Quasi-Orthogonal Space-Time Block Codes for Two Transmit Antennas and Three Time Slots

Zhongding Lei, *Senior Member, IEEE,* Chau Yuen, *Member, IEEE,* and Francois P.S. Chin, *Senior Member, IEEE*

***Abstract*—In this paper, a class of quasi-orthogonal space-time block codes (Q-STBC) is proposed for systems with two transmit antennas and three time slots, where the Alamouti code is not applicable due to the odd time slots. The proposed Q-STBC codes achieve rate one and full diversity with low complexity maximum likelihood decoding. The Q-STBC design also shows excellent properties in other practical aspects, such as the compatibility with the single antenna transmission mode, low power fluctuation, and low receiver decoding and transmitter encoding complexity.**

***Index Terms*—Quasi-orthogonal space-time block codes (Q-STBC); STBC; spatial diversity; fading.**

I. Introduction

SPACE-TIME block coding (STBC) is an efficient transmit diversity scheme to combat detrimental effects of wireless fading channels because of its simple decoding algorithm accomplishing full diversity at a radio receiver. Alamouti code is an elegant and seminal STBC design for a two-transmit-antenna system [1]. It achieves rate one, full-diversity transmission using two time slots for signals with complex constellations, which are employed in most current commercial wireless systems. The orthogonal code design from Tarokh et al is a generalization of Alamouti code for systems with an arbitrary number of transmit antennas [2][3]. It has been proved however, the orthogonal design for complex signals with linear decoding complexity achieving rate-one and full-diversity transmission is not available for the number of antennas more than two [2]. The system with a higher number of antennas has to either suffer from rate loss or put up with more decoding complexity. Quasi-orthogonal STBC (Q-STBC) codes are typically designed for more than two antenna systems with increased, but not exponentially, decoding complexity [4]-[10].

The 3rd Generation Partnership Project (3GPP) is a standard body standardizing cellular systems worldwide with most telecommunication manufacturers / vendors / operators involved. Recently, 3GPP has been working on the next generation wireless system (4G) under the project LTE-Advanced building upon the legacy standard LTE release 8 [11]. With the requirement of two transmit antennas imposed on the user equipment (UE, i.e. mobile station) in LTE-Advanced systems, STBC is one of the most popular candidates for the uplink transmission diversity schemes [12]. However, it has been identified afterwards that the LTE frame structure, which has been fixed and not likely to be changed makes it awkward to implement an orthogonal STBC design. The time slots (or symbols per slot in 3GPP language) in the frame structure for data transmission is not guaranteed to be an even number. In many cases, there are three time slots available for data transmission instead of two time slots as required in the orthogonal STBC for a two antenna UE [11][13]. This has brought up an interesting STBC design problem to achieve rate-one transmission with three time slots and two antennas. To the best of our knowledge, there is no such STBC code design available, including Q-STBC in the literature. In [13], an alternative hybrid scheme with two-time-slot STBC followed by one-time-slot repetition transmission has been proposed. The hybrid transmission scheme achieves reasonable performance with simple linear decoding at the receiver. However, it is not a full-diversity transmission scheme overall as the one-slot repetition transmission does not provide any transmit diversity gain. Inevitably, it suffers from significant performance loss as opposed to the orthogonal STBC transmission.

In this paper, we propose a set of Q-STBC codes for the three-time-slot and two-antenna transmission. The proposed codes can achieve rate-one and full diversity transmission and thus have much better performance than the hybrid scheme in [13]. An exemplary of such code design has been illustrated in our previous work in [14]. In this paper, we will provide the general set of the designed Q-STBC with complete analysis and proofs. The paper is organized as follows. The design problem is formulated in Section II. The proposed design is elaborated in Section III. The decoding complexity is analyzed in Section IV, followed by the simulation results in Section V. The paper is concluded in Section VI.

II. Problem Formulation

A. The System Model

We consider a communication system where the transmitter is equipped with two antennas and the receiver is equipped with M antennas as illustrated in Fig. 1. At each time slot t, signals x_t and y_t are transmitted simultaneously from antennas 1 and 2 respectively. Assuming the wireless channel is quasi static, i.e. constant over a frame of length L and varied from one frame to another. The random path gains are $\mathbf{h}^m = \begin{bmatrix} h_1^m & h_2^m \end{bmatrix}^{\mathrm{T}}$ $(m = 1 \ldots M)$, where h_1^m and h_2^m correspond to the gains from the transmit antennas 1 and 2,

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Z. Lei and F. Chin are with the Institute for Infocomm Research, Singapore (e-mail: {leizd, chinfrancois}@i2r.a-star.edu.sg).

C. Yuen is with Singapore University of Technology and Design (e-mail: yuenchau@sutd.edu.sg).

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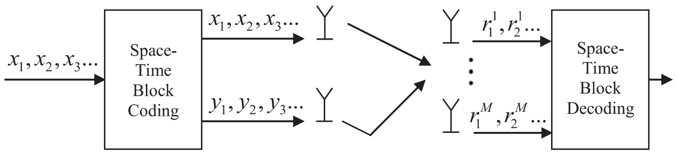

Fig. 1. The STBC system model with two transmit antennas.

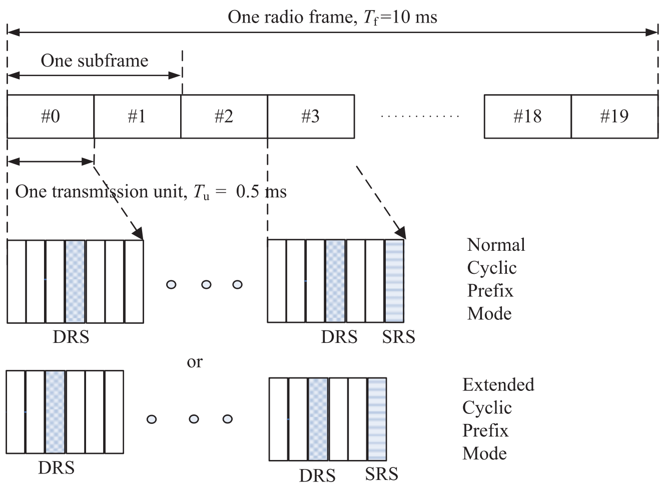

Fig. 2. Frame structure of LTE-Advanced uplink.

respectively, to the receive antenna m. At the receiver, the signal received by antenna m at time t is given by

$$r_t^m = h_1^m x_t + h_2^m y_t + n_t^m \tag{1}$$

where n_t^m are the samples of independent complex random Gaussian noise on the m^{th} antenna with zero-mean. Assuming perfect channel knowledge, the receiver computes the maximum likelihood decision statistics

$$\sum_{t=1}^{L} \sum_{m=1}^{M} |r_t^m - (h_1^m x_t + h_2^m y_t)|^2 \tag{2}$$

over all possible codewords $x_1 y_1 x_2 y_2 \ldots x_L y_L$ to decide in favor of the codeword minimizing (2).

B. LTE-Advanced Frame Structure

Fig. 2 illustrates the radio frame of uplink transmission in LTE-Advanced systems. Each radio frame is 10 ms long with 10 sub-frames or 20 transmission units[1]. Each transmission unit lasts 0.5 ms consisting of either 7 symbols in the normal cyclic-prefix mode or 6 symbols in the extended cyclic-prefix mode. One or two of the 7 or 6 symbols per transmission unit will be taken up by Demodulation Reference Signal (DRS) and Sounding Reference Signal (SRS) as shown (shaded symbols) in the figure. The remaining symbols will be used for data transmission. From the figure it can be seen that each transmission unit always, be it in the normal cyclic-prefix mode or the extended cyclic-prefix mode, contains odd number of (three symbols) time duration (or "slot" in STBC context).

