Distributed Spectrum Sensing in Cognitive Radio Networks with Fairness Consideration: Efficiency of Correlated Equilibrium

Sabita Maharjan[1,2], Yan Zhang[1,2], Chau Yuen[3] and Stein Gjessing[1,2]
[1] Simula Research Laboratory, Norway.
[2] University of Oslo, Norway.
[3] Singapore University of Technology and Design, Singapore.
Email: {sabita, yanzhang}@simula.no, yuenchau@sutd.edu.sg, steing@simula.no

***Abstract*—Cooperative spectrum sensing improves the reliability of detection. However, if the secondary users are selfish, they may not collaborate for sensing. In order to address this problem, Medium Access Control (MAC) protocols can be designed to enforce cooperation among secondary users for spectrum sensing. In this paper, we investigate this problem using game theoretical framework. We introduce the concept of correlated equilibrium for the cooperative spectrum sensing game among non-cooperative secondary users and formulate the optimization problem for the case where secondary users have heterogeneous traffic dynamics. We show that the correlated equilibrium improves the system utility, as compared to the mixed strategy Nash equilibrium. While maximizing system payoff is important, fairness is also equally important in systems with dissimilar users. In order to address fairness issue, we propose a new fair social welfare correlated equilibrium, which maximizes the system utility and ensures that the less well-off users do not starve. We employ a no-regret learning algorithm for distributed implementation of the correlated equilibrium. Finally, we propose a neighbourhood based learning algorithm and show that it achieves better performance than the no-regret algorithm.**

I. Introduction

Cognitive radio networks (CRNs) [1][2] have been proposed to address the problem of spectrum scarcity and to make effective use of the electromagnetic spectrum by opportunistically using the spectrum of the licensed users. The licensed users are called primary users and the users of the cognitive radio network are called secondary users. Spectrum sensing is a crucial function for cognitive radio networks to detect available channels from primary user spectra and also to prevent excessive interference to primary users from secondary users. Cooperative spectrum sensing [3]-[8] has been recognized as a powerful approach to improve detection reliability. In most of the existing literature on cooperative spectrum sensing, there is an inherent assumption that all secondary users cooperate with each other to maximize the payoff for the system [9]-[11]. However, if the secondary users are selfish, they will not collaborate with each other for cooperative sensing without any incentive [12]. This kind of selfish behavior comes into the picture in many situations e.g., if the secondary users belong to different systems, if their spectrum requirements are heterogeneous, if there is no central authority to assign the sensing responsibility etc. Under such situations, the decision of a secondary user to participate in cooperative sensing depends not only on the user's strategy but also on the strategies taken by other users. This framework can be modeled using non-cooperative game theory.

There is only a limited literature investigating the selfish behavior of secondary users for spectrum sensing using game theoretical framework. One of the pioneering works on modeling cooperation enforcement among selfish secondary users for cooperative spectrum sensing is [13]. In [13], the authors modeled cooperative sensing as an N-player horizontal infinite game and they proposed to use Carrot-and-Stick strategy, which results mutual cooperation as the Nash equilibrium (NE) of the game. In [14], the authors proposed mixed strategy NE as the solution of the non-cooperative game among secondary users for cooperative spectrum sensing. They deployed an evolutionary game [15] among secondary users for their strategies to converge to the equilibrium in distributed manner. However, the analytical framework was limited to the case of users with homogeneous traffic, and the mixed strategy NE does not always exist for the case when users are not homogeneous. In [19], we differentiated the heterogeneous users as heavy traffic and light traffic users, and designed the sensing game so as to encourage light traffic users take the sensing responsibility and allow heavy traffic users get a free ride. In this paper, we derive the analytical framework for a more general case with heterogeneous traffic dynamics and improve the system performance compared to the mixed strategy equilibrium, using correlated equilibrium (CE) as the solution of the game.

Medium Access Control (MAC) protocols can be designed to enforce cooperation among secondary users for spectrum sensing. In this paper, we investigate this problem using game theoretical framework. CE can improve the payoff of users in both cooperative and non-cooperative games. The concept of CE has been used extensively for spectrum access and transmission control [21]-[24]. These works focus on the issues of opportunistic spectrum access such as rate control/adaption, power allocation etc. but none of them dealt with the issues of spectrum sensing. In this paper, we apply the concept of CE to design the cooperative spectrum sensing game among secondary users in order to improve the payoff of the secondary users and the network. This is a novel approach in the context of opportunistic spectrum access based on spectrum sensing performed by the non-cooperative secondary users. In most of the studies on spectrum sharing and spectrum sensing, it is assumed that the secondary users always have data to transmit and they want to use the channel as long as possible. In a real network, the users may not necessarily have data to transmit all the time. Taking this into account, we analyze the sensing game for users with heterogeneous traffic: some of which have data to transmit all the time while others do not. We exploit the heterogeneity among the secondary users and use CE to improve the system payoff. While maximizing the system payoff is important, fairness is also equally important in systems with dissimilar users. To incorporate fairness into the utility maximization problem, we propose fair social welfare correlated equilibrium (f-SW CE), which maximizes system utility while ensuring that the users with less traffic do not suffer severely. In addition, we employ a no-regret (NR) learning algorithm for the game so that each secondary user's strategy converges to the CE using only local information. Furthermore, we propose a neighbourhood based learning (NBL) algorithm which yields considerable improvement in the performance compared to the NR algorithm. Thus, our three major contributions in this paper are as follows:

1) We formulate the optimization problem for obtaining CE of the spectrum sensing game among heterogeneous users and show that CE improves the system performance considerably.
2) We propose f-SW CE scheme, which maximizes the system utility and ensures fairness to all secondary users.
3) We propose a distributed NBL algorithm that needs exchange of some information among users but yields higher performance compared to the NR algorithm.

The rest of this paper is organized as follows. In section II, we discuss the system model and the energy detection mechanism for spectrum sensing. We derive the mixed strategy equilibrium for the game in section III. Our motivation to employ CE for the spectrum sensing game is presented in sections IV-A and IV-B. The optimization problem is formulated to maximize the system utility in section IV-C. We discuss schemes that incorporate fairness at different levels and propose the f-SW CE in section IV-D. We describe NR algorithm and propose an NBL algorithm in section V. The results are presented and analyzed in section VI. Section VII concludes the paper.

II. System Model

A. Network Model

We consider the sensing of M channels of the primary system. There are K secondary users, located far away from primary base station as shown in Figure 1. In this scenario, the received signal to noise ratio for secondary users from primary base station is very small.

