Performance of Alamouti Space-Time Coded OFDM with Carrier Frequency Offset

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***Abstract*—We investigate the error probability for Alamouti space-time coded orthogonal frequency-division multiplexing (OFDM) systems with carrier frequency offset (CFO) and multipath fading channels. Our main assumptions are: frequency-selective Rayleigh fading, a single receive antenna, and coherent detection. We derive new closed-form results for the (symbol or bit) error probability, for both spatially uncorrelated channels and spatially correlated channels. The proposed analysis shows that the error floor due to the CFO is strongly dependent on the power-delay profile of the multipath channel. Simulation results confirm the accuracy of the proposed theoretical analysis.**

Keywords—Alamouti space-time coding; carrier frequency offset (CFO); frequency-selective Rayleigh fading; OFDM

I. Introduction

Orthogonal frequency-division multiplexing (OFDM), combined with multiple-input multiple-output (MIMO) signal processing techniques, allow for both capacity increase and performance improvement of communication links. One way to achieve a performance gain is by means of orthogonal space-time block codes (STBC), such as the Alamouti code [1][2]. However, it is well known that OFDM-based systems are sensitive to the intercarrier interference (ICI) generated by a carrier frequency offset (CFO), which degrades the error probability performance for both single-antenna OFDM systems [3]-[6] and MIMO STBC-OFDM systems [7]-[10].

The error probability of coherently detected single-antenna OFDM systems with CFO has been widely analyzed, for additive white Gaussian noise (AWGN) channels [3] and for multipath channels with different fading statistics [4]-[6], using either the direct method or the characteristic function method. It has been shown that the ICI caused by the CFO produces two main effects: a performance degradation at low-to-medium signal-to-noise ratio (SNR), and an error floor at high SNR.

Unlike single-antenna OFDM systems, the error probability analysis for coherently detected orthogonal STBC-OFDM systems with CFO and Rayleigh fading has been considered only partially, using the direct method [7]-[10]. In particular, the analyses of [7] and [8] approximate the ICI power on a certain subcarrier as independent from the channel gain in that subcarrier. Despite the ICI power approximation of [7]-[8] may be acceptable for highly frequency-selective channels, it has been shown in [9] that the error floor is largely overestimated for flat-fading channels and inexact for moderately frequency-selective fading channels. On the other hand, the performance analysis of [10], which also includes different receiver impairments, models the dependency of the ICI power from the subcarrier gain, but unfortunately the error probability is evaluated only numerically, without providing a closed-form expression. In addition, the analyses of [7]-[10] assume that the two transmit channels are independent. However, in general, a certain level of spatial correlation may be present, especially when the two transmit antennas are very close to each other.

This paper presents a novel and accurate error probability analysis for coherently detected Alamouti-coded OFDM systems with CFO and multipath Rayleigh fading channels. *First*, we evaluate the signal-to-ICI-plus-noise ratio (SINR), conditioned on the subcarrier channel gain. The proposed analysis is valid for all multipath channels, i.e., with any degree of frequency selectivity. We show that our general SINR expression includes the specific SINR results previously obtained for flat-fading channels in [9] and for highly frequency-selective channels in [7]-[8]. *Second*, by exploiting a Gaussian approximation for the ICI included in the obtained SINR, we employ the direct method and derive a closed-form expression for the bit error probability of CFO-impaired Alamouti-coded OFDM systems, assuming spatially uncorrelated channels and coherent modulation with Gray coding. This analysis provides a novel theoretical solution, differently from the numerical result currently available in the recent literature [10]. The obtained theoretical results illustrate that the error floor due to the ICI is strongly dependent on the frequency selectivity of the channel, i.e., on its power-delay profile. *Third*, we extend the error probability analysis in order to include the effect of spatial correlation at the transmitter, which has not been considered in the previous literature [7]-[10]. Simulation results confirm that our theoretical analysis accurately models both the low-to-medium-SNR performance loss and the high-SNR error floor.

II. STBC-OFDM Systems with CFO

Let us consider an Alamouti-coded MIMO-OFDM system with N subcarriers, two transmit antennas, and a single receive antenna[1]. At the transmitter, the data bits are first multiplexed to the two transmit antennas and, after serial-to-parallel

[1] This is compatible with the downlink of current OFDM cellular systems, where the mobile user is often equipped with a single-antenna terminal. The multiple antenna case is not considered in this paper due to space limitations.