[1]The transmission unit refers to the term "slot" in 3GPP. We use the term "transmission unit" to avoid confusion of the "slot" concept in STBC

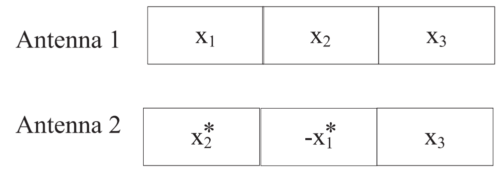

Fig. 3. A hybrid three-slot transmission scheme.

Alamouti code is designed for two transmission slots for wireless systems equipped with two transmit antennas. Unfortunately, they are not applicable to the systems with time slot restriction as described above. To achieve rate-one transmission, an alternative hybrid scheme has been proposed in [13], which is exemplified in Fig. 3 with three transmission time slots and two antennas. During the first two time slots, signals are transmitted following exactly the Alamouti code, whereas in the 3rd time slot, the signal transmitted on the 2nd antenna is a simple repetition of that on the 1st antenna. The hybrid transmission scheme achieves reasonable performance with simple linear decoding at the receiver. However, it is not a full-diversity transmission scheme overall as the one-slot repetition transmission does not provide any transmit diversity. Inevitably, it suffers from significant performance loss as opposed to orthogonal STBC transmission. In this paper, we will design a class of Q-STBC codes achieving rate-one and full-diversity transmission.

C. STBC Design Criteria

In the proposed STBC design for the three-time slot transmission, the following desirable criteria/conditions in real implementation are considered:

1) Rate one (three symbols transmitted in three time slots)
2) Full diversity (order $2M$ with two transmit antennas and M receive antennas)
3) Backward compatibility (with single antenna mode)
4) Receiver complexity
5) Transmitter complexity
6) Power fluctuation

The 1st criterion ensures that there will be no data rate loss employing STBC. We will meet this criterion by transmitting three data symbols during the three slot times. The 2nd criterion is to guarantee the superior performance of the designed STBC with the diversity order of $2M$ as opposed to the hybrid scheme, where one of the data slot transmission only achieves a diversity order of M. We will meet this criterion by ensuring full rank (rank two) of the difference matrix for any pair of distinct code sequences from the designed STBC codes [15][3]. We will elaborate it later.

The 3rd criterion refers to the backward compatibility to the single antenna system. This is not always a requirement in many orthogonal/non-orthogonal STBC designs, such as [3], [4] or [9]. However, it is a desired feature and could be a decisive factor for a STBC code design to be adopted in a practical wireless system. In many commercial systems such as 3GPP, WiMAX, WLAN etc., STBC transmission is only one of the transmission modes at the transmitter. The transmitter usually transmits in one of the multiple transmission modes at a time adapting to channel conditions, the user QoS requirement, and the resource availability etc. The backward compatibility ensures a smooth transition, in terms of overhead signaling and implementation complexity, between STBC modes and the single antenna mode. In our STBC design, we will impose that all the transmission from the 1st antenna will be exactly the same as that in the single antenna system.

The 4th criterion is addressing the issue of the decoding complexity at the receiver side [15]. As the full-rate full-diversity transmission with linear decoding complexity is proved to be impossible [3], we will design Q-STBC codes for low decoding complexity implementation of maximum likelihood (ML) detection (but has to be higher than linear complexity as in orthogonal codes cases).

The 5th criterion is addressing the encoding complexity at the transmitter side. It is preferable to have as low complexity as possible. Therefore, we will limit our Q-STBC transmission on the 2nd antenna to be a linear combination of the transmission (or the conjugate version) on the 1st antenna.

The 6th criterion is imposed to minimize the signal power fluctuation at the transmitter side. Similar to the 3rd criterion, this is a concern when come to the implementation of transmitters, especially for uplink. A less fluctuation of signal power will have smaller dynamic range and less demanding on the high linearity and costly analogue radio frequency components, such as power amplifiers [16]. Therefore, we will ensure the average transmission power of STBC is fixed during each time slot.

D. Problem Formulation

Fig. 4 shows a schematic drawing of the STBC design. Three data symbols are transmitted consecutively during the three time slots, i.e., x_1, x_2, x_3 (1st criterion above) and in the same format as in the case of single transmit antenna to keep the backward compatibility (3rd criterion). On the 1st antenna, the data symbols are transmitted with fixed power (6th criterion) or $E(|x_1|^2) = E(|x_2|^2) = E(|x_3|^2)$. On the 2nd antenna, the data symbols y_1, y_2, y_3 are transmitted consecutively in the three time slots and $E(|y_1|^2) = E(|y_2|^2) = E(|y_3|^2)$. To keep a linear complexity at the transmitter side (criterion 5), we restrict the data symbols y_1, y_2, y_3 linear combinations of the transmitted symbols on the 1st antenna, which can be represented mathematically as

$$\mathbf{y} = \mathbf{A} \cdot \mathbf{x}^* \tag{3}$$

where $\mathbf{y} = \begin{bmatrix} y_1 & y_2 & y_3 \end{bmatrix}^{\mathrm{T}}$, $\mathbf{x} = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^{\mathrm{T}}$, $\mathbf{A} = \begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 \end{bmatrix} = \begin{bmatrix} \mathbf{b}_1 & \mathbf{b}_2 & \mathbf{b}_3 \end{bmatrix}^{\mathrm{T}}$, $\mathbf{a}_t = \begin{bmatrix} a_{1t} & a_{2t} & a_{3t} \end{bmatrix}^{\mathrm{T}}$ is the t^{th} column of the 3-by-3 complex matrix $\mathbf{A}$ and $\mathbf{b}_n^{\mathrm{T}} = \begin{bmatrix} a_{n1} & a_{n2} & a_{n3} \end{bmatrix}$ is the *nth* row of $\mathbf{A}$ and the superscript "*" denotes the conjugate operation.

Stacking the transmitted symbols on two antennas in a matrix (*L=3*), the proposed STBC codes have a general form of

$$\mathbf{X} = \begin{bmatrix} \mathbf{x}^{\mathrm{T}} \\ \mathbf{y}^{\mathrm{T}} \end{bmatrix} = \begin{bmatrix} x_1 & x_2 & x_3 \\ \mathbf{b}_1^{\mathrm{T}}\mathbf{x}^* & \mathbf{b}_2^{\mathrm{T}}\mathbf{x}^* & \mathbf{b}_3^{\mathrm{T}}\mathbf{x}^* \end{bmatrix} \tag{4}$$

Now we show that the STBC code design problem can be transformed into the design of the complex matrix **A,** which

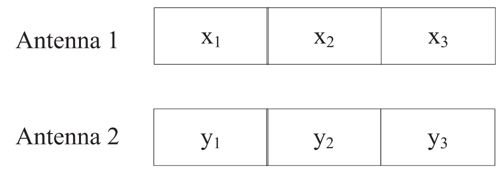

Fig. 4. Proposed three-slot transmission scheme.