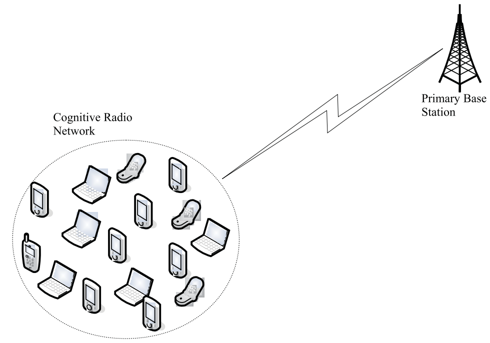

Fig. 1. Scenario and System Model

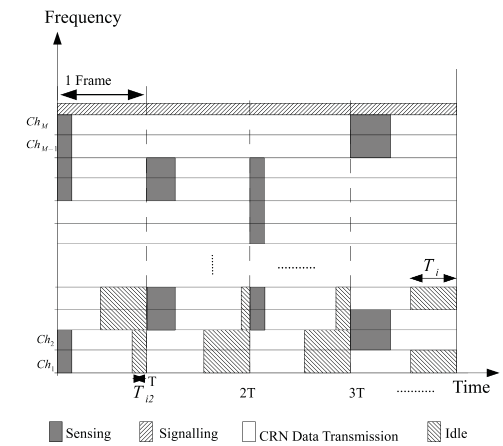

Fig. 2. Cooperative Spectrum Sensing Scheme

We consider the frame based sensing and transmission scheme as shown in Figure 2. Spectrum sensing is performed at the beginning of each frame. The duration of each frame is T and the sensing duration is T_s. The users in the network have heterogeneous traffic. Among them, some users always have data to transmit, while others have lower input data rate, thus, they need less time to transmit their data and have idle time slots in a frame duration. $T_i \geq 0$ is the idle duration for user i if it does not perform sensing, and $0 \leq T_{i2} < T_i$ is the idle duration after it performs sensing, if it does. The signaling channel is used to share the sensing results.

The secondary users that choose to perform sensing cannot transmit data while sensing. In order to achieve a higher throughput, secondary users will tend to decide not to sense if they are selfish, and take the advantage by overhearing the sensing results from other users. However, if no user contributes in sensing, then, no user will achieve any payoff. Thus, the users are fundamentally non-cooperative, but cooperation can emerge among them as none of them desires zero or very low payoff. For this kind of scenario, it is important to study their behavior dynamics such that cooperation can be enforced among the secondary users to maintain the required level of detection performance.

B. Spectrum Sensing Technique: Energy Detector

We employ energy detection mechanism to perform sensing. For energy detector, the received signal $x(n)$ is subject to a hypothesis test as following:

$$\begin{cases} H_0 : x(n) = w(n) \quad \text{primary user is absent,} \\ H_1 : x(n) = gs(n) + w(n) \quad \text{primary user is present,} \end{cases} \tag{1}$$

where g is the gain of the channel between primary user transmitter and secondary user receiver, $s(n)$ is the signal from the primary user, which is assumed to be an i.i.d. random process with zero mean and variance σ_s^2 and $w(n)$ is an additive white Gaussian noise with zero mean and variance σ_w^2, $n = 1, 2, ..., N$ is the discrete time index and N is the total number of samples collected during sensing period.

Neyman-Pearson detection approach for this hypothesis test has two errors. The first is the error that occurs when the primary signal is absent but the detector declares that the signal is present, namely probability of false alarm (P_{FA}). The second error occurs when the channel is occupied, but the detector decides that the channel is vacant. This error is called probability of miss detection (P_{MD}). Each of these errors is associated with the particular threshold γ for comparing the power of the received signal with, and can be calculated as follows [20]:

$$P_{\text{FA}} = Q(\frac{\gamma - N\sigma_w^2}{\sqrt{N\sigma_w^4}}), \tag{2}$$

$$P_{\text{D}} = Q(\frac{\gamma - N(\sigma_w^2 + |g|^2\sigma_s^2)}{\sqrt{N(\sigma_w^2 + |g|^2\sigma_s^2)^2}}), \tag{3}$$

where $P_{\text{D}} = 1 - P_{\text{MD}}$ is the probability of detection and $Q(.)$ is the standard complementary Gaussian function. Let $\overline{P}_{\text{D}}$ denote the target probability of detection. Then, substituting γ from (3) to (2), P_{FA} of sensing user i can be written as,

$$P_{\text{FA,i}} = Q(Q^{-1}(\overline{P}_{\text{D}})(1 + \lambda) + \sqrt{N}\lambda), \tag{4}$$

where $\lambda = \frac{|g|^2\sigma_s^2}{\sigma_w^2}$ is the received signal to noise ratio of the primary user under H_1.

We use *OR* rule to combine the result of sensing from each secondary user in the sensing group. If $\mathcal{J} = \{s_1, s_2, ..., s_J\}$ be the set of secondary users that participate in cooperative sensing, then, the probability of false alarm for cooperative sensing is,

$$\widehat{P}_{\text{FA}} = (1 - \prod_{i=1}^{J} P_{\text{FA,i}}). \tag{5}$$

III. Game Theoretical Framework for Cooperative Spectrum Sensing: Mixed Strategy Equilibrium

We model the spectrum sensing as a non-cooperative game. The players of the game are the K secondary users $\mathcal{K} = \{1, 2, ..., K\}$. As in [14], each user gets one channel if the primary channel is found vacant. The strategy of player i is to choose from the binary strategy set $A_i = \{0, 1\}$ at the beginning of each sensing interval, where 0 means *not-sense* and 1 means *sense*.

Let p_{H_0} be the probability that the primary user is absent. Let us define $0 \leq \tau_i \leq 1$ as the idle duration that user i will have in each frame after transmitting its data. The achievable throughput for a sensing user can be written as

$$R_i(J) = \begin{cases} p_{H_0}(1 - \widehat{P}_{\text{FA}})C_{H_0}(1 - \tau_i) \\ \quad \text{if } J \in [1, K] \text{ and } \frac{\tau}{J} <= \tau_i, \\ p_{H_0}(1 - \widehat{P}_{\text{FA}})C_{H_0}(1 - \frac{\tau}{J}) \\ \quad \text{if } J \in [1, K] \text{ and } \tau_i < \frac{\tau}{J} < 1 \end{cases} \tag{6}$$

where $\tau = \frac{T_s}{T}$ is the normalized sensing duration and C_{H_0} is the data rate that one channel offers the secondary user under H_0.

We define the payoff as the difference between the obtained throughput and the associated energy cost. Therefore, the average payoff for a sensing user is

$$U_{i,s}(J) = \begin{cases} U_0(1 - \tau_i) - \frac{\sigma\tau}{J} \\ \quad \text{if } J \in [1, K] \text{ and } \frac{\tau}{J} <= \tau_i, \\ U_0(1 - \frac{\tau}{J}) - \frac{\sigma\tau}{J} \\ \quad \text{if } J \in [1, K] \text{ and } \tau_i < \frac{\tau}{J} < 1 \end{cases} \tag{7}$$

where $U_0 = p_{H_0}(1 - \widehat{P}_{\text{FA}})C_{\text{H}_0}$, σ is the energy consumed per unit of time for sensing.