This research is partly supported by the Singapore-MIT International Design Center (grant no. IDG31100102 & IDD11100101).

conversion, successively mapped onto symbols using a modulation alphabet. Let us denote the generic uncoded OFDM block by $\mathbf{x}_t[k]=[x_t[k,0],...,x_t[k,N-1]]^T$, where $x_t[k,n]$ is the generic M-PSK (or M-QAM) symbol transmitted by the t th antenna in the n th subcarrier of the k th OFDM block. Using Alamouti STBC, the coded OFDM blocks transmitted in the $(2k)$ th and $(2k+1)$ th time intervals are expressed by

$$\mathbf{s}_1[2k]=\mathbf{x}_1[k], \quad \mathbf{s}_1[2k+1]=-\mathbf{x}_2^*[k],$$
$$\mathbf{s}_2[2k]=\mathbf{x}_2[k], \quad \mathbf{s}_2[2k+1]=\mathbf{x}_1^*[k], \tag{1}$$

where the subscript denotes the transmit antenna index. After Alamouti coding, an IDFT of size N is applied to $\mathbf{s}_1[k]$ and $\mathbf{s}_2[k]$, and a cyclic prefix (CP) of size L is appended, where L is the maximum channel length among the two channels. Hence, there is no ICI caused by insufficient CP length. The signal transmitted by the t th antenna can be expressed as

$$\mathbf{y}_t^{(\mathrm{TX})}[k]=\frac{1}{\sqrt{2}}\mathbf{I}_{\mathrm{CP}}\mathbf{F}^H\mathbf{s}_t[k] \tag{2}$$

where $\mathbf{F}$ is the N-size unitary DFT matrix, $\mathbf{I}_{\mathrm{CP}}=[\mathbf{J}^T\ \mathbf{I}_N]^T$ is the $P\times N$ CP insertion matrix [11], where $P=N+L$ and $\mathbf{J}$ contains the last L rows of $\mathbf{I}_N$, and $1/\sqrt{2}$ takes into account the presence of two transmit antennas.

The frequency-selective MIMO channel, assumed to be linear time-invariant, is modeled by $\mathbf{h}_t=[h_t[0],...,h_t[L-1]]^T$, $t=1,2$, where $h_t[l]$ is the l th discrete-time path of the channel link between the t th transmit antenna and the receiver. We assume that the elements $\{h_t[l]\}$ are independent zero-mean circularly complex Gaussian random variables, with variance $\sigma_l^2=E\{h_t[l]h_t^*[l]\}$. Therefore, we are assuming that the two transmit-receive channels have the same power-delay profile. This is compliant with the channel models proposed in [12]. We also assume that the spatial correlation is described by $\rho=E\{h_1[l]h_2^*[l]\}/\sigma_l^2$, independent from the path index l, which corresponds to a Kronecker model [12]. Without loss of generality, we assume ρ as real, and a subcarrier-power-normalized power-delay profile, where we have $\sum_{l=0}^{L-1}\sigma_l^2=N$. We define, for $t=1,2$, the $P\times P$ Toeplitz time-domain channel matrix $\mathbf{H}_{0,t}=\mathrm{Toep}_P(\mathbf{h}_t)$, with first column expressed by $[h_t[0],...,h_t[L-1],0,...,0]^T$, the $N\times N$ circulant time-domain channel matrix $\mathbf{H}_t=\mathbf{E}_{\mathrm{CP}}\mathbf{H}_{0,t}\mathbf{I}_{\mathrm{CP}}$, where $\mathbf{E}_{\mathrm{CP}}=[\mathbf{0}_{N\times L}\ \mathbf{I}_N]$ is the $N\times P$ CP removal matrix [11], and the $N\times N$ diagonal frequency-domain channel matrix $\mathbf{\Lambda}_t=\mathbf{F}\mathbf{H}_t\mathbf{F}^H$.

Assuming a CFO f_0, and defining $\varepsilon=f_0/\Delta_f$, where Δ_f is the subcarrier separation, the received vector, after CP elimination, can be expressed by [13]

$$\mathbf{y}^{(\mathrm{RX})}[k]=\frac{e^{j\theta[k]}}{\sqrt{2}}\mathbf{D}\sum_{t=1}^{2}\mathbf{H}_t\mathbf{F}^H\mathbf{s}_t[k]+\mathbf{n}[k], \tag{3}$$

where $\theta[k]=2\pi\varepsilon(kP+L)/N$, $\mathbf{D}$ is a diagonal matrix with elements $[D]_{n,n}=\exp(j2\pi\varepsilon n/N)$, $n=0,...,N-1$, and $\mathbf{n}[k]$ is the AWGN vector, with power σ_w^2. Without loss of generality, we have assumed that the phase offset (PO) caused by the CFO is zero at the beginning of the first OFDM block including the CP. Note that in (3) the CFO effect has been split into two parts: $e^{j\theta[k]}$ takes into account the PO of the k th OFDM block with respect to the first OFDM block, while $\mathbf{D}$ represents the effect of the CFO within the OFDM block. After DFT processing, we obtain $\mathbf{z}[k]=\mathbf{F}\mathbf{y}^{(\mathrm{RX})}[k]$, which by (3) leads to

