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NOVEL TOPOLOGICAL APPROACH TO DESIGNING FLOW CHANNELS

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ABSTRACT

Many natural systems that transport heat, energy or fluid from a distributed volume to a single flow channel exhibit an analogous appearance to trees. Examples include bronchial tubes, watersheds, lightening, and blood vessels. Commonly for natural and designed systems with this type of flow, the flow volume consists primarily of high resistance regimes with a smaller portion of interdigitated low resistance regions (flow channels). Perhaps, the most relevant design problem is cooling of a microchip. Since microchip performance is optimal at lower temperatures, bulk heat flow resistance for heat exiting the system should be minimized; therefore, it is critical to cleverly align the channel to maximize flow. Heat conduction from a microchip is typically simplified into heat conduction from a homogenously heat-generating plate. This simplified problem is used as a standard model with which engineering designers can compare the performance of various cooling configurations. Due to the apparent tree-fractal characteristics of empirically emerging systems (i.e. in nature), several authors have examined analogous, simplified fractal configurations. These fractal configurations appear to be the best solution in current publication. We present a novel topological analysis that provides insight into performance at a schematic level. This analysis leads to the development of a new configuration, *leaf-like,* which outperforms the current state-of-the-art. Performance is compared among the configurations with two parallels analyses: an extensive series of finite element models, covering a broad combinatory array of material properties and heating conditions; and topological analysis of path length, or *the average distance heat, energy or fluid, must travel through the substrate and channel media before exiting through the point flow channel.* There is a strong correlative mapping between the two analyses. Finite element modeling is employed because it provides a fundamental mechanics approach to assessment of heat transfer behavior, while the path length analysis provides an intuitive and computationally affordable means to predict performance. Novel contributions of this work include a configuration for conductive cooling in a plate for VTP flow, superior to the state-of-the-art, and a topological analysis of VTP flow that provides a generalized metric of bulk flow resistance and a schematic level conceptualization of the mechanics of VTP flow. Future advancements of our research could include enhancing the algorithm to automatically parse geometry into channel segments from an image or other external representation and eventually to even generate a suitable channel from arbitrary substrate geometry.

INTRODUCTION

A common feature among arteries, lightening, bronchial airways, leaves, and watersheds is truncated tree-like fractal organization. This commonality may be due to the fact that these systems solve a similar type of problem- the *transference of energy or matter from a distributed arrangement (area or volume) to a single point (sink)* [1].

The motivation of this paper is to examine whether the tree-like fractal found so frequently in nature is in fact optimal for volume-to-point flow. For the volume-to-point flow problem the amount of flow channel is constrained. Therefore, it is important to optimally arrange the available channel.

The approach of this paper is an in-depth look at theoretical analysis and experiments about one type of volume-to-point flow. This paper will focus on evaluating the problem of conduction from a heat generating plate to a sink point- analogous to cooling a microchip. In the past, tree like fractal channel geometry has been proposed for this type of cooling [4]. In order to evaluate this proposal, the performance of various geometries is compared via literature review and analyses.

NOMENCLATURE

Fractal, conduction, path length, finite element, volume to point flow, topology.

1 HYPOTHESIS

Channel design assessment may be accomplished through a novel method, *path length analysis;* this can be shown by developing a volume to point conductive heat flow channel with better performance (as verified with FEM) than the current ‘state-of-the-art’ configuration, using path length analysis.

2 INTRODUCTION

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2.1 INTRODUCTION TO FRACTAL GEOMETRY

The study of fractal geometry came about largely from the work of Mandelbrot. He developed a new form of geometry (Fig. 1) that describes the phenomena of nature (such as the shape of coastlines and trees) with more accuracy than traditional Euclidean geometry. Fractal geometry has a distinct set of characteristics (Table 1). Theory of fractal geometry is well understood, however, only a few projects have examined its application to engineered systems.

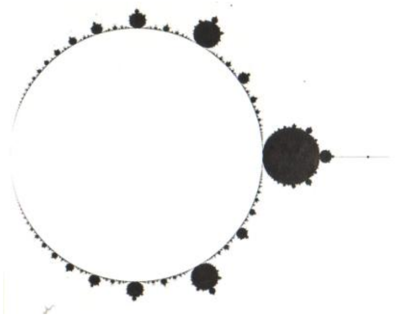


Fig. 1: Geometric representation of a true fractal [1]

2.2 TRUNCATED TREE-LIKE FRACTALS

The truncated tree-like fractal is the geometry of interest for this paper. Table 2 lists the characteristics of the tree-like fractal studied in this paper. Several authors have examined this particular tree-like fractal.