is named design matrix throughout the paper. It is notable that the 1st and 3rd design criteria are met automatically as long as x_1, x_2, x_3 are representing three transmitted data symbols. To meet the full diversity criterion (2nd criterion), the codeword difference matrix $\tilde{\mathbf{X}} = \mathbf{X}^{(1)} - \mathbf{X}^{(2)}$ should be full-rank (rank two) [15], or equivalently,

$$\det(\tilde{\mathbf{X}}\tilde{\mathbf{X}}^{\mathrm{H}}) = \det \begin{bmatrix} \tilde{\mathbf{x}}^{\mathrm{H}}\tilde{\mathbf{x}} & \tilde{\mathbf{y}}^{\mathrm{H}}\tilde{\mathbf{x}} \\ \tilde{\mathbf{x}}^{\mathrm{H}}\tilde{\mathbf{y}} & \tilde{\mathbf{y}}^{\mathrm{H}}\tilde{\mathbf{y}} \end{bmatrix} \neq 0 \tag{5}$$

where $\mathbf{X}^{(1)}$ and $\mathbf{X}^{(2)}$ are two distinct codeword matrixes constructed according to (4) with different data symbol vectors $\mathbf{x}^{(1)}$ and $\mathbf{x}^{(2)}$respectively, $\tilde{\mathbf{x}} = \mathbf{x}^{(1)} - \mathbf{x}^{(2)} \neq 0$, $\tilde{\mathbf{y}} = \mathbf{y}^{(1)} - \mathbf{y}^{(2)} = \mathbf{A} \cdot \tilde{\mathbf{x}}$ and det(.) denotes the determinant of a matrix.

The received signal at antenna m, $\mathbf{r}^m$, can be represented in the matrix form by

$$\mathbf{r}^m = \mathbf{X}^{\mathrm{T}} \cdot \mathbf{h}^m + \mathbf{n}^m \tag{6}$$

where $\mathbf{n}^m$ is the complex Gaussian noise vector. The ML decision metric (2) becomes

$$\sum_{\mathrm{m}=1}^{\mathrm{M}} \left| \mathbf{r}^m - \mathbf{X}^{\mathrm{T}} \cdot \mathbf{h}^m \right|^2 \tag{7}$$

As orthogonal STBC codes achieving ML detection with linear decoding complexity is not possible for three-time-slot transmission, we consider Q-STBC codes, where the ML decoding of the three transmitted symbols can be decoupled. It is notable that we also need to keep the first row of the code the same as in the single antenna mode to meet the backward compatible criterion. Our target is to decouple the decoding of one symbol from that of the other two symbols so that we need only do a joint ML decoding for two symbols, instead of three symbols. We will lower the ML decoding complexity by one exponential order. For example, the decoding searching set will be reduced by 4, 16 and 64 times for QPSK, 16-QAM and 64-QAM respectively.

In order to have the Q-STBC design with one-order lower ML sequence decoding complexity, we need the following Theorem.

Theorem I: *The ML sequence detection of STBC code* $\mathbf{X}$ *in* (2) *can be achieved with a lower complexity of joint decoding for* x_2, x_3 *and linear decoding for* x_1 *if and only if the following equations are fulfilled for the design matrix* ***A****,*

$$\begin{cases} a_{11}a_{12}^* + a_{21}a_{22}^* + a_{31}a_{32}^* = 0 \\ a_{11}a_{13}^* + a_{21}a_{23}^* + a_{31}a_{33}^* = 0 \\ a_{12} + a_{21} = 0 \\ a_{13} + a_{31} = 0 \end{cases} \tag{8}$$

The resulting STBC is Q-STBC.

Proof: The proof of the Theorem is given in Appendix A.

The constraints (8) are to ensure that the cost function of ML detection with respect to x_1, x_2, x_3 can be separated into two functions, one with the variable x_1 and the other with x_2, x_3. It is notable that the equation (8) can be adapted similarly to decouple x_2 and x_1, x_3 or decouple x_3 and x_1, x_2.

To meet the 6th criterion as discussed above, we have an additional set of equations to shape the design matrix and ensure the STBC transmission with fixed average power during each time slot. They are obtained by solving $E(\mathbf{y}_1^{\mathrm{H}}\mathbf{y}_1) = E(\mathbf{y}_2^{\mathrm{H}}\mathbf{y}_2) = E(\mathbf{y}_3^{\mathrm{H}}\mathbf{y}_3) = c$ as follows

$$\begin{cases} a_{11}a_{11}^* + a_{12}a_{12}^* + a_{13}a_{13}^* = c \\ a_{21}a_{21}^* + a_{22}a_{22}^* + a_{23}a_{23}^* = c \\ a_{31}a_{31}^* + a_{32}a_{32}^* + a_{33}a_{33}^* = c \end{cases} \quad (9)$$

where *c* is the average transmission power in one slot. Without loss of generality, we will set *c=1* in the remaining of the paper.

In a summary to our STBC design, our proposed Q-STBC codes take the form of (4) with parameters a_{nt} $(n, t = 1, 2, 3)$ being solutions to the equations (8) and (9), subject to the constraint in (5).

III. Proposed Q-STBC Codes

As formulated in last section, our proposed Q-STBC codes are the solutions to the equations (8) and (9) subject to the conditions (5). Unfortunately, it is extremely difficult, if not impossible to find the complete set of solutions. The condition (5) involves a fourth-order multivariate polynomial and is a non-convex function. Solving the multivariate quadratic equations (8) and (9) is an NP-hard problem in general. In the following, we will impose an additional set of conditions to the problem so as to find a subset of solutions analytically. It is notable that finding a subset of the solutions does not diminish the importance of the Q-STBC code design, as one single solution is sufficient to solve the practical problem.

In equations (8) and (9), there are seven equations but nine unknown complex variables (18 real variables). It is an underdetermined system of equations. To obtain the potential solutions, we have added in two more equations to further limit our solutions with the design matrices having equal-amplitude diagonal elements, i.e.

$$a_{11}a_{11}^* = a_{22}a_{22}^* = a_{33}a_{33}^* \quad (10)$$

From (10) and (9), it immediately follows

$$\begin{cases} a_{12}a_{12}^* = a_{32}a_{32}^* \\ a_{13}a_{13}^* = a_{23}a_{23}^* \end{cases} \quad (11)$$

Substituting (10) and (11) in (9), it can be seen that (9) reduces to one equation.

To solve the equations in (8) and (9), we will need the following Lemmas.

Lemma I: *If matrix* $\mathbf{A}$ *is a solution to equations in* (8) *and* (9) *and subject to the constraint in* (3)*, the off-diagonal entries of the first row and the first column of* $\mathbf{A}$ *are non-zero.*

Proof: The proof of Lemma I is given in Appendix B.

Lemma II: *If matrix* $\mathbf{A}$ *is a solution to equations in* (8) *and* (9) *and the amplitudes of the* diagonal elements *of matrix* $\mathbf{A}$ *satisfy* (10)*, the diagonal amplitudes are non-zero.*

Proof: The proof of Lemma II is given in Appendix C.