$$\mathbf{z}[k]=\frac{e^{j\theta[k]}}{\sqrt{2}}\mathbf{\Phi}\sum_{t=1}^{2}\mathbf{\Lambda}_t\mathbf{s}_t[k]+\mathbf{w}[k], \tag{4}$$

where $\mathbf{\Phi}=\mathbf{F}\mathbf{D}\mathbf{F}^H$ is the ICI matrix and $\mathbf{w}[k]=\mathbf{F}\mathbf{n}[k]$ is the frequency-domain AWGN. Using (1), (4) becomes

$$\mathbf{z}[2k]=\frac{e^{j\theta[2k]}}{\sqrt{2}}\mathbf{\Phi}(\mathbf{\Lambda}_1\mathbf{x}_1[k]+\mathbf{\Lambda}_2\mathbf{x}_2[k])+\mathbf{w}[2k],$$
$$\mathbf{z}[2k+1]=\frac{e^{j\theta[2k+1]}}{\sqrt{2}}\mathbf{\Phi}(\mathbf{\Lambda}_2\mathbf{x}_1^*[k]-\mathbf{\Lambda}_1\mathbf{x}_2^*[k])+\mathbf{w}[2k+1]. \tag{5}$$

Since the residual CFO is unknown, the receiver cannot compensate for the joint effect of $e^{j\theta[k]}\mathbf{\Phi}$. Indeed, the receiver can compensate only for the PO introduced by $e^{j\theta[k]}\mathbf{\Phi}$, since the PO is contained inside the channel estimate. We assume that the receiver is able to perfectly compensate for the average PO $\varphi[k]=\theta[k]+\pi\varepsilon(1-1/N)$ within the OFDM block, where the term $\pi\varepsilon(1-1/N)$ is average PO of $\mathrm{diag}(\mathbf{\Phi})$. Therefore the residual PO is zero and, as in [10], does not increase with the block index k. In practice, $\varphi[k]$ should be affected by estimation error. Similarly, we assume perfect knowledge of the channel matrices $\{\mathbf{\Lambda}_t\}$ at the receiver. Under these hypotheses, the Alamouti decoding operation can be expressed as [1][2]

$$\hat{\mathbf{x}}_1[k]=\mathbf{\Lambda}_1^*\hat{\mathbf{z}}[2k]+\mathbf{\Lambda}_2\hat{\mathbf{z}}^*[2k+1],$$
$$\hat{\mathbf{x}}_2[k]=\mathbf{\Lambda}_2^*\hat{\mathbf{z}}[2k]-\mathbf{\Lambda}_1\hat{\mathbf{z}}^*[2k+1], \tag{6}$$

where $\hat{\mathbf{z}}[k]=e^{-j\varphi[k]}\mathbf{z}[k]$ represents the received signal after PO compensation. By (5) and (6), we obtain

$$\hat{\mathbf{x}}_1[k]=\frac{1}{\sqrt{2}}(\mathbf{\Lambda}_1^*\mathbf{M}\mathbf{\Lambda}_1+\mathbf{\Lambda}_2\mathbf{M}^*\mathbf{\Lambda}_2^*)\mathbf{x}_1[k]+\mathbf{\Lambda}_1^*\boldsymbol{\omega}[2k]$$
$$+\frac{1}{\sqrt{2}}(\mathbf{\Lambda}_1^*\mathbf{M}\mathbf{\Lambda}_2-\mathbf{\Lambda}_2\mathbf{M}^*\mathbf{\Lambda}_1^*)\mathbf{x}_2[k]+\mathbf{\Lambda}_2\boldsymbol{\omega}^*[2k+1],$$
$$\hat{\mathbf{x}}_2[k]=\frac{1}{\sqrt{2}}(\mathbf{\Lambda}_1\mathbf{M}^*\mathbf{\Lambda}_1^*+\mathbf{\Lambda}_2^*\mathbf{M}\mathbf{\Lambda}_2)\mathbf{x}_2[k]+\mathbf{\Lambda}_2^*\boldsymbol{\omega}[2k]$$
$$+\frac{1}{\sqrt{2}}(\mathbf{\Lambda}_2^*\mathbf{M}\mathbf{\Lambda}_1-\mathbf{\Lambda}_1\mathbf{M}^*\mathbf{\Lambda}_2^*)\mathbf{x}_1[k]+\mathbf{\Lambda}_1\boldsymbol{\omega}^*[2k+1], \tag{7}$$

where $\mathbf{M}=\exp(-j\pi\varepsilon(N-1)/N)\mathbf{\Phi}$ is the PO-compensated ICI generating matrix, with elements expressed by [4]