Table 1: Characteristics of fractal geometry [2]

It has a fine structure at arbitrarily small scales
It is too irregular to be easily described in traditional Euclidean geometric language
It is self-similar (at least approximately or stochastically)
It has a Hausdorff dimension (fractal dimension) which is greater than its topological dimension
It has a simple and recursive definition

The ‘fractal’ evaluated in this paper cannot be considered a complete fractal; it is truncated. Truncation is given by termination of the tree after a finite number of branches. Natural fractals are truncated.

The particular tree fractal described in Table 2 was first proposed for conductive cooling by Bejan [4]. It is considered the optimal form of tree fractal for cooling from a volume-to-point flow [4,7,11]. The performance of this geometry was optimized with respect to branching angle, branch length, number of branches and fractal dimension. Hence the tree-like fractal (Fig. 2) proposed by Bejan is employed.

The Hausdorf (fractal) dimension is calculated as follows:

$$D = \frac{Log\ (N)}{Log\ (l)}\ [2] \qquad (1)$$

where N is the number of self similar pieces in the fragmentation and l is the magnification factor (or size ratio of fragments to the original), and D is the fractal dimension

Table 2: Characteristics of the tree fractal

Iterative construction of nodes, each connected by a line
At each node the ‘parent’ branch line divides into two or more new branches, ‘children’
The branching angle and ratio of side lengths stays the same through each iteration
The fractal dimension and branching number for this example are both 2 [1]
The branching angle for the tree used in this paper is 90 degrees [1]

3 LITERATURE REVIEW

Review of the relevant literature is used to examine the performance of the tree-like fractal and other configurations for conductive cooling. Additional literature review is used to examine whether the metric path length (particle travel distance) is relevant for volume-to-point conduction problems.

3.1 FRACTAL COOLING PERFORMANCE

Several analytical works have analyzed the conductive cooling performance of tree-like fractal channel configuration in a heat generating plate. Each of the studies included in this review utilizes the same problem definition outlined in this paper: volume-to-point flow via conduction for a plate with homogenous heating, interdigitated channel of high thermal conductivity and variance of channel configuration without constant volume and surface area of channel.

Many authors have examined fractals from a geometric modeling perspective. However, only a few studies have examined tree-like geometry in engineered systems. Only one author attempted to explain the instantiation of so many fractals in nature from an engineering perspective. This explanation is known as the "*constructal theory*".

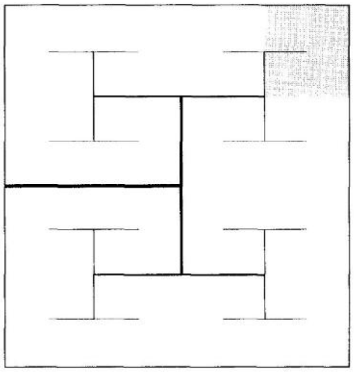


Fig. 2: Bejan's optimized tree fractal [4]

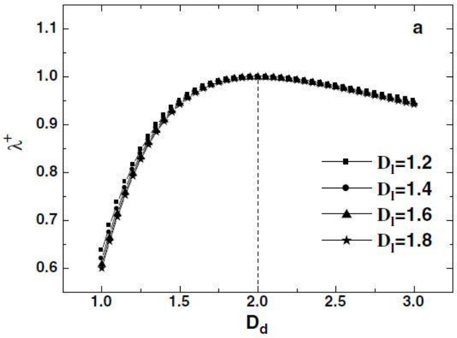


Fig. 3: λ, dimensionless thermal conductivity, versus D_d, fractal dimension- theoretical [8]

3.1.1 THE CONSTRUCTAL THEORY

Adrian Bejan proposed the constructal theory in 1997. The constructal theory states that *the tree-like structures observed in nature are deterministic. They result from a directional optimization process- one that starts at a smaller scale and moves towards a system level scale* [4]. In other words tree-like structures are the result of an iterative optimization process for the described volume-to-point flow problem [4].