In order to simplify and solve the equations in (8) and (9), it is beneficial to represent the complex numbers in polar forms. Assuming $a_{nt} = \rho_{nt} \cdot e^{j\phi_{nt}}(\rho_{nt} \geq 0$ and $n, t \in \{1, 2, 3\})$ and replacing a_{21} and a_{31} with $a_{21} = -a_{12}$ and $a_{31} = -a_{13}$ respectively, we may rewrite the first two equations in (8) as

$$\begin{aligned} &\rho_{11}\rho_{12} - \rho_{12}\rho_{22}e^{j(-\phi_{11}+2\phi_{12}-\phi_{22})} \\ &\qquad - \rho_{13}\rho_{32}e^{j(-\phi_{11}+\phi_{12}+\phi_{13}-\phi_{32})} = 0 \\ &\rho_{11}\rho_{13} - \rho_{12}\rho_{23}e^{j(-\phi_{11}+\phi_{12}+\phi_{13}-\phi_{23})} \\ &\qquad - \rho_{13}\rho_{33}e^{j(-\phi_{11}+2\phi_{13}-\phi_{33})} = 0 \end{aligned} \quad (12)$$

Further denoting

$$\begin{aligned} \theta_1 &= -\phi_{11} + \phi_{12} + \phi_{13} - \phi_{32} \\ \theta_2 &= -\phi_{11} + 2\phi_{12} - \phi_{22} \\ \theta_3 &= -\phi_{11} + \phi_{12} + \phi_{13} - \phi_{23} \\ \theta_4 &= -\phi_{11} + 2\phi_{13} - \phi_{33} \end{aligned} \quad (13)$$

and writing real parts and imaginary parts in (12) separately, we obtain

$$\begin{aligned} \rho_{11}\rho_{12} - \rho_{12}\rho_{22}\cos\theta_2 - \rho_{13}\rho_{32}\cos\theta_1 &= 0 \\ \rho_{12}\rho_{22}\sin\theta_2 + \rho_{13}\rho_{32}\sin\theta_1 &= 0 \\ \rho_{11}\rho_{13} - \rho_{12}\rho_{23}\cos\theta_3 - \rho_{13}\rho_{33}\cos\theta_4 &= 0 \\ \rho_{12}\rho_{23}\sin\theta_3 + \rho_{13}\rho_{33}\sin\theta_4 &= 0 \end{aligned} \quad (14)$$

It is noted that (10) and (11) can be written equivalently as

$$\begin{aligned} \rho_{11} &= \rho_{22} = \rho_{33} \\ \rho_{12} &= \rho_{32} \\ \rho_{13} &= \rho_{23} \end{aligned} \quad (15)$$

According to Lemma I and Lemma II, all terms in (15) are non-zero. Therefore, the equations in (14) can be simplified to

$$\begin{aligned} \rho_{11}(\cos\theta_2 - 1) + \rho_{13}\cos\theta_1 &= 0 \\ \rho_{11}\sin\theta_2 + \rho_{13}\sin\theta_1 &= 0 \\ \rho_{11}(\cos\theta_4 - 1) + \rho_{12}\cos\theta_3 &= 0 \\ \rho_{11}\sin\theta_4 + \rho_{12}\sin\theta_3 &= 0 \end{aligned} \quad (16)$$

For convenience, we rewrite (9) below as

$$\rho_{11}^2 + \rho_{12}^2 + \rho_{13}^2 = 1 \quad (17)$$

Examining (16) and (17), there are 7 unknown variables and 5 equations involving trigonometric functions. Fortunately, the equation set in (16) and (17) are resolvable. The solutions have been given below with detailed derivation attached in the Appendix D.

$$\begin{aligned} &\rho_{11} = \frac{1}{\sqrt{1+4(\cos^2\theta_1+\cos^2\theta_3)}} \\ &\rho_{12} = 2\rho_{11}\cos\theta_3 = \frac{2\cos\theta_3}{\sqrt{1+4(\cos^2\theta_1+\cos^2\theta_3)}} \\ &\rho_{13} = 2\rho_{11}\cos\theta_1 = \frac{2\cos\theta_1}{\sqrt{1+4(\cos^2\theta_1+\cos^2\theta_3)}} \\ &\theta_2 = 2\theta_1 - \pi + 2k\pi \ \ (k = 0, \pm 1 \ldots) \\ &\theta_4 = 2\theta_3 - \pi + 2k\pi \ \ (k = 0, \pm 1 \ldots) \\ &\theta_1 \ or \ \theta_3 \neq \tfrac{\pi}{2} + k\pi \ \ (k = 0, \pm 1 \ldots) \end{aligned} \quad (18)$$

It is noted that in the solution (18), we have two degrees of freedom, i.e., θ_1 and θ_3 are two free variables except the constraint that $\cos\theta_1 \neq 0$ and $\cos\theta_3 \neq 0$. Different choices of θ_1 and θ_3 do not have any difference in satisfying the equations in (14) and (15). They could be randomly selected or fixed to certain preferred values, e.g. to maximize the minimum determinant of (5) in order to maximize the coding gain. Once they are chosen, all amplitudes in the design matrix **A**, i.e. ρ_{nt} (*n, t = 1, 2, 3*) are uniquely determined.

As far as the phase values of the design matrix **A** are of concerned, they can be determined through (13) which defines their relationship to θ_i (*i = 1...4*). θ_2 and θ_4 can be determined through θ_1 and θ_3 in (18), thus θ_i (*i = 1...4*) are considered known variables in (13). The equations of (13) can be taken as a set of four linear equations with 7 unknown phases ϕ_{nt}. Rewriting (13) in a matrix form, we have

$$\mathbf{T}\cdot\boldsymbol{\varphi} = \boldsymbol{\vartheta} \tag{19}$$

where $\boldsymbol{\varphi} = [\phi_{11}\ \phi_{12}\ \phi_{13}\ \phi_{22}\ \phi_{23}\ \phi_{32}\ \phi_{33}]^{\mathrm{T}}$, $\boldsymbol{\vartheta} = [\theta_1\ \ \theta_2\ \ \theta_3\ \ \theta_4]^{\mathrm{T}}$ and

$$\mathbf{T} = \begin{bmatrix} -1 & 1 & 1 & 0 & 0 & -1 & 0 \\ -1 & 2 & 0 & -1 & 0 & 0 & 0 \\ -1 & 1 & 1 & 0 & -1 & 0 & 0 \\ -1 & 0 & 2 & 0 & 0 & 0 & -1 \end{bmatrix} \tag{20}$$

As the coefficient matrix **T** in (20) is full rank (rank 4), there are three degrees of freedom when determining the unknown phases $\boldsymbol{\varphi}$. We may randomly select three phases and resolve others with (19) or follow for example the following to obtain the phases

$$\boldsymbol{\varphi} = pinv(\mathbf{T})\cdot\boldsymbol{\theta} \tag{21}$$

where $pinv(\cdot)$ denotes the pseudo inverse function. It is notable that the selection of the phases $\boldsymbol{\varphi}$ might affect the diversity order of the designed STBC codes, the condition (5) must be verified to make sure the full diversity order achieved. We will discuss this further in the later part of this section.

In a summary, we may determine the design matrix ***A*** and the desired STBC codes in the following steps:

1) Select any pair of θ_1 and θ_3 that satisfy $\cos\theta_1 \neq 0$ and $\cos\theta_3 \neq 0$
2) Compute θ_2 and θ_4, and $\rho_{11}, \rho_{12}, \rho_{13}$ according to (18)
3) Form equation sets (19) or use (21) to determining $\boldsymbol{\varphi}$
4) Determining the remaining elements using $a_{21} = -a_{12}$, $a_{31} = -a_{13}$, and (15) and form the design matrix ***A***
5) Form the STBC codes according to (3) and (4) with the design matrix ***A*** in the last step

At last, we should verify the designed STBC codes satisfy (5) to make sure the full diversity order achieved. Otherwise, we need to repeat step 3 with different $\boldsymbol{\varphi}$.