$$M_{n,m}=[\mathbf{M}]_{n,m}=\frac{\sin(\pi(m-n+\varepsilon))}{N\sin(\pi(m-n+\varepsilon)/N)}e^{j\pi\frac{N-1}{N}(m-n)}, \tag{8}$$

and $\boldsymbol{\omega}[k]=e^{-j\varphi[k]}\mathbf{w}[k]$ is the rotated AWGN. In (7), it is clearly shown that the estimated signal vectors suffer from ICI, since the matrices $\mathbf{\Lambda}_1^*\mathbf{M}\mathbf{\Lambda}_1+\mathbf{\Lambda}_2\mathbf{M}^*\mathbf{\Lambda}_2^*$ and $\mathbf{\Lambda}_1\mathbf{M}^*\mathbf{\Lambda}_1^*+\mathbf{\Lambda}_2^*\mathbf{M}\mathbf{\Lambda}_2$ are not diagonal. In addition, (7) highlights the presence of joint interference between $\mathbf{x}_1[k]$ and $\mathbf{x}_2[k]$, because the matrices $\mathbf{\Lambda}_1^*\mathbf{M}\mathbf{\Lambda}_2-\mathbf{\Lambda}_2\mathbf{M}^*\mathbf{\Lambda}_1^*$ and $\mathbf{\Lambda}_2^*\mathbf{M}\mathbf{\Lambda}_1-\mathbf{\Lambda}_1\mathbf{M}^*\mathbf{\Lambda}_2^*$ are not zero.

III. Conditional SINR Analysis

We evaluate the SINR on the n th subcarrier, conditioned on $\lambda_{t,n}=[\mathbf{\Lambda}_t]_{n,n}$ $t=1,2$. The conditional SINR will be used in Section IV in order to derive the error probability. For the sake of conciseness, we focus on $\mathbf{x}_1[k]$; similar results are valid for $\mathbf{x}_2[k]$. For the n th subcarrier, we obtain

$$\hat{x}_1[k,n]=u_n x_1[k,n]+I_1[k,n]+I_2[k,n]+\tilde{\omega}[k,n], \tag{9}$$

$$u_n=\frac{M_0}{\sqrt{2}}(|\lambda_{1,n}|^2+|\lambda_{2,n}|^2), \tag{10}$$

$$I_1[k,n]=\sum_{m=0,m\neq n}^{N-1}i_{1,m,n}x_1[k,m], \tag{11}$$

$$i_{1,m,n} = \frac{M_{n,m}\lambda_{1,n}^{*}\lambda_{1,m} + M_{n,m}^{*}\lambda_{2,n}\lambda_{2,m}^{*}}{\sqrt{2}}, \quad (12)$$

$$I_2[k,n] = \sum_{m=0}^{N-1} i_{2,m,n} x_2[k,m], \quad (13)$$

$$i_{2,m,n} = \frac{M_{n,m}\lambda_{1,n}^{*}\lambda_{2,m} - M_{n,m}^{*}\lambda_{2,n}\lambda_{1,m}^{*}}{\sqrt{2}}, \quad (14)$$

$$\tilde{\omega}[k,n] = \lambda_{1,n}^{*}\omega[2k,n] + \lambda_{2,n}\omega^{*}[2k+1,n], \quad (15)$$

where $M_0 = M_{n,n}$, $\lambda_{t,n} = [\mathbf{\Lambda}_t]_{n,n}$, and $\omega[k,n] = [\boldsymbol{\omega}[k]]_n$. Note that in (9) there are two interference terms: $I_1[k,n]$, defined by (11)-(12), stands for the CFO-generated ICI due to the useful vector $\mathbf{x}_1[k]$, while $I_2[k,n]$, defined by (13)-(14) contains the CFO-generated ICI caused by the "interfering" vector $\mathbf{x}_2[k]$.

The conditional SINR, which is useful for the derivation of the error probability, is obtained by assuming $\lambda_{1,n}$ and $\lambda_{2,n}$ in (9)-(15) as fixed and by averaging over the joint probability density function (pdf) of the other random variables. Since the frequency-domain channel values $\{\lambda_{1,n}, \lambda_{1,m}, \lambda_{2,n}, \lambda_{2,m}\}$ are correlated jointly Gaussian, we can express [4][10]

$$\lambda_{t,m} = C_{m,n}\lambda_{t,n} + v_{t,m,n}, \qquad t = 1,2, \quad (16)$$

with $C_{m,n} = E\{\lambda_{t,m}\lambda_{t,n}^{*}\} = [\mathbf{C}]_{m,n}$, where the frequency-domain correlation matrix is $\mathbf{C} = \mathbf{F}\,\mathrm{Diag}(\sigma_0^2, ..., \sigma_{L-1}^2, 0, ..., 0)\mathbf{F}^H$, and $v_{t,m,n}$ is Gaussian with zero mean and variance $1 - |C_{m,n}|^2$. Note that $C_{n,n} = E\{|\lambda_{t,n}|^2\} = 1$. It can be shown that $v_{1,m,n}$ and $v_{2,m,n}$ in (16) are correlated with each other, but are uncorrelated (and hence independent) from $\{\lambda_{1,n}, \lambda_{2,n}\}$. By inserting (16) into (12), we obtain