3.1.2 ANALYTICAL EVALUATION OF CONSTRUCTAL TREES

Performance of tree like fractals for cooling has been examined. Bejan employs an optimization process to show that the tree-like fractal (Fig. 2) is the optimal fractal configuration, as discussed in the previous section. This is true regardless of the magnitude of heat generation in the plate or thermal conductivities of the plate and channel- as long as the channel's thermal conductivity is significantly higher than the plate's. The characteristics of this configuration have been optimized. However, it remains unknown if it is the best possible configuration [4].

Zhang compared performance of serpentine channels (Fig. 6b) to performance of tree-like fractals [12]. He found that the tree-like fractal results in a much lower maximum plate temperature than the serpentine configuration (Fig. 4) [12]. Neither he nor other authors have compared tree-like fractals to other geometries.

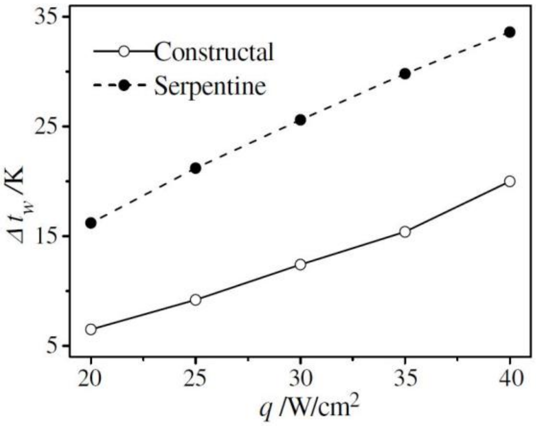


Fig. 4: Temperature increase at the channel wall as a function of heat generation for constructal (fractal) versus serpentine [12]

3.1.3 EXPERIMENTAL EVALUATION OF CONSTRUCTAL TREES

Authors have compared the performance of serpentine and fractal configurations via empirical experiments. These experiments have confirmed that the tree-like geometry performed more effectively at cooling than serpentine [5]. One paper reports that tree fractals are superior to parallel lines. However, the experimenters used an external and unreferenced source for the data on performance of parallel channels [11]. Therefore it is not certain that all of the necessary conditions (consistent channel volume and material properties, etc.) for a direct comparison were met.

3.1.4 SUMMARY OF LITERATURE REVIEW

It has been conclusively shown that tree-like fractal geometry is superior to serpentine geometry in cooling performance in a volume-to-point conduction problem [xx]. It has also been shown that the tree-like configuration shown in Fig. 2 is optimal among tree configurations for cooling. It has not been shown that the truncated tree-like fractal is a global optimum for the volume-to-point conduction problem.

3.2 FOUNDATIONS FOR USING PATH LENGTH.

Analysis of configuration topology, using the metric path length, is employed to better understand performance. To visualize path length, suppose that a grid of particles have been set on a plate with interdigitated channel. Each particle is then transported to the exit (located in the same position as the sink in the conduction problem). Allow the assumption that each particle first moves to the channel, then along the channel to the exit. This assumption is standard in the analogous volume-to-point conduction problem since thermal conductivity of the channel is much higher than that of the plate [2]. The average distance travelled by each particle is the particle travel distance or *path length*. Several reasons for using this metric are discussed below.

3.2.1 THE NEED FOR MULTIPLE METRICS

Two analyses are used in this paper. First, a finite element model is used to simulate real world performance, i.e. max. plate temperature. Then path length is used to examine the configurations topologically. This twofold approach allows a comprehensive insight into the systems. Thus both inductive (FEM) and deductive (path length) reasoning are used to examine the hypothesis [14].

3.2.2 THERMAL RESISTANCE MINIMIZATION

Several authors have evaluated configurations for the volume-to-point flow problem in terms of their thermal resistance [4,5,10]. Thermal resistance is given in Equation 2. Thermal resistance is proportional to path length.

To efficiently conduct heat from a distributed volume to a single point, the over-all thermal flow resistance must be minimized [1]. The thermal resistance between two points will be proportional to the distance between those two points [13]:

$$\dot{q} = -\frac{kA}{L}(T_H - T_C)$$
$$R_C = -\frac{L}{kA}$$
$$R_C \ \alpha \ L \qquad (2)$$

where A is area, L is path length and R_C is thermal resistance

A cooling channel with a minimum path length is configured to ensure that the transport distance between each particle (heat generating point) and the exit (sink) is as short as possible. In this way, such a configuration will have a minimum thermal resistance.

