. A way to automate this procedure is by using a weighted compensated design strategy [14-17]. This compensating strategy is based on the trading off performance parameters cooperatively to gain in the overall performance. This is achieved by raising each goal considering to a power quantifying its importance level, in a product expression.

Mathematically, the weighted compensated design [14], is formulated as:

$$\mu(d^*) = \sup\{ \prod_{k=1}^{N} \mu_k^{\omega_k} \} \tag{21}$$

d*= best compensated solution
μ_k= preference of parameter k
ω_k= weighted importance for parameter k

The problem is to define the weights for each design parameter. This weight must have relation with the importance of the parameter to the final design. In the case of cost, we can quantify the importance of each parameter for the final design. For instance, in the example of the labeled fuzzy set of figure 10, the final cost is defined by the cost of the functions at each α-cut. From figures 4 y 5, we can see that N represents 80% of the total cost of the gear, while P represents the 20% left. Therefore, we can directly define a weigh of 0.8 for N and a weigh of 0.2 for P.

$$\omega_N=0.8 \quad ; \quad \omega_P=0.2 \tag{22}$$

Substituting these values in the equation for weighted compensated design, we obtain the final preference values for the performance parameter d of table 3. Notice that the highest compensated preference of 0.76 corresponds to the best design cost of $575 obtained before.

Table 3: Weighted Compensated Design Preference for an 1 inch Planet Gear

N	μ_N	P	μ_p	d_p	μ_d	C_d	$\mu(d)$
12	0.00	12	1.00	1	0.00	900.00	0.00
13	0.17	13	0.88	1	0.17	845.83	0.23
14	0.33	14	0.75	1	0.33	791.67	0.39
15	0.50	15	0.63	1	0.50	737.50	0.52
15.43	0.57	15.43	0.57	1	0.57	714.21	0.57
16	0.67	16	0.50	1	0.50	683.33	0.63
17	0.83	17	0.38	1	0.38	629.17	0.71
18	1.00	18	0.25	1	0.25	575.00	0.76

6. Conclusions

This article describes the development of procedures for the trade-off of fuzzy design parameters of unequal cost. The decision-making in classical fuzzy theory based on Zadeh's extension principle is not able to perform design compensation. Instead, the strategy followed here is to solve the inverse design problem by the back propagation of functional requirements into design parameters. The possible design solutions for a specific performance value are located along a vertical line drawn from the support to the maximum preference found using the extension principle. However, the most economical solution does not necessarily correspond to the most preferred solution using the extension principle. An example shows to demonstrate this fact. The procedure shown can be followed to perform trade-off between dynamic criteria for future applications to be developed.

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