$$i_{1,m,n} = \frac{M_{n,m}C_{m,n}|\lambda_{1,n}|^2 + M_{n,m}^{*}C_{m,n}^{*}|\lambda_{2,n}|^2}{\sqrt{2}} + \frac{M_{n,m}v_{1,m,n}\lambda_{1,n}^{*} + M_{n,m}^{*}v_{2,m,n}^{*}\lambda_{2,n}}{\sqrt{2}}. \quad (17)$$

Therefore, when conditioned on $\lambda_{1,n}$ and $\lambda_{2,n}$, $i_{1,m,n}$ is a random variable with mean and variance expressed by

$$\eta_{1,m,n} = \frac{M_{n,m}C_{m,n}|\lambda_{1,n}|^2 + M_{n,m}^{*}C_{m,n}^{*}|\lambda_{2,n}|^2}{\sqrt{2}} \quad (18)$$

$$\sigma_{1,m,n}^2 = \frac{1}{2}|M_{n,m}|^2(1 - |C_{m,n}|^2)(|\lambda_{1,n}|^2 + |\lambda_{2,n}|^2). \quad (19)$$

Similarly, by inserting (16) into (14), it is easy to show that $i_{2,m,n}$ is a random variable with conditional mean

$$\eta_{2,m,n} = j\sqrt{2}\,\mathrm{Im}\{M_{n,m}C_{m,n}\}\lambda_{1,n}^{*}\lambda_{2,n} \quad (20)$$

and conditional variance

$$\sigma_{2,m,n}^2 = \sigma_{1,m,n}^2. \quad (21)$$

Therefore, taking into account that $I_1[k,n]$ and $I_2[k,n]$ defined in (11) and (13) are uncorrelated, from (9) the conditional SINR on the nth subcarrier is expressed as

$$\gamma_n(\lambda_{1,n}, \lambda_{2,n}) = \frac{u_n^2\sigma_x^2}{(P_{1,n} + P_{2,n})\sigma_x^2 + \sigma_{\tilde{\omega}_n}^2}, \quad (22)$$

$$P_{t,n} = \sum_{m=0, m\neq n}^{N-1}(|\eta_{t,m,n}|^2 + \sigma_{t,m,n}^2), \quad (23)$$

$$\sigma_{\tilde{\omega}_n}^2 = (|\lambda_{1,n}|^2 + |\lambda_{2,n}|^2)\sigma_w^2. \quad (24)$$

By inserting (18)-(21) into (23), $P_{1,n} + P_{2,n}$ in (22) becomes

$$P_{1,n} + P_{2,n} = \frac{1}{2}\left(\sum_{m=0, m\neq n}^{N-1}|M_{n,m}C_{m,n}|^2\right)G_n^2(\lambda_{1,n}, \lambda_{2,n}) + \left[\sum_{m=0, m\neq n}^{N-1}|M_{n,m}|^2(1 - |C_{m,n}|^2)\right]G_n(\lambda_{1,n}, \lambda_{2,n}), \quad (25)$$

where the conditional gain $G_n(\lambda_{1,n}, \lambda_{2,n})$ is expressed by

$$G_n(\lambda_{1,n}, \lambda_{2,n}) = |\lambda_{1,n}|^2 + |\lambda_{2,n}|^2. \quad (26)$$

Hence, by (10), (24)-(26), the conditional SINR (22) becomes

$$\gamma_n(\lambda_{1,n}, \lambda_{2,n}) = \frac{M_0^2\,G_n(\lambda_{1,n}, \lambda_{2,n})}{A\,G_n(\lambda_{1,n}, \lambda_{2,n}) + 2B + 2\sigma_w^2/\sigma_x^2}, \quad (27)$$

$$A = \sum_{m=0, m\neq n}^{N-1}|M_{n,m}C_{m,n}|^2, \quad B = \sum_{m=0, m\neq n}^{N-1}|M_{n,m}|^2(1 - |C_{m,n}|^2). \quad (28)$$

Note that both A and B do not depend on the subcarrier index n, since both $\mathbf{M}$ and $\mathbf{C}$ are circulant. Equation (27) clearly shows that the conditional SINR $\gamma_n(\lambda_{1,n}, \lambda_{2,n})$ contains two types of (normalized) ICI power terms: the ICI power $A\,G_n(\lambda_{1,n}, \lambda_{2,n})$, which is proportional to the useful power $M_0^2 G_n(\lambda_{1,n}, \lambda_{2,n})$, and the ICI power $2B$, which behaves as an additional noise whose power does not depend on the conditional gain $G_n(\lambda_{1,n}, \lambda_{2,n})$ in (26).