3.2.3 PRECEDENCE OF USE

The correlation between one-dimensional path length and the performance of cooling systems has been reported in the literature [12]. The performance of a cooling system as a function of path length is illustrated in Fig. 5.

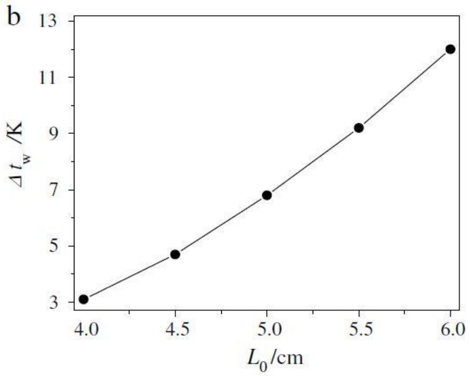
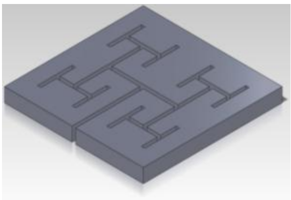


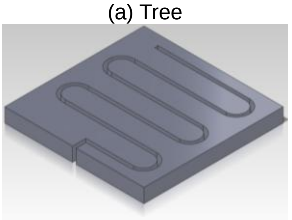
Fig. 5: Temperature increase in a channel wall as a function of intrachannel path length [12]

4 ANALYTICAL METHOD

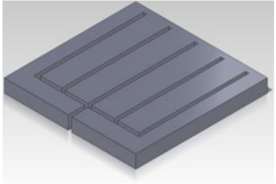
Two analyses are used in this paper. The purpose of these analyses is to rank performance of several configurations for the volume-to-point flow problem. The first analysis is a finite element heat transfer model; the second is a topological analysis.



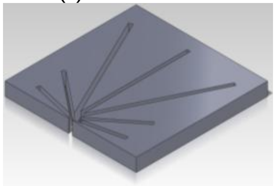
(a) Tree



(b) Serpentine



(c) Parallel Lines



(d) Leaf

Fig. 6: Solidworks rendering of each configuration. In the FEM voids are filled with cooling channel. The cooling channel is kept at constant temperature at the small opening on the side.

The analyzed geometries can be seen in Fig. 6. The tree geometry is a truncated fractal, and to date considered to be the optimal fractal for volume-to-point flow on a square plate (Fig. 6a) [4]. The serpentine and parallel line geometries were also extracted from relevant literature (Fig 6b,c) [6,11,12]. The leaf geometry is novel to this paper and was developed based on path length analysis (Fig. 6d).

4.0.1 METRICS

The given configurations are ranked based on performance metrics (Table 3). Path length and maximum plate temperature measure performance. The metrics plate volume and channel volume are held constant to remove their effect on performance. The material properties and heat generation values are varied to generalize the results.

Table 3: Metrics for analysis

Path Length – performance measure
Maximum Plate Temperature – performance measure
Volume and Surface Area of Cooling Channel – held constant
Plate Volume – held constant
Plate and Channel Material Properties – varied
Volumetric Heating Values – varied

4.1 FINITE ELEMENT MODEL

For the finite element model (Table 4) heat is conducted through two materials to a sink. The first material is the heat generating plate (substrate) and the second is cooling channel (located at the voids in Fig. 6). In ANSYS, the channel component is given a high constant of thermal conductivity. The small opening where the channel meets the wall is kept at a constant temperature (zero degrees Celsius). The channels of various geometrical configurations conduct heat to this sink area. The plate is adiabatic at all surfaces except for this sink. Every geometric configuration has the same volume and surface area of cooling channel. Various values of volumetric heating and thermal conductivity are employed. Each model is run to steady state conditions. The model is effectively two-dimensional as there is no variation in the z dimension.

Table 4: Properties of the FEM

Geometry	10X10X1 cm^3 plate
Thermal Conductivity of Plate	K_1
Thermal Conductivity of Channel	K_2, *where* $(K_2 > K_1)$
Mesh Size (after convergence)	0.003 m
Cooling Channel Volume	1,245 cm^3
Cooling Channel Surface Area	12,250 cm^2

All geometries were constructed in Solidworks and imported into ANSYS for evaluation. Solidworks analysis confirms that all geometries have the same volume and surface area to within one percent. Lossless contact between the cooling channel and the plate is assumed. The simplification from this assumption is overcome by widely varying the conduction coefficients to effectively model different thermal resistances within the cooling system.