It is remarkable that there are two degrees of freedom in choosing θ_1 and θ_3 in step 1 and three degrees of freedom in choosing $\boldsymbol{\varphi}$ in step 3. Any such choice of θ_1, θ_3 and $\boldsymbol{\varphi}$ may generate codes achieving full diversity as long as the codes fulfill the diversity constraint in (3). It is notable however codes based on different choices of θ_1, θ_3 and $\boldsymbol{\varphi}$ may have different coding gains. To further elaborate and gain insights on the relationship between the diversity/coding gain and the choices of these parameters, we consider $\theta_1 = \theta_3 = 0$ for example.

A. STBC Design for $\theta_1 = \theta_3 = 0$

When $\theta_1 = \theta_3 = 0$, it is easy to compute from (18) that $\theta_2 = \theta_4 = \pi$ (or $-\pi$). From (15) and (18), all the amplitudes of the design matrix **A** are uniquely determined, i.e.

$$\begin{aligned} \rho_{11} = \rho_{22} = \rho_{33} &= \tfrac{1}{3} \\ \rho_{12} = \rho_{13} = \rho_{21} = \rho_{23} = \rho_{31} = \rho_{32} &= \tfrac{2}{3} \end{aligned} \tag{22}$$

As to the phases of the design matrix **A,** the constraint equations (13) or (19) can be simplified to (without loss of generality, assuming $\phi_{11} = 0$)

$$\begin{aligned} &\phi_{23} = \phi_{32} = \phi_{12} + \phi_{13} \\ &\phi_{22} = 2\phi_{12} + \pi \\ &\phi_{33} = 2\phi_{13} + \pi \end{aligned} \tag{23}$$

It is obvious from (23) that there are two degrees (besides ϕ_{11}) of freedom, i.e. free variables ϕ_{12} and ϕ_{13}, in the determination of (phases of) **A**. The selection of ϕ_{12} and ϕ_{13} will have an impact to the achieved diversity order and coding gain. We will elaborate as follows.

With (22) and (23), the design matrix **A** has a general form of

$$\mathbf{A} = \frac{1}{3}\begin{bmatrix} 1 & 2e^{j\phi_{12}} & 2e^{j\phi_{13}} \\ -2e^{j\phi_{12}} & -e^{j2\phi_{12}} & 2e^{j(\phi_{12}+\phi_{13})} \\ -2e^{j\phi_{13}} & 2e^{j(\phi_{12}+\phi_{13})} & -e^{j2\phi_{13}} \end{bmatrix} \tag{24}$$

It is easy to verify that

$$\mathbf{A}\mathbf{A}^{\mathrm{H}} = \mathbf{I} \tag{25}$$

$$\tilde{\mathbf{y}}^{\mathrm{H}}\tilde{\mathbf{y}} = \tilde{\mathbf{x}}^{\mathrm{H}}\mathbf{A}\mathbf{A}^{\mathrm{H}}\tilde{\mathbf{x}} = \tilde{\mathbf{x}}^{\mathrm{H}}\tilde{\mathbf{x}} \tag{26}$$

Therefore, the determinant of the codeword difference matrix $\tilde{\mathbf{X}}$ in (5) can be simplified to

$$\det(\tilde{\mathbf{X}}\tilde{\mathbf{X}}^{\mathrm{H}}) = \left|\tilde{\mathbf{x}}^{\mathrm{H}}\tilde{\mathbf{x}}\right|^2 - \left|\tilde{\mathbf{x}}^{\mathrm{H}}\mathbf{A}\tilde{\mathbf{x}}^*\right|^2 \tag{27}$$

1) Diversity Gain: Equation (27) shows that the determinant of $\tilde{\mathbf{X}}$ with non-zero $\tilde{\mathbf{x}}$ will only be zero when $\left|\tilde{\mathbf{x}}^{\mathrm{H}}\mathbf{A}\tilde{\mathbf{x}}^*\right| = \left|\tilde{\mathbf{x}}^{\mathrm{H}}\tilde{\mathbf{x}}\right|$. From (24), it may be derived with some manipulations that

$$\begin{aligned} &\left|\tilde{\mathbf{x}}^{\mathrm{H}}\mathbf{A}\tilde{\mathbf{x}}^*\right| \\ &= \tfrac{1}{3}[\tilde{x}_1^*\tilde{x}_1^* - e^{j2\phi_{12}}\tilde{x}_2^*\tilde{x}_2^* \\ &\qquad - e^{j2\phi_{13}}\tilde{x}_3^*\tilde{x}_3^* + 4e^{j(\phi_{12}+\phi_{13})}\tilde{x}_2^*\tilde{x}_3^*] \\ &\le \tfrac{1}{3}\left(|\tilde{x}_1|^2 + |\tilde{x}_2|^2 + |\tilde{x}_3|^2 + 4|\tilde{x}_2||\tilde{x}_3|\right) \\ &\le \tfrac{1}{3}\left(|\tilde{x}_1|^2 + 3|\tilde{x}_2|^2 + 3|\tilde{x}_3|^2\right) \\ &\le |\tilde{x}_1|^2 + |\tilde{x}_2|^2 + |\tilde{x}_3|^2 = \left|\tilde{\mathbf{x}}^{\mathrm{H}}\tilde{\mathbf{x}}\right| \end{aligned} \tag{28}$$

The conditions that the equalities hold in the three inequalities in (28) are

$$\begin{aligned} &\phi_{13} - \phi_{12} = \alpha_3 - \alpha_2 \pm (2k+1)\pi \quad (k = 0, 1, 2\ldots) \\ &|\tilde{x}_2| = |\tilde{x}_3| \\ &\tilde{x}_1 = 0 \end{aligned} \tag{29}$$

respectively, where α_t is the phase of $\tilde{x}_t$, i.e. $\tilde{x}_t = |\tilde{x}_t|\,e^{j\alpha_t}$ $(t = 1, 2, 3)$. The first condition implies that

$$\mathbf{X}=\left[\begin{array}{ccc} x_1 & x_2 & x_3 \\ \frac{x_1^*+2e^{j\frac{2\pi}{5}}x_2^*+2e^{-j\frac{2\pi}{5}}x_3^*}{3} & \frac{-2e^{j\frac{2\pi}{5}}x_1^*+e^{-j\frac{\pi}{5}}x_2^*+2x_3^*}{3} & \frac{-2e^{-j\frac{2\pi}{5}}x_1^*+2x_2^*+e^{j\frac{\pi}{5}}x_3^*}{3}\end{array}\right] \quad (31)$$

we may guarantee the full diversity gain of the proposed design with full rank matrix $\widetilde{\mathbf{X}}$ as long as the phases ϕ_{12} and ϕ_{13} (free variables) are set to $\phi_{13}-\phi_{12}\neq\alpha_3-\alpha_2\pm(2k+1)\pi \quad (k=0,1,2\ldots)$. For example, if we set $\phi_{12}=\frac{2\pi}{5}$ and $\phi_{13}=-\frac{2\pi}{5}$ then $\phi_{13}-\phi_{12}=-\frac{4\pi}{5}\neq$ any phase difference of constellations for QPSK and QAM signals. Therefore such STBC design achieves full diversity with full rank matrix $\widetilde{\mathbf{X}}$. The corresponding design matrix is

$$\mathbf{A}=\frac{1}{3}\left[\begin{array}{ccc} 1 & 2e^{j\frac{2\pi}{5}} & 2e^{-j\frac{2\pi}{5}} \\ -2e^{j\frac{2\pi}{5}} & e^{-j\frac{\pi}{5}} & 2 \\ -2e^{-j\frac{2\pi}{5}} & 2 & e^{j\frac{\pi}{5}}\end{array}\right] \quad (30)$$

Substituting (30) into (4) we obtain the STBC design (31) as shown at the top of the page. It is noted that the STBC design (31) may not have an optimized coding gain which will be discussed in the following section.