If user i decides not to sense, its payoff can be derived in the similar manner and is given by

$$U_{i,ns}(J) = \begin{cases} U_0(1 - \tau_i) \quad \text{if } J \in [1, K - 1], \\ 0 \quad \text{if } J = 0. \end{cases} \tag{8}$$

For a finite K user game in strategic form $\mathcal{G} = \{\mathcal{K}, (A_i)_{i\in\mathcal{K}}, (u_i)_{i\in\mathcal{K}}\}$, where u_i is the utility function of user i, the strategy space for all users other than i is denoted as A_{-i}. Let us denote the actions taken by user i and the rest of the users as a_i and a_{-i}, respectively. Then, for all $i \in \mathcal{K}$, the strategy $a_i^* \in A_i$ is a *Nash Equilibrium* (NE) for user i if for every alternative strategy $a_i \in A_i$, ($a_i^* \neq a_i$), the following relation holds:

$$u(a_i^*, a_{-i}^*) \geq u(a_i, a_{-i}^*). \tag{9}$$

Equation (9) indicates that at NE, a player cannot improve its payoff by deviating alone from the equilibrium, given the strategies of all other players.

If the achievable payoff for being a free rider is greater than the achievable payoff for contributing in cooperative sensing, each secondary user will tend to be a free rider [14]. Conversely, if the payoff of being a sensing user is greater, all users will take strategy *sense*. One of the equilibria for the game is the point where the obtainable payoff for being a sensing user is same as the obtainable payoff of being a not-sensing user. This point is called the *mixed strategy NE*. Let, $K_1, K_2, ..., K_L$ ($\sum_{l=1}^{L} K_l = K$) be the number of secondary users which choose to contribute in sensing with probabilities p_1, p_2,....,p_L respectively. Then, for user i which belongs to the group of K_l users, if it decides to contribute in sensing, the achievable average utility $\overline{U}_{i,s}$ will be,

$$\overline{U}_{i,s} = \begin{array}{l} \sum_{j_L=0}^{K_L} .. \sum_{j_l=0}^{K_l-1} .. \sum_{j_1=0}^{K_1} \left(\begin{array}{c} K_L \\ j_L \end{array} \right) p_L{}^{j_L} (1-p_L)^{(K_L-j_L)} .. \\ \left(\begin{array}{c} K_l - 1 \\ j_l \end{array} \right) p_l{}^{j_l} (1-p_l)^{(K_l-1-j_l)} .. \\ \left(\begin{array}{c} K_1 \\ j_1 \end{array} \right) p_1{}^{j_1} (1-p_1)^{(K_1-j_1)} U_{i,s}((\sum_{l=1}^{L} j_l)+1), \end{array} \tag{10}$$

The average utility for user i for taking pure strategy *not-sense* is given by $\overline{U}_{i,ns}$,

$$\overline{U}_{i,ns} = \begin{array}{l} \sum_{j_L=0}^{K_L} .. \sum_{j_l=0}^{K_l-1} .. \sum_{j_1=0}^{K_1} \left(\begin{array}{c} K_L \\ j_L \end{array} \right) p_L{}^{j_L} (1-p_L)^{(K_L-j_L)} .. \\ \left(\begin{array}{c} K_l - 1 \\ j_l \end{array} \right) p_l{}^{j_l} (1-p_l)^{(K_l-1-j_l)} .. \\ \left(\begin{array}{c} K_1 \\ j_1 \end{array} \right) p_1{}^{j_1} (1-p_1)^{(K_1-j_1)} U_{i,ns}(\sum_{l=1}^{L} j_l). \end{array} \tag{11}$$

Now, using equality of payoff theorem, the mixed strategy NE p_i for $i \in \mathcal{K}$ can be calculated by solving

$$\overline{U}_{i,s} = \overline{U}_{i,ns}. \tag{12}$$

We can get L such equations, solving which we can obtain p_1, p_2,....,p_L.

IV. Game Theoretical Framework for Cooperative Spectrum Sensing: Correlated Equilibrium

A. Preliminary

In this paper, we introduce the concept of *correlated equilibrium* (CE) for the cooperative spectrum sensing game among secondary users. The CE [16] is defined in a context where the players are able to access certain common signals. These signals allow players to coordinate their actions and to perform joint randomization over their strategies according to a certain distribution. This probability distribution is called CE.

A probability distribution P_d over $A_1 \times ..... \times A_K$ is a CE of game $\mathcal{G}$ if and only if for all $i \in \mathcal{K}$, for every strategy $a_i \in A_i$ and for every alternative strategy $a_i{}' \in A_i$,

$$\sum_{a_{-i} \in A_{-i}} p(a_i, a_{-i})[u(a_i{}', a_{-i}) - u(a_i, a_{-i})] \leq 0, \tag{13}$$

where $p(a_i, a_{-i}) \geq 0$, represents the probability that user i takes action a_i and the rest of the users take actions a_{-i}. The inequality (13) indicates that when the recommendation to user i is to choose action a_i, then choosing action $a_i{}'$ instead of a_i cannot give a higher expected payoff to user i. The set of correlated equilibria is nonempty, closed and convex in every finite game. Moreover, it may include the distribution that is not in the convex hull of the NE distributions. In fact, every NE is a CE and Nash equilibria correspond to the special case where $p(a_i, a_{-i})$ is a product of each individual users probability for different actions. If we replace the right-hand side of inequality (13) by $\epsilon \geq 0$, the CE is called correlated ϵ-equilibrium.

TABLE I
Payoff table for a homogeneous 2 user sensing game
($\overline{P}_D = 0.95$, $SNR = -14$ dB, $\tau = 0.5$, $f_s = 1$ MHz $C_{H_0} = 10$ Mbps)

	sense	not-sense
sense	$(6.75, 6.75)$	$(4.5, 9.0)$
not-sense	$(9.0, 4.5)$	$(0, 0)$

The constraints (13) can be solved by using linear programming. This gives a set of equilibria. Thus, there can be multiple set of the solutions of correlated equilibria. An appropriate objective function should be defined in order to obtain a refined CE. In this paper, we focus on social welfare and fairness related objective functions, considering the unique characteristics of the spectrum sensing game.

B. Motivation

In this section, we explain the motivation behind introducing the concept of CE for the cooperative spectrum sensing game and present examples to demonstrate that the system performance can be improved by using CE. There are two reasons why CE is an important solution for such games. Firstly, some games do not have a pure NE while some games have multiple NE. Even the mixed strategy NE does not always exist but the set of correlated equilibria is nonempty, closed and convex in every finite game. Secondly, CE can achieve better utilities than NE. The following examples illustrate the efficiency of CE.