4.2 VERIFICATION OF THE FEM

A simple configuration was modeled to verify that the FEM and analytical results [14] match. The configuration consists of a plate with fixed temperatures on each side, internal heat generation and adiabatic conditions on the top, bottom, front and back. The plate is 1X1X.1 cm^3. The plate has temperature variance in the horizontal axis only. The left surface is kept at a constant 80 degrees Celsius and the right surface is kept at a constant 20 degrees Celsius. The thermal conductivity of the plate is 1 $\frac{W}{mK}$. The maximum plate temperature at several values of heat generation is given for the FEM and the analytical solution in Table 5.

The analytical solution for temperature as a function of penetration depth between the two walls is taken as the following:

$$T = \frac{gL^2}{2k}\left[\frac{x}{L} - \left(\frac{x}{L}\right)^2\right] - \frac{(T_H - T_C)}{L}x + T_H \qquad (3)$$

where $\dot{g}$ stands for the heat generation rate, L the width, k the thermal conductivity in $\frac{\mathrm{W}}{\mathrm{mK}}$, x the penetration depth, T_{H} the hot wall temperature, and T_{C} the cooler wall temp. [13]

The maximum plate temperature is found analytically using Matlab and computationally using ANSYS. The difference between the analytical and computational solution is found to be 0.007%, confirming that the two methods exhibit strong relative accordance.

Additionally, convergence to a steady state condition in the FEM is confirmed by the characteristic shape of the plot in Fig. 7 (for an arbitrary point in the leaf configuration).

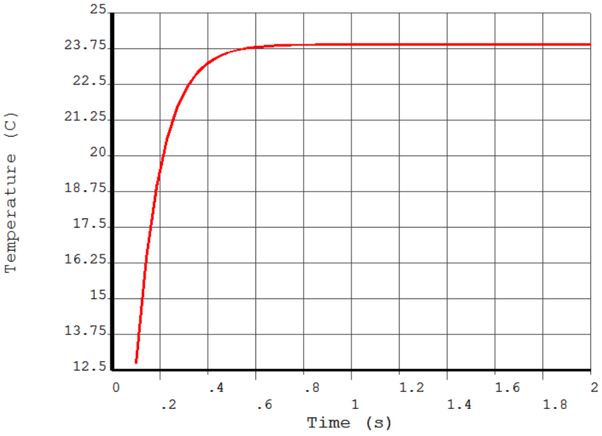


Fig. 7: Temperature convergence at an arbitrary point in the leaf configuration with K_1 = 149 $\frac{W}{mK}$, K_2 = 401 $\frac{W}{mK}$ and $\dot{g}$ = 5X10^5 $\frac{W}{m^3}$

Table 5: Maximum plate temperature in a volumetrically heated plate, comparison of analytical and FEM

Heat Generation	$0\frac{W}{m^3}$	$5X10^5\frac{W}{m^3}$	$1X10^6\frac{W}{m^3}$	$2x10^6\frac{W}{m^3}$	$5X10^6\frac{W}{m^3}$	$1X10^7\frac{W}{m^3}$
Analytical	80.0000°C	80.0000°C	80.0000°C	84.0000°C	116.1000°C	176.8000°C
FEM	80.0000°C	80.0000°C	80.0000°C	83.9965°C	116.1110°C	176.8680°C
Error	0.0000%	0.0000%	0.0000%	0.0040%	0.0090%	0.0300%

The results are given at only one value of sink temperature. The validity of this approach was tested. The temperature of the sink area was varied in a validation experiment. It was found that only the magnitude of the maximum plate temperature changed, while the morphology of the temperature contour plots and the performance rankings remained constant.

To verify that mesh granularity was sufficient, the sensitivity of maximum temperature in each model was determined by employing five mesh sizes of increasing granularity for each model. These variations spanned a full order of magnitude in mesh size, and the maximum temperature never varied by more than one percent of the values reported in the results section.

The results of these verification procedures support that the FEM is accurate.