2) Coding Gain: The coding gain of a STBC design is closely related to the determinant of the codeword difference matrix $\tilde{\mathbf{X}}$ or Equation (27). The larger the determinant, the greater the coding gain. In order to achieve the optimal coding gain for a STBC design, it is desirable to maximize the minimum of the determinant, i.e.

$$\max_{\tilde{\mathbf{x}},\phi_{12},\phi_{13}}\min\det(\tilde{\mathbf{X}}\tilde{\mathbf{X}}^{\mathrm{H}})>0 \quad (32)$$

The determinant can be expanded as

$$\begin{aligned}&\det(\tilde{\mathbf{X}}\tilde{\mathbf{X}}^{\mathrm{H}})=\left|\tilde{\mathbf{x}}^{\mathrm{H}}\tilde{\mathbf{x}}\right|^2-\left|\tilde{\mathbf{x}}^{\mathrm{H}}\mathbf{A}\tilde{\mathbf{x}}^*\right|^2\\&=\left(|\tilde{x}_1|^2+|\tilde{x}_2|^2+|\tilde{x}_3|^2\right)^2\\&-\frac{1}{9}\left|\tilde{x}_1^*\tilde{x}_1^*-e^{j2\phi_{12}}\tilde{x}_2^*\tilde{x}_2^*-e^{j2\phi_{13}}\tilde{x}_3^*\tilde{x}_3^*+4e^{j(\phi_{12}+\phi_{13})}\tilde{x}_2^*\tilde{x}_3^*\right|^2\end{aligned} \quad (33)$$

It is obvious that the minimum is dependent on the values of $\tilde{x}_t$ (*t=1, 2, 3*). It is not straightforward to obtain a close form solution to (32). Although a brute force solution is possible, the optimum is in general dependent on modulation signals employed. As the design criterion and the focus of the paper is diversity gain, we leave the coding gain issue open for the future work.

IV. Decoding of the Proposed Q-STBC Codes

To illustrate the decoding complexity of the proposed STBC codes with ML performance, we will give the decision metric used for the ML detection.

The ML decoding is to find the optimal $\mathbf{X}$ or x_1, x_2, x_3 that minimizes the metric (7) amongst all the possibilities.

Substituting $\mathbf{X}$ in (7) with (4) and noticing Appendix A and the properties of (8) and (9) in the proposed design matrix, the ML metric (7) can be expanded and simplified to a sum of two items, after removal of irrelevant items, i.e.

$$\sum_{\mathrm{m}=1}^{\mathrm{M}}\left|\mathbf{r}^m-\mathbf{X}^{\mathrm{T}}\cdot\mathbf{h}^m\right|^2=f_1(x_1)+f_{23}(x_2,x_3) \quad (34)$$

where

$$\begin{aligned}f_1(x_1)=&\sum_{\mathrm{m}=1}^{\mathrm{M}}\left(|h_1^m|^2+|h_2^m|^2-1\right)|x_1|^2\\&+\sum_{\mathrm{m}=1}^{\mathrm{M}}\left|x_1^2+(h_1^m)^*h_2^m a_{11}\right|^2-\mathrm{M}\cdot|x_1|^4\\&+\sum_{\mathrm{m}=1}^{\mathrm{M}}\left|x_1-\left[(h_1^m)^*r_1^m+h_2^m(\mathbf{r}^m)^{\mathrm{H}}\mathbf{a}_1\right]\right|^2\end{aligned}$$

$$\begin{aligned}f_{23}(x_2,x_3)=&\sum_{\mathrm{m}=1}^{\mathrm{M}}\left|x_2x_3+(h_1^m)^*h_2^m(a_{23}+a_{32})\right|^2\\&+\sum_{\mathrm{m}=1}^{\mathrm{M}}\left(|h_1^m|^2+|h_2^m|^2-1\right)\left(|x_2|^2+|x_3|^2\right)\\&+\sum_{\mathrm{m}=1}^{\mathrm{M}}\sum_{t=2,3}\left[\left|x_t^2+(h_1^m)^*h_2^m a_{tt}\right|^2-|x_t|^4\right]\\&+\sum_{\mathrm{m}=1}^{\mathrm{M}}\sum_{t=2,3}\left|x_t-\left[r_t^m(h_1^m)^*+h_2^m(\mathbf{r}^m)^H\mathbf{a}_t\right]\right|^2\\&-\mathrm{M}\cdot|x_2|^2|x_3|^2\end{aligned}$$

As f_1 is independent of x_2 and x_3 and f_{23} is independent of x_1, the minimization of the metric in (7) is equivalent to minimizing f_1 and f_{23} independently. This reduces the complexity of decoding without sacrificing the performance.

V. Simulations

We have conducted numerical simulations to evaluate the performance of the designed Q-STBC codes. We use the STBC code shown in (31) for example. We have transmitted 300 blocks of QPSK symbols for each of 10000 two-transmit-one-receive channel realizations.

The simulation results for the simple flat Rayleigh fading cases have been plotted in Fig. 5. For comparison, we have included the performance of the hybrid transmission scheme [13], together with the benchmark performance of single antenna transmission and Alamouti STBC transmission. It is noted that only two out of three slots are used for transmission in Alamouti STBC transmission in order to obtain performance bounds for comparison. Two performance bounds are considered: one is for QPSK modulation, i.e. the same modulation but less overal spectrum efficiency (1.33 bits per channel use) as compared to the proposed transmission. The other is for 8-PSK, which corresponds to the case with same spectrum efficiency, 2 bits per channel use. As can be seen from Fig. 5, the proposed STBC design reveals the same diversity order of two as Alamouti code (parallel to the two bounds). Its performance far exceeds the hybrid transmission scheme, especially at a higher SNR. The performance gap between the proposed STBC transmission and the benchmark

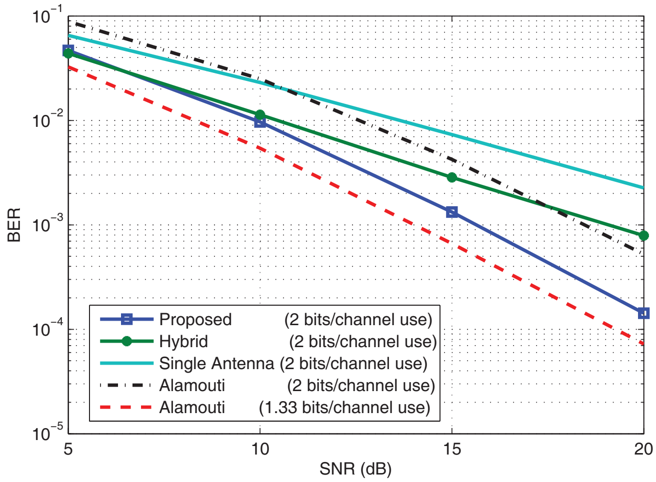

Fig. 5. Performance comparison for three-slot transmissions.