1) Spectrum Sensing Game Among Homogeneous Users: Let us consider the sensing game between two homogeneous users, which always have data to transmit. The payoff table for the two players is shown in Table I. The first element of each cell is the payoff of the first(/row) user and the second element is the payoff for the second(/column) user. The payoffs were calculated for $\sigma = 0$. The game has two pure Nash equilibria: $(4.5, 9.0)$ and $(9.0, 4.5)$. A mixed strategy equilibrium is that both user take the action *sense* with a probability of $\frac{2}{3}$, that yields average utilities of 6.0 for both of them and the total utility is 12.

In terms of CE concept, let, (q_1, q_2, q_3, q_4) be the probabilities of taking action pairs $\{0,0\}, \{0,1\}, \{1,0\}$, and $\{1,1\}$, respectively. The CE for this case to maximize the system utility is $(q_1 = 0.0, q_2 = 0.4702, q_3 = 0.4702, q_4 = 0.0596)$. Thus, the achievable utilities for both users will be 6.75, giving the system utility of 13.5, higher than in the case of mixed strategy equilibrium. Figure 3 shows the average utilities obtained at mixed strategy NE and CE. It is clear that CE improves the average utility of each user and hence, it improves the system utility.

2) Spectrum Sensing Game Among Heterogeneous Users: When the users have heterogeneity in traffic dynamics, the heterogeneity can be utilized to design a suitable approach for the case of cooperative spectrum sensing. The secondary users with lower traffic intensity do not need the full frame duration

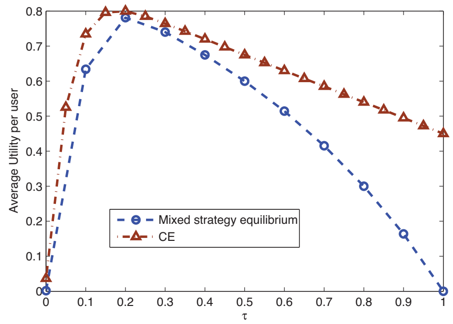

Fig. 3. Average utilities ($K = 2$)

TABLE II
PAYOFF TABLE FOR A HETEROGENEOUS 2 USER SENSING GAME

	sense	not-sense
sense	$(6.75, 6.75)$	$(4.5, 7.2)$
not-sense	$(9.0, 4.5)$	$(0, 0)$

for their data transmission. The idle duration of such users can be exploited for sensing to increase the network utility, as illustrated in the following example.

Let us consider the spectrum sensing game between two users with $\tau_1 = 0.0, \tau_2 = 0.2$. The payoff table for the two players is shown in Table II with other parameters same as for Table I. The game has two pure Nash equilibria: $(4.5, 7.2)$ and $(9.0, 4.5)$. A mixed strategy equilibrium is that user 1 takes the action 1 with a probability of 0.91 and user 2 takes the same action with a probability of 0.67. The average utilities obtainable at mixed strategy are 6.0 for user 1 and 6.5475 for user 2, respectively with the total utility of 12.5475. The CE for maximizing the system utility is $(q_1 = 0.0, q_2 = 1.0, q_3 = 0.0, q_4 = 0.0)$ and the achievable utilities for user 1 and user 2 will be 9.0 and 4.5 respectively, giving the system utility of 13.5. Unlike the case of homogeneous users, the CE probability distribution may be asymmetric in case of heterogeneous users as we saw in this example.

Always cooperating i.e. $\{1, 1\}$ is not an equilibrium here because of the lack of cooperation among selfish users. However, since the users adopt CE strategy, they jointly optimize their actions by considering the low utility of mutual non-cooperation and the high utility when both cooperate, thus the degree of cooperation is increased and the system utility improves.

C. Problem Formulation

There are 2^K possible combinations of strategies for the sensing game with K secondary users. Let us represent all possible combination of actions as matrix $B = [B_{i,j}]$, where the rows $i \in [1 \quad 2^K]$ represent the combination of actions of each user and the columns $j \in [1 \quad K]$ indicate the action chosen by each user, the matrix rows starts from $\{0, 0, .., 0\}$ and ends at $\{1, 1, .., 1\}$, the leftmost element being for user 1. For instance, $\mathrm{B}_{4,1}$ indicates the action of user 1 for strategy combination 0011 for a 4 user game. When a user, say k^{th} user, chooses an action: 0 or 1, it will be removed from matrix B and the number of possible combination of actions for the rest of the users will be 2^{K-1}. Let us represent this matrix as $C = [C_{i,j}] : \forall i \in [1 \quad 2^{(K-1)}], \forall j \in [1 \ K], j \neq k$, which is of size $2^{K-1} \times (K-1)$. The optimization problem ($\mathbf{P_1}$) for the cooperative spectrum sensing game to obtain CE that maximizes the *social welfare* (SW), can be defined as

<u>$\mathbf{P_1}$ (CE that maximizes social welfare)</u>

$$\max_{p} \sum_{i=1}^{K} \sum_{l=1}^{2^{K-1}} \{p(a_i = 0, [a_k]_{k=1,k\neq i}^{K} = [C_{l,k}]_{k=1}^{K-1}) U_{i,ns}(K_s) + p(a_i = 1, [a_k]_{k=1,k\neq i}^{K} = [C_{l,k}]_{k=1}^{K-1}) U_{i,s}(K_s+1)\} \tag{14}$$

s.t.

$$\sum_{l=1}^{2^{K-1}} p(a_i = 0, [a_k]_{k=1,k\neq i}^{K} = [C_{l,k}]_{k=1}^{K-1}) (U_{i,ns}(K_s) - U_{i,s}(K_s+1)) \geq 0 \quad \forall i \in \mathcal{K}, \tag{15}$$

$$\sum_{l=1}^{2^{K-1}} p(a_i = 1, [a_k]_{k=1,k\neq i}^{K} = [C_{l,k}]_{k=1}^{K-1}) (U_{i,s}(K_s+1) - U_{i,ns}(K_s)) \geq 0 \quad \forall i \in \mathcal{K}, \tag{16}$$

$$p(a_i = 0, ..) \geq 0, \quad p(a_i = 1, ..) \geq 0.$$

where $K_s = \sum_{k=1,k\neq i}^{K} a_k$, is the total number of users performing cooperative sensing, for each combination of strategies, $U_{i,s}(.)$ and $U_{i,ns}(.)$ are the utilities given by (7) and (8) respectively. $\mathbf{P_1}$ is a linear programming problem and it can be solved in a centralized fashion by using either the simplex method (SM) or the interior point method (IPM).

It is noteworthy that the optimization problem $\mathbf{P_1}$ looks similar for the case of homogeneous as well as heterogeneous users. However, the difference in the two cases lies in the utility functions: $U_{i,s}(K_s + 1)$ and $U_{i,ns}(K_s)$. For the case of homogeneous users, the utility achievable for each user is the same for one value of K_s, irrespective of which users are contributing in sensing. This implies that the probabilities of choosing the combinations of strategies for which K_s is the same, are equal. On the other hand, when we consider users with heterogeneous traffic dynamics, the value of τ_i is different for each user. This means, different users can get different utilities even for the same value of K_s. Consequently, the probability distribution may also be asymmetric.