We now discuss the extreme cases when either $A = 0$ or $B = 0$ (they cannot be both zero, except in the trivial no-CFO case, since from (8) $\varepsilon = 0$ implies $M_{n,m} = 0$ for $m \neq n$). When $A = 0$, the conditional SINR (27) is proportional to the conditional gain $G_n(\lambda_{1,n}, \lambda_{2,n})$. Therefore, the SINR would be equivalent, apart for a constant scale factor, to the classical case for MRC combining of Rayleigh-fading diversity branches with AWGN, in the absence of interference. The condition $A = 0$ happens when $C_{m,n} = 0$ for all $m \neq n$, i.e., the subcarrier channel gains $\{\lambda_{1,n}\}$ (and $\{\lambda_{2,n}\}$) are uncorrelated. In this first case, from (28), $B = 1 - M_0^2$, and therefore the conditional SINR (27) coincides with the conditional SINR in Equation 23 of [8], obtained assuming spatially and frequency uncorrelated channel gains. However, we remark that N subcarrier gains $\{\lambda_{1,n}\}$ (or $\{\lambda_{2,n}\}$) can be frequency uncorrelated only for highly frequency-selective channels, when the power-delay profile of the channel has a sufficient number of channel taps, i.e., $L \geq N$. This is unusual for OFDM systems, where the CP length is often a small fraction of the number of subcarriers (typically, $L \leq N/4$). On the other hand, $B = 0$ only in flat-fading channels. Indeed, in flat-fading channels, $\sigma_l^2 = 0$ for $l \neq 0$, which implies $C_{m,n} = 1$ for all couples (m,n). In this second case, from (28), $A = 1 - M_0^2$, and (27) boils down to Equation 33 of [9]. However, we remind that, in general frequency-selective channels, both $A \neq 0$ and $B \neq 0$.

IV. Error Probability Analysis

Herein we derive the bit error probability, assuming QPSK with Gray coding. As in [4], we use the direct method, which leverages on the conditional SINR. Specifically, the bit error probability $P_{\mathrm{BE},n}$ on the nth subcarrier is obtained as

$$P_{\mathrm{BE},n} = \int_0^{+\infty} P_{\mathrm{BE}}(\gamma_n) f_\Gamma(\gamma_n)\,d\gamma_n, \quad (29)$$

where $P_{\mathrm{BE}}(\gamma_n)$ is the bit error probability conditioned on the SINR per symbol γ_n, which is expressed by (27), and $f_\Gamma(\gamma_n)$ is the pdf of γ_n. In order to obtain $P_{\mathrm{BE}}(\gamma_n)$, we approximate the interference term $I_1[k,n]+I_2[k,n]$ in (9) as Gaussian, with power expressed by (25). We presume that this approximation will be quite accurate for STBC-OFDM, since a similar approximation has been shown to be very accurate for single-antenna OFDM systems [4]. Therefore, for QPSK we have

$$P_{\mathrm{BE}}(\gamma_n)=Q\left(\sqrt{\gamma_n}\right), \tag{30}$$

where $Q(\cdot)$ is the Gaussian Q-function. The approach would be identical for general M-PSK and M-QAM, since the bit error probability with Gray coding can always be expressed as the weighted sum of few Q-functions. Since the conditional SINR γ_n in (27) depends on G_n, $P_{\mathrm{BE},n}$ in (29) becomes

$$P_{\mathrm{BE},n}=\int_0^{+\infty} P_{\mathrm{BE}}(\gamma_n(G_n))f_{\mathrm{G}}(G_n)dG_n\ , \tag{31}$$

where $f_{\mathrm{G}}(G_n)$ is the pdf of G_n.

A. Spatially Uncorrelated Channels

When $\rho=0$, the two antenna gains $|\lambda_{1,n}|^2$ and $|\lambda_{2,n}|^2$ experience spatially uncorrelated Rayleigh fading. In this case, G_n in (26) is a chi-square distributed random variable with four degrees of freedom, with pdf expressed by [14]

$$f_{\mathrm{G}}(G)=Ge^{-G}\,\mathrm{u}(G), \tag{32}$$

where $\mathrm{u}(\cdot)$ represents the unit step function. Therefore, using (30) and (32), the bit error probability (31) can be expressed as

$$P_{\mathrm{BE},n}=\int_0^{+\infty} Q\left(\sqrt{\gamma_n(G_n)}\right)G_n e^{-G_n}dG_n\ , \tag{33}$$

where the relation between γ_n and G_n is expressed by (27). To solve (33), we use a series expansion of the error function (see Equation 7.1.5 of [15]), which leads to

$$Q\left(\sqrt{\gamma_n}\right)=\frac{1}{2}-\frac{1}{\sqrt{2\pi}}\sum_{k=0}^{+\infty}(-1)^k\frac{\gamma_n^{k+1/2}}{2^k\,k!(2k+1)}\,. \tag{34}$$