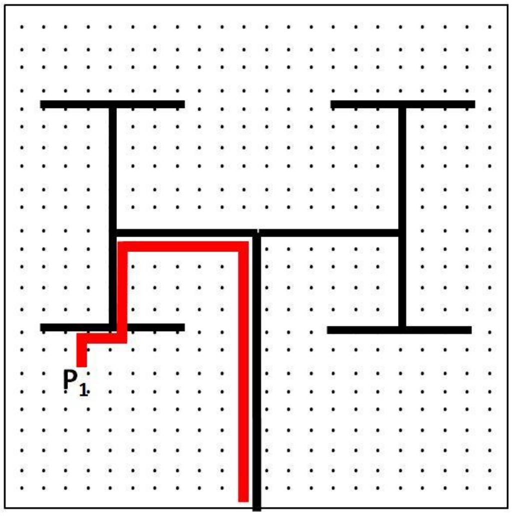


Fig. 8: Path length for an arbitrary point, P_1, in the tree configuration, highlighted in red

4.3 PATH LENGTH ANALYSIS

The path length of each configuration is computed. Path length is a generic transport metric. This generic analysis provides insight into the superior performance of some geometries.

Path length represents the distance travelled by an imaginary particle located on the substrate which must travel to the channel and then along the channel to the small exit area- the area kept at constant temperature in the FEM model. An example of path length can be seen in Fig. 8.

Steps to calculate path length:

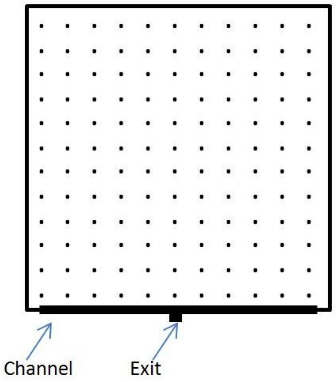
(1) Create a matrix of the x, y coordinates of points representing the position of a set of 100X100 points, evenly distributed over a square of 10X10 cm^2.

(2) Construct a second matrix using a set of points to approximate the location of cooling channel in each configuration- place a point in the matrix along a line at the center of each segment of cooling channel at 0.01 cm intervals.

(3) Select one point in the substrate matrix. Calculate the distance to every point in the channel from that substrate point. Take the minimum distance.

(4) Compute the distance along the channel to the exit (cooling area) from the nearest channel point, found in (3), and add this to the distance found in (3) to find the path length for that point.

(5) Steps (3) and (4) are repeated for each point in the substrate matrix and averaged- this average value is the path length for the configuration.



(6)

Fig. 9: Arbitrary geometry used to test the path length algorithm

4.4 VERIFICATION OF PATH LENGTH.

An example is used to validate that the numerical algorithm for calculating path length is accurate. Algorithmic and analytical computations of path length are made for an arbitrary geometry (Fig. 9). The cooling channel in this configuration is a straight line along the base of a square. The square is 10X10 cm^2. The average path length before the channel is half of the height, 5 cm. The average distance to the exit along the channel is half of half of the width, 2.5 cm. Thus the analytically determined path length is 7.5 cm while the Matlab code used to determine the path length for other configurations finds the path length to be 7.5025. Thus the algorithm exhibits a small error, around 0.02%. This error is due to the discretization.

Table 6: Maximum plate temperature across various model parameters

	$\dot{g} = 5X10^5 \frac{W}{m^3}$, K_1/K_2 *vary and are given below*		
Configuration	**1/2**	**1/10**	**1/100**
Tree	5110 °C	2870 °C	772 °C
Serpentine	5130 °C	3400 °C	1540 °C
Parallel	5030 °C	2590 °C	476 °C
Leaf	**4300 °C**	**1650 °C**	**355 °C**
	$K_1 = 149 \frac{W}{mK}$ (silicon), $K_2 = 401 \frac{W}{mK}$ (copper), $\dot{g}$ *varies and is given below*		
	1X10^3	**5X10^5**	**1X10^7**
Tree	0.316 °C	31.6 °C	3160 °C
Serpentine	0.323 °C	32.3 °C	3230 °C
Parallel	0.310 °C	31.0 °C	3100 °C
Leaf	**0.245 °C**	**24.5 °C**	**2450 °C**

5 RESULTS

Results of the various analyses are provided in this section.