D. Fairness Consideration and Utility Maximization

When the users are heterogeneous, social welfare, although yields the highest system utility, it may lead users that have worst conditions or those that demand less resources to starve. The loss that the less well-off users suffer when the system utility is maximized, may be undesirable in some situations. Let's consider the example in section IV-B2. User 2 has 20% less traffic compared to user 1. But the utility of user 1 is twice the utility of user 2. Even for smaller values of τ_2 e.g., $\tau_2 = 0.05$, the payoffs that the users obtain will be 9.0 and 4.5 respectively. The difference in the utilities they get, increases further for higher values of τ. Eg., when $\tau = 0.9$, user 1 still gets a utility of 9.0 but the utility of user 2 will be 0.9. The fairness issue is critical in this case and it is one of the worst consequences of maximizing the system utility. In such systems, the objective function should be fairness centric rather than social welfare. Next, we discuss two schemes that consider two standard definition of fairness, and then we propose a new scheme to maximize the social welfare incorporating fairness.

1) Max-min Fairness (MMF): One of the common approaches to focus on fairness, is to use max-min fairness as the objective function. The optimization problem (**P₂**) for the cooperative spectrum sensing game to obtain CE that maximizes the utility of the worst user, can be defined as
P₂ (CE that maximizes the utility of the worst user)

$$\max_{p} \min_{i} \sum_{l=1}^{2^{K-1}} \{p(a_i = 0, [a_k]_{k=1,k\neq i}^{K} = [C_{l,k}]_{k=1}^{K-1}) U_{i,ns}(K_s) + p(a_i = 1, [a_k]_{k=1,k\neq i}^{K} = [C_{l,k}]_{k=1}^{K-1}) U_{i,s}(K_s+1)\} \tag{17}$$

s.t.

$$\sum_{l=1}^{2^{K-1}} p(a_i = 0, [a_k]_{k=1,k\neq i}^{K} = [C_{l,k}]_{k=1}^{K-1}) (U_{i,ns}(K_s) - U_{i,s}(K_s+1)) \geq 0 \quad \forall i \in \mathcal{K}, \tag{18}$$

$$\sum_{l=1}^{2^{K-1}} p(a_i = 1, [a_k]_{k=1,k\neq i}^{K} = [C_{l,k}]_{k=1}^{K-1}) (U_{i,s}(K_s+1) - U_{i,ns}(K_s)) \geq 0 \quad \forall i \in \mathcal{K}, \tag{19}$$

$$p(a_i = 0, ..) \geq 0, \quad p(a_i = 1, ..) \geq 0.$$

P₂ is also a linear programming problem and it can be solved in a centralized fashion by using either the SM or the IPM.

When the difference in user requirements is not significant, using max-min fairness as the objective function in **P₂** yields almost the same total utility as in **P₁** but it improves the utility of the less-well off users significantly. This means, the max-min fairness greatly improves the fairness without considerably degrading the system utility. On the other hand, if the achievable utilities of the users have a vast difference, and we try to optimize the utility of the worst user(s), the total utility will be affected significantly.

2) Proportional Fairness (PF): There are several studies on rate control algorithms and dynamic spectrum allocation with fairness [25]-[27]. PF ensures more fairness than the SW scheme, while achieving better performance than the MMF scheme. The proportionally fair distribution is a feasible vector x such that if one element of x is increased by $y\%$, the total percentage of reduction that has to be applied to the other elements of x in order to get another feasible distribution vector must be more than $y\%$. The optimization problem (**P₃**) for the cooperative spectrum sensing game to obtain CE that maximizes proportional fairness, can be defined as
P₃ (CE that maximizes proportional fairness)

$$\max_{p} \sum_{i=1}^{K} \prod_{l=1}^{2^{K-1}} \{p(a_i = 0, [a_k]_{k=1,k\neq i}^{K} = [C_{l,k}]_{k=1}^{K-1}) U_{i,ns}(K_s) + p(a_i = 1, [a_k]_{k=1,k\neq i}^{K} = [C_{l,k}]_{k=1}^{K-1}) U_{i,s}(K_s+1)\} \tag{20}$$

s.t.

$$\sum_{l=1}^{2^{K-1}} p(a_i = 0, [a_k]_{k=1,k\neq i}^{K} = [C_{l,k}]_{k=1}^{K-1}) (U_{i,ns}(K_s) - U_{i,s}(K_s+1)) \geq 0 \quad \forall i \in \mathcal{K}, \tag{21}$$

$$\sum_{l=1}^{2^{K-1}} p(a_i = 1, [a_k]_{k=1,k\neq i}^{K} = [C_{l,k}]_{k=1}^{K-1}) (U_{i,s}(K_s+1) - U_{i,ns}(K_s)) \geq 0 \quad \forall i \in \mathcal{K}, \tag{22}$$

$$p(a_i = 0, ..) \geq 0, \quad p(a_i = 1, ..) \geq 0.$$

P₃ is also a linear programming problem and it can be solved in a centralized fashion by using either the SM or the IPM.

3) Fair Social Welfare (f-SW): System efficiency and fairness are two contradictory objectives. The key motivation behind deploying CE for the cooperative spectrum sensing game is to improve the overall system utility by utilizing the heterogeneity among the secondary users. However, addressing the fairness issue among the secondary users is of great importance too. We combine these two aspects and propose a new fair and efficient CE for the game, and we name it f-SW CE.

The average utility of user i at CE is

$$\left\{ \begin{array}{l} \overline{U}_i = \sum_{l=1}^{2^{K-1}} p(a_i = 0, [a_k]_{k=1,k\neq i}^{K} = [C_{l,k}]_{k=1}^{K-1}) U_{i,ns}(K_s) \\ \quad + \sum_{l=1}^{2^{K-1}} p(a_i = 1, [a_k]_{k=1,k\neq i}^{K} = [C_{l,k}]_{k=1}^{K-1}) U_{i,s}(K_s+1). \end{array} \right. \tag{23}$$

Among K secondary users, let $\overline{U}_j$ be the utility of the highest traffic user(s) i.e. $\overline{U}_j = \max(\overline{\mathrm{U}}_\mathrm{i})$. In the proposed scheme, the highest traffic user gets the highest utility and the difference in the utility obtained by user i is related to their traffic dynamics by an additional constraint as following:

$$\frac{\overline{U}_j - \overline{U}_i}{\overline{U}_j} \leq d\tau_i \qquad \forall i, j \in \mathcal{K},\ i \neq j, \tag{24}$$

where $d \geq 1$ is a scaling parameter such that $d\tau_i \leq 1\ \forall i$. Equation (24) can be written as

$$\overline{U}_j(1 - d\tau_i) - \overline{U}_i \leq 0 \qquad \forall i, j \in \mathcal{K},\ i \neq j. \tag{25}$$