By inserting (27) and (34) into (33), we obtain

$$P_{\mathrm{BE},n}=\frac{1}{2}-\frac{1}{\sqrt{2\pi}}\sum_{k=0}^{+\infty}\frac{(-1)^k\beta^{k+1/2}}{2^k\,k!(2k+1)}\int_0^{+\infty}\frac{G_n^{k+3/2}}{(\alpha G_n+1)^{k+1/2}}e^{-G_n}dG_n\ , \tag{35}$$

where we have defined

$$\beta=\frac{M_0^2}{2B+2\sigma_w^2/\sigma_x^2}\,,\qquad \alpha=\frac{A}{2B+2\sigma_w^2/\sigma_x^2}\,. \tag{36}$$

The integral in (35) can be solved using Equation 3.383.5 of [16], which yields

$$P_{\mathrm{BE},n}=\frac{1}{2}-\sqrt{\frac{\beta}{32\alpha^5}}\sum_{k=0}^{+\infty}q_k\left(\frac{\beta}{\alpha}\right)^k U\left(k+\frac{5}{2}\,,3\,,\frac{1}{\alpha}\right), \tag{37}$$

$$q_k=\left(-\frac{1}{2}\right)^k(2k+3)\frac{(2k-1)!!}{(2k)!!}\,, \tag{38}$$

where !! denotes the double factorial and $U(a,b,z)$ is the Tricomi confluent hypergeometric function defined in Equation 13.2.5 of [15]. The summation in (37) can be easily calculated using standard software packages, such as Mathematica®, and by exploiting the recurrence relations

$$U\left(k+\frac{5}{2},3,z\right)=\frac{(2k+z)U\left(k+\frac{3}{2},3,z\right)-U\left(k+\frac{1}{2},3,z\right)}{k^2+k-3/4}, \tag{39}$$

which is obtained from Equation 13.4.16 of [15], and

$$q_{k+1}=-\frac{1}{2}\left(1+\frac{2k-1}{(2k+2)(2k+3)}\right)q_k\,. \tag{40}$$

Note that the bit error probability in (37) does not depend on the subcarrier index n, since the signal power on the N subcarriers is the same. In the presence of guard subcarriers, the error probability would be different from subcarrier to subcarrier, since the edge subcarriers receive less ICI than the middle subcarriers [4]. The result in (37) is compliant with Equation 19 of [17] for the symbol error probability of STBC with a nonlinear power amplifier, in the absence of CFO.

B. Spatially Correlated Channels

When $\rho\neq 0$, the two antenna gains $|\lambda_{1,n}|^2$ and $|\lambda_{2,n}|^2$ are correlated, and the pdf of G_n in (26) can be expressed as [14]

$$f_{\mathrm{G}}(G)=\frac{1}{2\rho}\left(e^{-\frac{G}{1+\rho}}-e^{-\frac{G}{1-\rho}}\right)\mathrm{u}(G)\,. \tag{41}$$

In this case, the bit error probability (31) is the difference of two integrals, which can be solved using the same procedure employed for spatially uncorrelated channels. The final result is

$$P_{\mathrm{BE},n}=\sqrt{\frac{\beta}{32\rho^2\alpha^3}}\sum_{k=0}^{+\infty}\left(-\frac{1}{2}\right)^k\frac{(2k-1)!!}{(2k)!!}\left(\frac{\beta}{\alpha}\right)^k \times\left[U\left(k+\frac{3}{2}\,,2\,,\frac{1}{(1-\rho)\alpha}\right)-U\left(k+\frac{3}{2}\,,2\,,\frac{1}{(1+\rho)\alpha}\right)\right]. \tag{42}$$

V. Simulation Results

We consider an STBC-OFDM system with $N=64$ subcarriers, CP length $L=16$, and QPSK with Gray coding. We assume a truncated-exponential power-delay profile $\sigma_l^2=\Gamma e^{-\delta l}$, for $l=0,\ldots,L-1$. $\delta\geq 0$ is the decay factor that controls the amount of frequency selectivity, and Γ is a normalization factor. The bit-error rate (BER) is evaluated as a function of the SNR per bit, defined[2] as $E_b/N_0=\sigma_x^2/(4\sigma_w^2)$.