Alamouti codes with QPSK is mainly due to the difference in spectral efficiency, i.e. 1.33 bits per channel use in the Alamouti code case vs. 2 bits per channel use as in the proposed transmission.

VI. Conclusions

In this paper, a class of quasi-orthogonal STBC code design has been proposed for three-slot transmission. The STBC design achieves rate-one full-diversity transmission with superior performance. The maximum likelihood decoding of the designed codes has one order lower complexity.

Appendix A
Proof of Theorem I

The ML cost function (7) can be expanded as

$$\begin{aligned}&\sum_{m=1}^{M}\left|\mathbf{r}^m-\mathbf{X}^{\mathrm{T}}\cdot\mathbf{h}^m\right|^2\\&=\sum_{m=1}^{M}\left[(\mathbf{r}^m)^{\mathrm{H}}\mathbf{r}^m-h_1^m(\mathbf{r}^m)^{\mathrm{H}}\mathbf{x}-h_2^m(\mathbf{r}^m)^{\mathrm{H}}\mathbf{y}\right]\\&+\sum_{m=1}^{M}\left[|h_1^m|^2\mathbf{x}^{\mathrm{H}}\mathbf{x}-(h_1^m)^*\mathbf{x}^{\mathrm{H}}\mathbf{r}^m-(h_2^m)^*\mathbf{y}^{\mathrm{H}}\mathbf{r}^m\right]\\&+\sum_{m=1}^{M}\left[|h_2^m|^2\mathbf{y}^{\mathrm{H}}\mathbf{y}+h_1^m(h_2^m)^*\mathbf{y}^{\mathrm{H}}\mathbf{x}+h_1^*h_2^m\mathbf{x}^{\mathrm{H}}\mathbf{y}\right]\end{aligned}\tag{35}$$

Only the last sum containing $\mathbf{y}^{\mathrm{H}}\mathbf{y}$ or $\mathbf{y}^{\mathrm{H}}\mathbf{x}$ (or $\mathbf{x}^{\mathrm{H}}\mathbf{y}$) has cross terms of x_1, x_2, x_3. We will zoom into it as follows.

First, we expand its last two terms and keep only those relevant to the cross terms of x_1 and x_2, x_3, as

$$\begin{aligned}&h_1^m(h_2^m)^*\mathbf{y}^{\mathrm{H}}\mathbf{x}+(h_1^m)^*h_2^m\mathbf{x}^{\mathrm{H}}\mathbf{y}\\&\sim(h_1^m(h_2^m)^*)\cdot x_1\cdot\begin{bmatrix}x_2 & x_3\end{bmatrix}\begin{bmatrix}a_{21}^*+a_{12}^*\\a_{31}^*+a_{13}^*\end{bmatrix}\\&+[(h_1^m)^*h_2^m]\cdot x_1^*\cdot\begin{bmatrix}x_2^* & x_3^*\end{bmatrix}\begin{bmatrix}a_{21}+a_{12}\\a_{31}+a_{13}\end{bmatrix}\end{aligned}\tag{36}$$

To decouple $x_1$from x_2, x_3, the terms in (36) shall be zero for any channel coefficients and values of x_1, x_2, x_3. This amounts to the conditions that $\begin{bmatrix}a_{21}+a_{12}\\a_{31}+a_{13}\end{bmatrix}$ should be zero or the last two equations in (8).

Second, we expand the remaining term $|h_2^m|^2\mathbf{y}^{\mathrm{H}}\mathbf{y}$ and again keep only those relevant to the cross terms of x_1 and x_2, x_3

$$\begin{aligned}&|h_2^m|^2\mathbf{y}^{\mathrm{H}}\mathbf{y}\\&\sim|h_2^m|^2x_1\begin{bmatrix}x_2^* & x_3^*\end{bmatrix}\\&\quad\cdot\left(a_{11}^*\begin{bmatrix}a_{12}\\a_{13}\end{bmatrix}+\begin{bmatrix}a_{22} & a_{32}\\a_{23} & a_{33}\end{bmatrix}\begin{bmatrix}a_{21}^*\\a_{31}^*\end{bmatrix}\right)\\&+\left(|h_2^m|^2\right)x_1^*\begin{bmatrix}x_2 & x_3\end{bmatrix}\\&\quad\cdot\left(a_{11}\begin{bmatrix}a_{12}^*\\a_{13}^*\end{bmatrix}+\begin{bmatrix}a_{22}^* & a_{32}^*\\a_{23}^* & a_{33}^*\end{bmatrix}\begin{bmatrix}a_{21}\\a_{31}\end{bmatrix}\right)\end{aligned}\tag{37}$$

To decouple x_1 from x_2, x_3, the terms in (37) shall be zero for any channel coefficients and values of x_1, x_2, x_3. This amounts to the conditions that $a_{11}\begin{bmatrix}a_{12}^*\\a_{13}^*\end{bmatrix}+\begin{bmatrix}a_{22}^* & a_{32}^*\\a_{23}^* & a_{33}^*\end{bmatrix}\begin{bmatrix}a_{21}\\a_{31}\end{bmatrix}$ should be zero or the first two equations in (8).

With zero values in (36) and (37), there will be no cross term of x_1 and x_2, x_3 in the cost function (35). Therefore the corresponding STBC is quasi-orthogonal and the proof is complete.

Appendix B
Proof of Lemma I

Due to the relationship of a_{12} and a_{21} or a_{13} and a_{31} in (8), we need only to prove $a_{12} \neq 0$ and $a_{13} \neq 0$. We will use Reductio ad Absurdum to prove this lemma.

Assuming $a_{12} = 0$ and substituting it into (8), we have

$$\begin{cases}a_{31}a_{32}^* & = & 0\\a_{11}a_{13}^*+a_{31}a_{33}^* & = & 0\\a_{21} & = & 0\\a_{13}+a_{31} & = & 0\end{cases}\tag{38}$$

On one hand, if $a_{31} = 0$, we will have $a_{13} = 0$ and the design matrix $\mathbf{A}$ will become

$$\mathbf{A}=\begin{bmatrix}a_{11} & 0 & 0\\0 & a_{22} & a_{23}\\0 & a_{32} & a_{33}\end{bmatrix}\tag{39}$$

This corresponds to the case where x_1(or x_1^*) is only transmitted in the 1st time slot on both antennas and is equivalent to the repetition transmission of x_1 in the 1st time slot. There is no spatial transmit diversity achieved in this case. In other words, the solution does not satisfy the full diversity criterion in (5).

On the other hand, if $a_{31} \neq 0$, then $a_{32} = 0$. The design matrix $\mathbf{A}$ will become

$$\mathbf{A}=\begin{bmatrix}a_{11} & 0 & a_{13}\\0 & a_{22} & a_{23}\\a_{31} & 0 & a_{33}\end{bmatrix}\tag{40}$$

In this case, x_2^* is only transmitted in the 2nd time slot on both antennas. Similarly there is no spatial transmit diversity achieved and it does not satisfy the full diversity criterion in (5).