The proposed CE aims at maximizing the system utility and incorporates fairness criteria in it. The optimization problem (**P₄**) for the f-SW CE scheme can be defined as

P4 (CE that maximizes system utility ensuring fairness)

$$\max_p \sum_{i=1}^{K} (\sum_{l=1}^{2^{K-1}} p(a_i = 0, [a_k]_{k=1,k\neq i}^{K} = [C_{l,k}]_{k=1}^{K-1}) U_{i,ns}(K_s) + \sum_{l=1}^{2^{K-1}} p(a_i = 1, [a_k]_{k=1,k\neq i}^{K} = [C_{l,k}]_{k=1}^{K-1}) U_{i,s}(K_s+1)) \quad (26)$$

s.t.

$$\sum_{l=1}^{2^{K-1}} p(a_i = 0, [a_k]_{k=1,k\neq i}^{K} = [C_{l,k}]_{k=1}^{K-1}) (U_{i,ns}(K_s) - U_{i,s}(K_s+1)) \geq 0 \quad \forall i \in \mathcal{K}, \quad (27)$$

$$\sum_{l=1}^{2^{K-1}} p(a_i = 1, [a_k]_{k=1,k\neq i}^{K} = [C_{l,k}]_{k=1}^{K-1}) (U_{i,s}(K_s+1) - U_{i,ns}(K_s)) \geq 0 \quad \forall i \in \mathcal{K}, \quad (28)$$

$$\overline{U}_j(1 - d\tau_i) - \overline{U}_i \leq 0 \quad \forall i, j \in \mathcal{K}, i \neq j \quad (29)$$

$$p(a_i = 0, ..) \geq 0, \quad p(a_i = 1, ..) \geq 0.$$

As (25) is a linear constraint, **P4** is a linear program, and thus it can also be solved using either the SM or the IPM.

Note that the proposed scheme is different and more flexible than PF CE scheme. In f-SW CE scheme, fairness can be controlled through parameter d for given τ and τ_i, which is not the case in PF CE.

V. Distributed Implementation of CE

A. No-Regret Algorithm

In order to solve the optimization problem for computing the CE, each player needs to know the payoff matrix of other players as well. This makes the concept of CE difficult to be implemented distributedly. We propose to use a *No-Regret* (NR) algorithm [17] so that the secondary users' strategy converges to the CE, by using only local information of each secondary user.

The sensing game is played repeatedly through time $t = 1, 2, 3, ...$ Given the history of play of each user until time t: $h(t) = (a^m)_{m=1}^t$, where a^m means the strategy taken at time index m, each player $i \in \mathcal{K}$ chooses $(a_i^{t+1}) \in A_i$ according to a probability distribution $p_i^{t+1} \in \Delta(A_i)$, where $\Delta(A_i) = A_1 \times A_2 \times .... \times A_K$, which can be calculated based only on the local information of each player.

Let $r, r^{'} \in A_i$ be two different strategies of player i. Then, for every action r that i took until time t, if it had taken strategy $r^{'}$ instead, the resulting difference in utility would have been

$$D_i^t(r, r^{'}) = \frac{1}{t} \sum_{m \leq t : a_i^m = r} [u_i(r^{'}, a_{-i}^m) - u_i(a^m)]. \quad (30)$$

The average regret until time t for not playing strategy $r^{'}$ every time it played strategy r in the past is

$$R_i^t(r, r^{'}) = max(D_i^t(r, r^{'}), 0). \quad (31)$$

TABLE III
The No-regret learning algorithm

Choose the probability distribution $p_i^1 \in \Delta(A_i)$ randomly for each user i at the beginning of the game i.e. at $t = 1$.
for $t = 1, 2, , ...$ do Compute $D_i^t(r, r^{'})$ using (30). Compute $R_i^t(r, r^{'})$ using (31). If $r \in A_i$ be the strategy chosen by user i at time t i.e. $a_i^m = r$, then, the probability distribution $p_i^{t+1} \in \Delta(A_i)$ used by user i at time $t+1$ can be calculated using $p_i^{t+1}(r^{'}) = \frac{1}{\mu} R_i^t(r, r^{'})$ for all $r^{'} \neq r$ $p_i^{t+1}(r) = 1 - \sum_{r^{'} \in A_i, r^{'} \neq r} p_i^{t+1}(r^{'})$, where $\mu > 2M_i(n_i - 1)$ $\forall i \in \mathcal{K}$, M_i is the upper bound for $\lvert u_i(.)\rvert$ and n_i is the number of strategies of user i.

The algorithm is shown in detail in Table III. The average regret in (31) can be calculated based only on the local information, which enables the NR algorithm to be implemented in a distributed manner.

At each stage of the game t, let $z^t \in \Delta(A)$ be the empirical distribution of the Q-tuples of strategies played until time t. Then, for every $a \in A$,

$$z^t(a) = \frac{1}{t} |m \leq t : a^m = a|, \quad (32)$$

where $|.|$ is the number of times the event inside $||$ occurs.

If every player plays according to NR learning algorithm, then the empirical distributions of play z_t converge almost surely (with probability one) to the set of correlated equilibria of the game as $t \to \infty$ [17]. A necessary and sufficient condition for this is that, all regrets converge to zero. In a more general form, this condition can be represented as the following proposition:

Proposition:Let $(a^t)_{t=1,2,..}$ be a sequence of plays ($a^t \in A\ \forall t$) and let $\epsilon \geq 0$. Then, $\forall i \in \mathcal{K}$ and every $r, r^{'} \in A_i$ with $r \neq r^{'}$,

$$\lim_{t \to \infty} \sup R_i^t(r, r^{'}) \leq \epsilon \quad (33)$$

if and only if the sequence of empirical distributions z_t converge to the set of correlated ϵ-equilibria.

Proof: For each player i and every $r, r^{'} \in A_i$, using (30) and (32), we get,

$$D_i^t(r, r^{'}) = \sum_{a \in A, a_i = r} z^t(a)[u_i(r^{'}, a_{-i}^m) - u_i(a^m)]. \quad (34)$$

On any subsequence where z^t converges, say $z^{t'} \to \psi \in \Delta(A)$, (30) can be written as,

$$D_i^t(r, r^{'}) = \sum_{a \in A, a_i = r} \psi(a)[u_i(r^{'}, a_{-i}^m) - u_i(a^m)]. \quad (35)$$

The result is immediate from the definition of correlated ϵ-equilibrium and (31). This completes the proof.