5.1 FEM RESULTS

Results of the twenty-four unique finite element models can be seen in Table 6. Table 6 shows the steady state maximum plate temperature occurring in each of the geometries over a variety of different thermal conductivities and applied volumetric heat generation values. Interestingly, shifts in heat generation values result in proportional changes to the maximum temperature; likely, adding more heat generation to the system does not change flow dynamics in the way that changing the conductivities does or, generally, **flow contour is independent of flow intensity when internal resistance values are constant**. The leaf geometry performs the best while the parallel lines configuration performs the second best. The results match results from the literature in that the tree configuration performs better than the serpentine configuration [6,12]; however the direct comparison of other configurations such as the leaf configuration to the tree configuration is novel to this analysis (Fig. 10). **The leaf and parallel lines geometry perform better than the tree geometry.** On average the leaf geometry cools the plate to a thirty six percent lower temperature than the basic, serpentine configuration, while the tree geometry only cools an average fourteen percent more than the standard.

5.2 PATH LENGTH RESULTS.

It is found that path length performance (Table 7) corresponds directly to cooling performance (Table 6). The analyses show that the geometry with the minimum path length also has the best cooling performance. This observation has been confirmed after development and testing of the leaf configuration. The leaf configuration has a near minimum path length and the best cooling performance. The absolute minimum path length is a straight line from each point to the exit (5.95 cm).

The leaf geometry is a design novel to this paper. The shortest possible path length for each point is a straight line to the exit. If the minimum path for every point on the plate were traced, a fan-like geometry would appear. Shorter path length corresponds to better cooling performance. Therefore, geometry that closely maps to this trace of the minimum path should cool effectively. The leaf configuration is an attempt at creating fan-like geometry using only a finite volume of cooling channel.

Table 7: Results of the Path Length Calculations

Configuration	**Average Total Path Length**
Tree	10.90 cm
Serpentine	25.26 cm
Parallel	8.01 cm
Leaf	**6.50 cm**

6 DISCUSSION OF HYPOTHESIS

Both the literature and finite element analysis support that the tree fractal is superior to the serpentine configuration, as was also predicted by the path length analysis. As hypothesized, however, it has been clearly shown that more effective geometry can be designed through the computationally inexpensive topological path length analysis method.

6.1 LIMITATIONS

This paper suffers from several limitations. Only finite element modeling has been conducted rather than empirical experimentation. There are several limitations to the finite element model, e.g. lack of consideration of convective scenarios, adiabatic wall conditions, lack of contact resistances between the channel and the plate. The path length analysis may also limited by the assumption that movement is to the nearest channel segment, and the discretization. Moreover, only a few geometries have been analyzed.

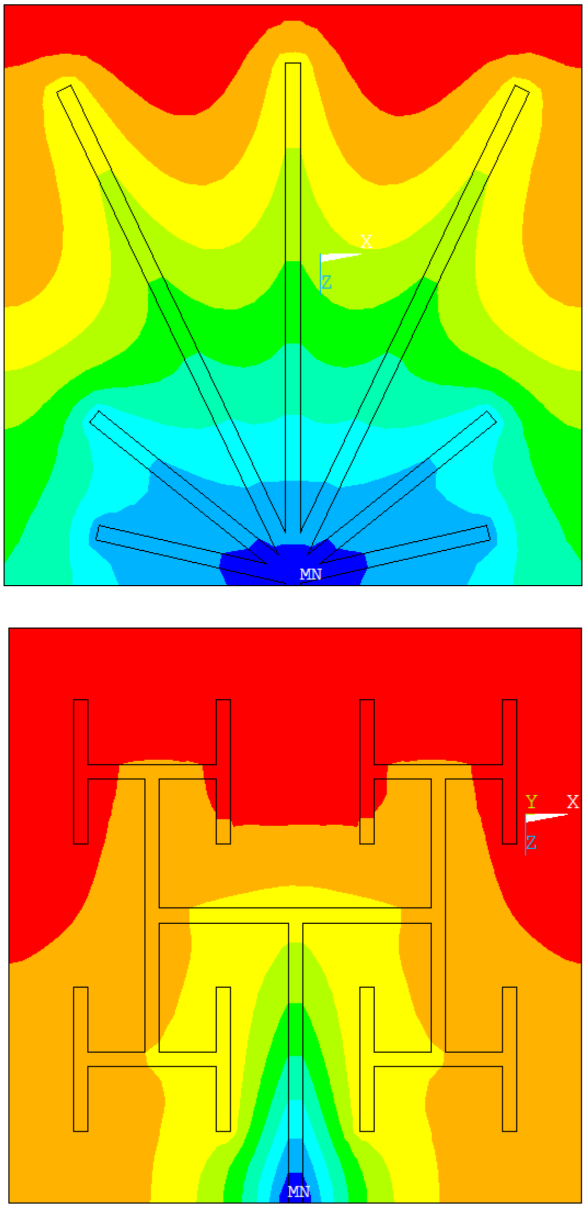


Fig. 10: FEM, contour plots of temperature with K_1 = 1 $\frac{W}{mK}$ and K_2 = 100 $\frac{W}{mK}$ and $\dot{g}$ = 5X10^5 $\frac{W}{m^3}$ for the Leaf (top)- max temperature 355 °C; and Tree (bottom)- max temperature 772 °C