Therefore, we must have $a_{12} \neq 0$. Similarly, it can be proved in the same way that $a_{13} \neq 0$.

Appendix C
Proof of Lemma II

Again we use Reductio ad Absurdum to prove the lemma. We assume $a_{11} = a_{22} = a_{33} = 0$ and provide the proof of contradiction. Substituting the above into (8) and noticing $a_{21} \neq 0$ and $a_{31} \neq 0$ in Lemma 1, we have $a_{32} = a_{23} = 0$. Substituting it into (9), we will arrive at the following contradictory equations

$$\begin{cases} |a_{12}|^2 + |a_{13}|^2 &= 1 \\ |a_{12}|^2 &= 1 \\ |a_{13}|^2 &= 1 \end{cases} \tag{41}$$

Therefore we complete the proof.

Appendix D
Solving Equations (16) and (17)

The first two equations in (16) can be taken as two linear equations with respect to ρ_{11} and ρ_{13}. From Lemmas I and II, ρ_{11} and ρ_{13} are non-zero. To have non-zero solutions, the coefficient matrix must be deficient in rank. It amounts to

$$(\cos\theta_2 - 1)\sin\theta_1 + \cos\theta_1 \sin\theta_2 = 0 \tag{42}$$

Expanding the equation and applying twice the *addition and subtraction theorems* for sine, we obtain

$$2\cos(\theta_1 - \frac{\theta_2}{2})\sin\frac{\theta_2}{2} = 0 \tag{43}$$

If $\sin\frac{\theta_2}{2} = 0$ then $\cos\theta_2 = 1$ and $\sin\theta_2 = 0$. Substituting them into the first two equations in (16) yields to $\cos\theta_1 = 0$ and $\sin\theta_1 = 0$ which are contradictory. Therefore $\sin\frac{\theta_2}{2} \neq 0$.

From (43), $\sin\frac{\theta_2}{2} \neq 0$ leads to $\cos(\theta_1 - \frac{\theta_2}{2}) = 0$ or

$$\theta_2 = 2\theta_1 - \pi + 2k\pi \quad (k = 0, \pm 1 \ldots) \tag{44}$$

Therefore we have

$$\begin{aligned} \cos\theta_2 &= 1 - 2\cos^2\theta_1 \\ \sin\theta_2 &= -2\sin\theta_1\cos\theta_1 \end{aligned} \tag{45}$$

Substituting (45) into the first two equations in (16) gives

$$\begin{aligned} -2\rho_{11}\cos^2\theta_1 + \rho_{13}\cos\theta_1 &= 0 \\ -2\rho_{11}\sin\theta_1\cos\theta_1 + \rho_{13}\sin\theta_1 &= 0 \end{aligned} \tag{46}$$

It is noted that $\cos\theta_1 \neq 0$ otherwise the second equation in (46) will yield the contradictory $\sin\theta_1 = 0$. Therefore we arrive at

$$\rho_{13} = 2\rho_{11}\cos\theta_1 \tag{47}$$

Similarly, we may obtain the following from the third and fourth equations in (16)

$$\begin{aligned} &\rho_{12} = 2\rho_{11}\cos\theta_3 \quad (\cos\theta_3 \neq 0) \\ &\theta_4 = 2\theta_3 - \pi + 2k\pi \quad (k = 0, \pm 1 \ldots) \end{aligned} \tag{48}$$

Substituting (47) and (48), we may obtain the three amplitudes in (18) easily. With (44), (47) and (48), we solve the equations.

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Zander (Zhongding) Lei (M'00-SM'01) received his B.S. (Honor) and the Ph.D degrees from Beijing Jiaotong University, Beijing, China in 1991 and 1997, and the M.Eng degree from Tong Ji University, Shanghai, China in 1994. From 1997 to 1999, he was a Post-doctoral Fellow in the National University of Singapore. Since 1999, he has been with Institute for Infocomm Research Singapore, where he is a Project Manager and Research Scientist, leading a team working on 3GPP LTE-Advanced standard. He is a technical leader in IEEE 802.22 standard for wireless regional area network (WRAN) standardizing the first cognitive radio system. His research interests include next generation/emerging wireless communication systems/technologies, cooperative communications, cognitive radio etc. He has nine patents granted and has published over 90 papers in international refereed journals or conferences.

Dr. Lei is a co-recipient of Institution Engineering of Singapore Prestigious Engineering Achievement Awards in both 2005 and 2007. He has been the PHY Technical Editor for IEEE 802.22 standard since 2006. He has served as Co-Chairs/Panel Chairs and in Technical Program Committee for various IEEE international conferences on wireless communications.

Chau Yuen received the B. Eng and PhD degree from Nanyang Technological University (NTU), Singapore in 2000 and 2004 respectively respectively. He is the recipient of Lee Kuan Yew Gold Medal, Institution of Electrical Engineers (IEE) Book Prize, Institute of Engineering of Singapore (IES) Gold Medal, Merck Sharp & Dohme (MSD) Gold Medal and twice the recipient of Hewlett Packard (HP) Prize. He was a Post Doc Fellow in Lucent Technologies Bell Labs, Murray Hill during 2005. Dr. Yuen was a Visiting Assistant Professor of Hong Kong Polytechnic University on 2008. During the period of 2006-2010, he worked at Institute for Infocomm Research (I2R, Singapore) as a Senior Research Engineer, where he involved in an industrial project on developing an 802.11n Wireless LAN system, and participated actively in 3Gpp Long Term Evolution (LTE) and LTE-Advanced (LTE-A) standardization. He serves as an Associate Editor for IEEE Transactions on Vechicular Technology. He has published over 70 research papers at international journals or conferences. His present research interests include green communications, cognitive radio network, cooperative transmissions, network coding, wireless positioning, and wireless network. He joined Singapore University of Technology and Design as an assistant professor from June 2010.

Francois P. S. Chin received his Ph.D. degrees in electrical engineering from National University of Singapore. Since 1995, he has been at the Institute for Infocomm Research, a government-funded research institute, where he is now a Programme Manager, leading a group of R&D teams in future broadband wireless access technologies, including MIMO, UWB, mmWave, Cognitive Radio and Optical Access Networks. His research focus include signal processing techniques for wireless communications system capacity enhancement, network optimization, system design/performance evaluation for future communications systems, including beyond 3G cellular systems and Ultra-wideband (UWB) based high-speed short range radio networks. He is actively contributing to various standardization activities/efforts, including 3GPP Long Term Evolution, WiMedia Alliance for UWB Wireless Personal Area Network (WPAN), IEEE 802.15.3a UWB Alternate PHY for High Rate, IEEE 802.15.4a UWB Alternate PHY for Low-Rate Low-power WPAN, IEEE 802.22 Wireless Regional Area Networks (WRAN). He has published over 180 papers in well-known international journals and technical conferences, over 50 technical documents in wireless communications related Standards and has 30 pending patents.

He is a co-recipient of the 50th IEEE Vehicular Technology and Communications Conference (IEEE VTC 1999-Fall) Best Paper Award; and also received Institution Engineering of Singapore (IES) Prestigious Engineering Achievement Awards in both 2005 and 2007 for his contribution in IEEE 802.15.4b and IEEE 802.22 Standards respectively. He is a senior member of IEEE and a member of Radio Standard Technical Committee, Infocomm Development Authority (Singapore).