This algorithm requires player i to know its own payoff matrix but not those of other players, and at time $t+1$, the history of play $h(t)$. In terms of computation, player i needs to keep a record of time t along with $2(2-1) = 2$ values of

TABLE IV
THE NEIGHBOURHOOD BASED LEARNING ALGORITHM

Choose the probability distribution $p_i^1 \in \Delta(A^i)$ randomly for each user i at the beginning of the game i.e. at $t = 1$.
for $t = 1, 2, , ...$ do Compute $D^t_{i,\text{net}}(r, r^{'})$ using (36). Compute $R^t_{i,\text{net}}(r, r^{'})$ using (37). If $r \in A_i$ be the strategy chosen by user i at time t i.e. $a_i^m = r$, then, the probability distribution $p_i^{t+1} \in \Delta(A_i)$ used by user i at time $t+1$ can be calculated using $p_i^{t+1}(r^{'}) = \frac{1}{\mu} R^t_{i,\text{net}}(r, r^{'})$ for all $r^{'} \neq r$ $p_i^{t+1}(r) = 1 - \sum_{r^{'} \in A_i, r^{'} \neq r} p_i^{t+1}(r^{'})$, where μ is a sufficiently large number.

$D^t_i(r, r^{'})$ for $r \neq r^{'}$ in A_i, and has to update these numbers after each frame.

B. Neighbourhood Based Learning Algorithm

We propose a new algorithm based on learning from the neighbours to improve the performance of the NR algorithm. We call it *Neighbourhood Based Learning* (NBL) algorithm. In this algorithm, the strategy of each user is calculated based on the total regret rather than the user's individual regret.

Let $r, r^{'} \in A_i$ be two different strategies of player i. Then, for every action r that user i took until time t, if it had taken strategy $r^{'}$ instead its total utility would have been different. The resulting difference in total utility would have been

$$D^t_{i,\text{net}}(r, r^{'}) = \frac{1}{t} \sum_{m \leq t : a_i^m = r} \sum_{l \in \mathcal{K}} \{u_l(a_i^m = r^{'}, a_{-i}^m) - u_l(a^m)\}. \quad (36)$$

To compute $D^t_{i,\text{net}}(r, r^{'})$, at time $t+1$, each user should have the information about the history $h(t)$, as in NR algorithm. In addition, each user needs to compute the resulting difference in the utilities of all other users for not having played $r^{'}$ everytime it played r in the past . Each user gets the information required to compute this difference through its neighbours. This information exchange accounts for an additional cost for the proposed algorithm. However, in the spectrum sensing game, the only extra parameter that each user needs to exchange in the NBL algorithm is $\tau_i \forall i \in \mathcal{K}$, only once at the beginning of the game. Thus, the overhead due to this information exchange is negligible.The average net regret until time t for not having played $r^{'}$ every time it played r in the past is

$$R^t_{i,\text{net}}(r, r^{'}) = max(D^t_{i,\text{net}}(r, r^{'}), 0). \quad (37)$$

The detailed algorithm is described in Table IV. In terms of computation in the NBL algorithm, player i needs to keep a record of time t along with $2(2-1) = 2$ values of $D^t_{i,\text{net}}(r, r^{'})$ for $r \neq r^{'}$ in A_i and has to update these numbers after each frame. But calculating $D^t_{i,\text{net}}(r, r^{'})$, needs $K - 1$ times more computation in this case.

Computing correlated equilibria for multi-player games becomes rapidly intractable in the general case when the number of players is large because the input length is exponential [18].

TABLE V
PARAMETERS USED FOR ANALYSIS

Parameters	Values
$\overline{P}_{\text{D}}$	0.95
SNR	-14 dB
τ	0.5
f_s	1 MHz
C_{H_0}	10 Mbps

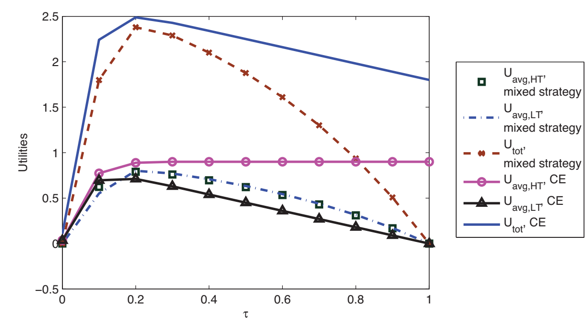

Fig. 4. Comparison of average utilities (CE and mixed strategy)

However, using the NBL algorithm, the users can start with an arbitrary strategy and then, converge to the CE, with some extra computations which is only linearly proportional to the number of users.

VI. NUMERICAL RESULTS AND ANALYSIS

The parameters used for evaluation are shown in Table V. Different values of the parameters will give different CE probability distributions. However, for the purpose of illustration, we use a set of realistic parameters in all analyses in this section. We consider $K = 3$ with $\tau_1 = \tau_2 = 0.0, \tau_3 = 0.2$, unless otherwise stated. We refer to users 1 and 2 as heavy traffic users and user 3 a light traffic user. In addition, for all plots in this section we consider $\sigma = 0$, but please note that, similar analysis can be done for $\sigma \neq 0$ as well using our analytical framework.

A. Utility Maximization for Heterogeneous Users: SW CE

The optimal CE probability distribution for this game can be obtained by solving $\mathbf{P_1}$. The utilities obtained are depicted in Figure 4. This figure indicates that the system utility is maximized if light traffic user(s) is(are) selected to contribute more in sensing. When SW is considered, the light traffic user may get even less utility than in mixed strategy NE but the heavy traffic users get higher utilities compared to the mixed strategy NE. Thus the total utility is higher in SW CE than in mixed strategy NE.

B. Comparison of SW CE, MMF CE and PF CE Schemes

We plot the utilities obtained by each user and the total utility in Figure 5, for the three schemes. The figure depicts that the utility of the heavy traffic users (users 1 and 2) and the total utility are high for the case of SW. In contrast, for the light traffic user (user 3), the utility is better when MMF is considered. The performance of PF is close to that of SW for smaller values of τ and it approaches MMF for higher values of τ.

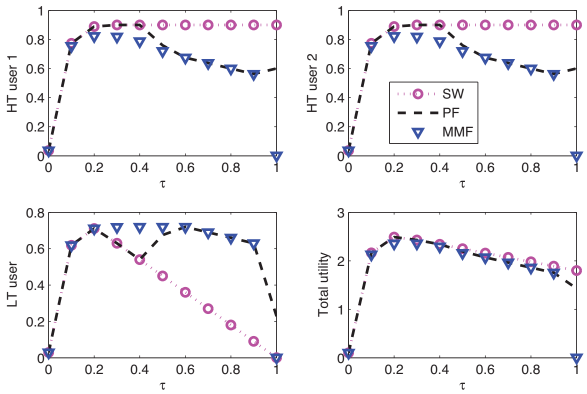

Fig. 5. Comparison of utilities (SW, PF, MMF)