Figure 1 shows the BER for different values of the normalized CFO ε, when the decay factor is $\delta=0.2$ and the spatial correlation coefficient is $\rho=0$. The proposed theoretical approach is able to capture both the twofold diversity gain at medium SNR and the error floor at high SNR, for any value of ε. A slight mismatch between theory and simulations is recorded at high CFO values, e.g., $\varepsilon=0.15$, where the Gaussian approximation of the ICI is less accurate.

Figure 2 illustrates the BER for different values of the decay factor δ, when the normalized CFO is $\varepsilon=0.07$ and the spatial correlation coefficient is $\rho=0$. Also in this case the proposed analysis is quite accurate for any value of δ. From Figure 2, the error floor increases for lower values of the decay δ, which correspond to an increased frequency selectivity. Specifically, a great decay, e.g., $\delta=2$, corresponds to almost-flat fading channels, where the error floor is significantly reduced with respect to highly frequency-selective channels [9]. In addition, the approximation of [8], valid for independent subcarrier gains, overestimates the error floor even for $\delta=0$ (uniform power-delay profile and high frequency selectivity).

Figure 3 displays the effect of the spatial correlation coefficient ρ when $\varepsilon=0.07$ and $\delta=0.2$. The proposed analysis, which agrees with the simulations, proves that an augmented level of spatial correlation produces an increased error floor.

[2] The factor four takes into account both the presence of two transmit antennas and of two bits per symbol in QPSK.

VI. CONCLUSIONS

We have derived new closed-form solutions for the bit error probability of Alamouti-coded OFDM systems subject to both CFO and frequency-selective Rayleigh fading. Unlike the previous literature, our theoretical results are able to describe the effect of both the frequency-domain channel correlation and the spatial channel correlation coefficient. Simulation results have confirmed that the proposed analysis is very accurate for a wide range of parameters.

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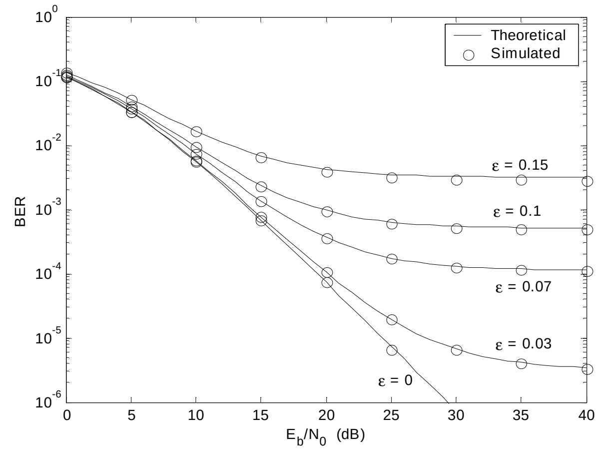

Figure 1. BER for different CFO values ($\delta = 0.2$, $\rho = 0$).

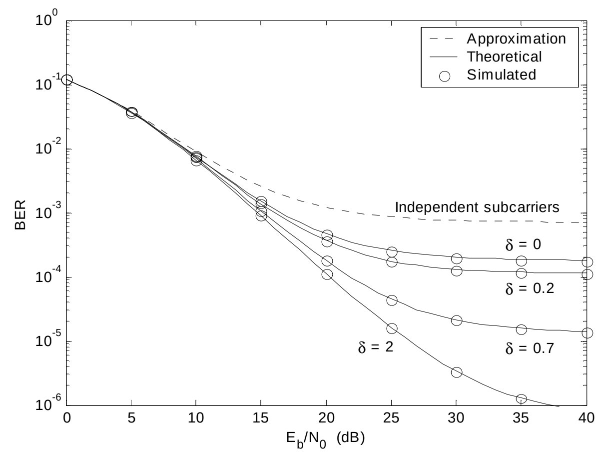

Figure 2. BER for different power-delay profiles ($\varepsilon = 0.07$, $\rho = 0$).

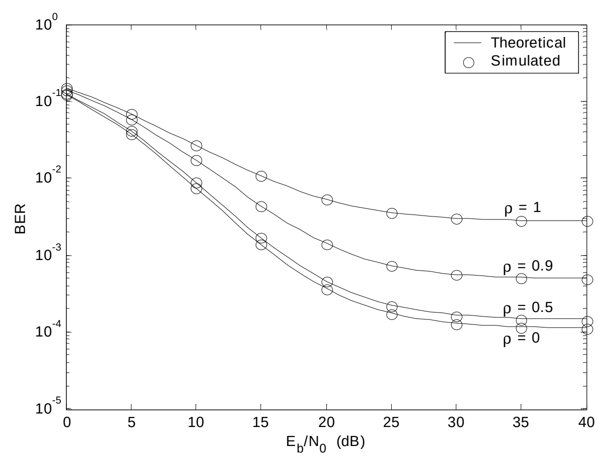

Figure 3. BER for different spatial correlation values ($\varepsilon = 0.07$, $\delta = 0.2$).