7 CONCLUSIONS

This paper compared several geometric cooling channel configurations for the volume-to-point conductive cooling problem. A finite element model was used to measure performance of the conductive cooling channel geometries. The best configuration is the leaf geometry followed by parallel lines. The tree configuration is the second worst while the serpentine configuration is the worst. These results are consistent with the extant literature. Finally, the same order of performance has been observed in a second and novel topological analysis- path length. Additionally, to confirm the correlation between the Path Length analysis and FEM model, a Pearson’s correlation coefficient was determined between the average (across conditions) FEM results and the Path Length model. The correlation coefficient is .99, or nearly perfect agreement.

8 CONTRIBUTIONS AND FUTURE WORK

The high performing leaf-like geometry is a promising discovery. Moreover, both literature review and analysis also support the metric path length as a useful tool to benchmark design performance in volume-to-point flow problems.

In future efforts, the path length metric might easily be incorporated into a numerical analysis to determine the globally optimal cooling channel for arbitrary substrate geometry. Literature review also revealed that the minimum spanning tree is analogous to a configuration with minimum path length [4]. This observation may be leveraged in future work by employing known methods to find the minimum spanning tree (and thus the minimum path length configuration)- eliminating the need to develop a new optimization routine to find the minimum path length configuration.

ACKNOWLEDGMENTS

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REFERENCES

[1] Mandelbrot, B.B., *The Fractal Geometry of Nature*, W.H. Freeman and Company, 1982

[2] Falconer, Kenneth, *Fractal Geometry: Mathematical Foundations and Applications*, John Wiley & Sons, Ltd. xxv, 2003

[3] Masters, B., “Fractal Analysis of the Vascular Tree in the Human Retina”, Annual Review of Biomedical Engineering, 2004 vol. 6, pp. 427-452

[4] Ledezma, G., Bejan, A., “Constructal Tree Networks for Heat Transfer”, Journal of Applied Physics 1997 vol. 82, no. 1

[5] Poulikakos, S., “Laminar Mixing, Heat Transfer and Pressure Drop in Tree-like Microchannel nets and their Application for Thermal Management in Polymer Electrolyte Fuel Cells”, Journal of Power Sources, 2004 Vol. 130, pp. 178-191-

[6] Pence, D., “Reduced Pumping Power and Wall Temperature in Microchannel Heat Sinks with Fractal-like Branching Channel Networks”, Microscale Thermophysical Engineering, 2002 vol. 6, pp. 319-330

[7] Bejan, A., “Constructal-theory Network of Conducting Paths for Cooling a Heat Generating Volume”, International Journal of Heat and Mass Transfer, 1997 Vol. 40, no. 4 pp. 799-816

[8] Xu, P., Yu, B., "Heat Conduction in Fractal Tree-like Branched Networks", International Journal of Heat and Mass Transfer, 2006 Vol 49, pp. 3746-3751

[9] ALGOR development staff, "The Eight-node Hexahedral "Brick" Element in Finite Element Analysis", Autodesk incorporated, 2011

[10] Neagu, M., Bejan, A., "Constructal-theory Tree Networks of "Constant" Thermal Resistance", Journal of Applied Physics, 1999 vol. 86, no.2 pp. 1136-1144

[11] Chen, Y., Cheng, P., "An Experimental Investigation on the Thermal Efficiency of Fractal Tree-like Microchannel Nets", International Communications in Heat and Mass Transfer, 2005 vol. 32, pp. 931-938

[12] Zhang, C., Chen, Y., "Flow Boiling in Constructal Tree-Shaped Microchannel Network", International Journal of Heat and Mass Transfer, 2011 vol. 54, pp. 202-209

[13] Nellis, G., Klein, S., *Heat Transfer*, Cambridge University Press, 2009

[14] Frankenberger, E., *Designers: The Key to Successful Product Development*, Springer-Verlag London, 19