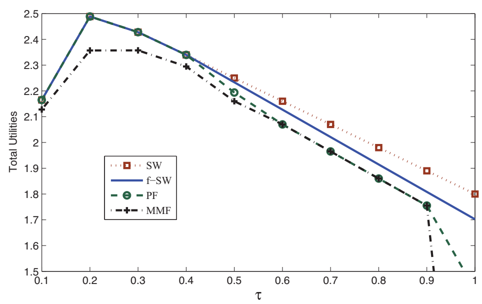

Fig. 6. Comparison of total utilities (SW, f-SW, PF and MMF)

C. Performance of the Proposed f-SW CE Scheme

We compare the total utilities obtained at f-SW CE with SW CE, PF CE and MMF CE schemes in Figure 6 for the scaling parameter $d = 2$. From the figure we see that the total utility at f-SW CE is less than in SW CE scheme, but it is higher than in MMF CE scheme for all values of τ. The total utility is better than in PF CE scheme as well for $\tau > 0.4$. Figure 7 depicts the utilities per user obtained using SW CE, MMF CE and f-SW CE schemes. One of the issues with MMF scheme is that the utility of users 1 and 2 is less than that of user 3 after $\tau > 0.5$. This is not fair to users 1 and 2. In the proposed scheme, this problem is eliminated as the utility of users 1 and 2 is always higher than that of user 3. The utilities of users 1 and 2 is lower than in SW CE but higher than in MMF CE. Moreover, the utility of user 3 is slightly less in the proposed scheme than in MMF CE but improves considerably compared to SW CE scheme. Thus, fairness is achieved in the proposed scheme keeping the total utility higher than in MMF CE scheme.

D. Proposed NBL algorithm

The probability distribution for NBL algorithm is computed in similar manner as in case of the NR algorithm. Here, we consider a two user game with $\tau_1 = 0.0, \tau_2 = 0.2$. q_1, q_2, q_3, q_4 are the probabilities of the strategy combinations: $\{0,0\}, \{0,1\}, \{1,0\}$ and $\{1,1\}$, respectively. Figure 8 shows the performance of the NBL algorithm. The game converges to the probability distribution $(q_1, q_2, q_3, q_4) = (0.0, 0.37, 0.08, 0.55)$, indicating that the second user is involved more in sensing. Although not shown here, the sensing contribution from second user i is more if τ_i is higher.

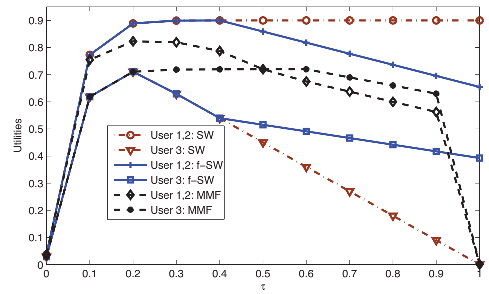

Fig. 7. Comparison of utilities per user (SW, f-SW and MMF)

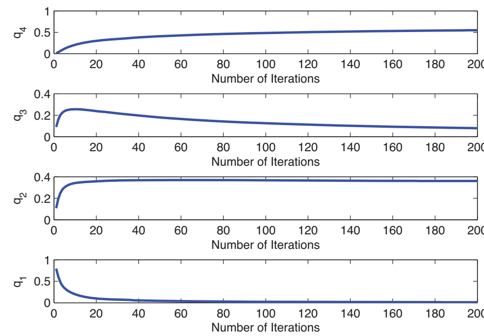

Fig. 8. Probability distribution obtained using NBL algorithm

Figure 9 compares the performance of the NR algorithm and the NBL algorithm for the case of two users with $\tau_1 = 0.0, \tau_2 = 0.2$ for different values of τ. The figure shows that the total utility obtained using the NBL algorithm is much closer to the total utility at SW CE compared to the total utility obtained using the NR algorithm. The improvement in the performance is more visible for larger values of τ.

One of the common questions for such schemes is, when each user is selfish, why would it exchange information and make more computations for social welfare if this does not benefit itself. However, if the users act based on net the regret instead of the individual regret, the utility of each individual user also increases. This is what motivates the users to exchange information and to make extra computations. To consider more general case, we show the performance (average utility of each user) of the proposed algorithm for the case of 10 users in Figure 10 with $\tau_1, \tau_2, \tau_3 = 0.0, \tau_4, \tau_5 = 0.1, \tau_6, \tau_7, \tau_8 = 0.2, \tau_9, \tau_{10} = 0.3$. The figure shows that the performance of the NBL algorithm is close to that of the NR algorithm until about $\tau = 0.3$. The improvement brought by the proposed algorithm is significant for higher values of τ. As the average utility of each user improves in NBL algorithm, it is obvious that the total utility in NBL algorithm is higher compared to the NR algorithm.

VII. Conclusion and Future Work

We introduced the concept of correlated equilibrium for the cooperative spectrum sensing game among non-cooperative secondary users of heterogeneous traffic dynamics and il-

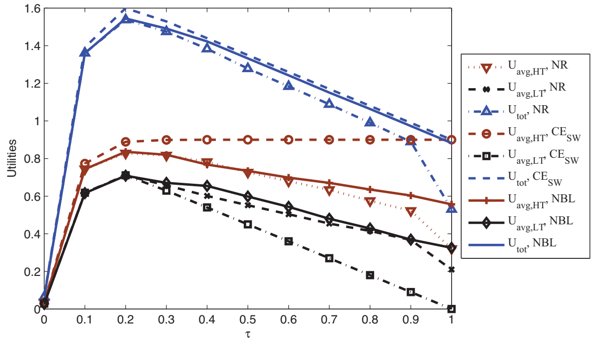

Fig. 9. Comparison of average utilities for 2-heterogeneous users game

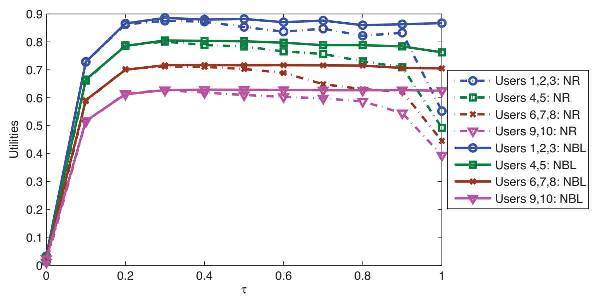

Fig. 10. Comparison of average utilities for 10-heterogeneous users game

lustrated that the correlated equilibrium improves the payoff of the users compared to the mixed strategy equilibrium. We proposed fair social welfare correlated equilibrium to maximize the social welfare without letting some users starve. We showed that the proposed scheme always performs better than the max-min fairness scheme in terms of total utility and ensures more fairness than the social welfare scheme. In order to implement correlated equilibrium in distributed manner, we proposed the neighbourhood based learning algorithm and the results show that this algorithm improves the performance compared to the no-regret algorithm significantly. As a future work, we would like to extend our study by focussing on the aspect of dynamic formation/splitting of the users for cooperative spectrum sensing and the infrastructure requirement for the same.

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