# 3-Time-Slot Group-Decodable STBC with Full Rate and Full Diversity

Tian Peng Ren, Chau Yuen, Yong Liang Guan, and Kun Hua Wang

***Abstract*—In this paper, we propose a generic method to construct group-decodable space-time block codes (STBC) with arbitrary code dimensions, including odd time slot. Based on the proposed code construction method, 3-time-slot STBC for two transmit antennas with full or even higher code rate can be obtained. The full-rate 3-time-slot STBC obtained can achieve full diversity and symbol-wise decoding complexity. It serves as a solution to the orphan-symbol (3-time-slot) transmit diversity issue raised in 3rd Generation Partnership Project (3GPP) standards.**

***Index Terms*—Space-time block codes/coding (STBC), Long Term Evolution-Advanced (LTE-A), group-decodability (quasi-orthogonality), orphan-symbol transmission.**

## I. INTRODUCTION

THE 3rd Generation Partnership Project (3GPP) has been working on the next generation wireless system (4G) under the project Long Term Evolution-Advanced (LTE-A), building upon the legacy standard LTE release 8 [1]. Space-time block coding (STBC) is a popular candidate for the uplink transmission scheme [2]. However, it has been identified that the LTE frame structure, which has been fixed and not likely to be changed, makes it awkward to implement an orthogonal STBC design, because the time slots in the LTE frame structure for data transmission are not guaranteed to be an even number. In many cases, there are 3 time slots available for data transmission, instead of 2 time slots as required by the orthogonal Alamouti STBC for a two-antenna user equipment (i.e. mobile station) [1] [3]. This has brought up an interesting STBC design problem: 3 time slots for two-antenna full-rate (code rate 1) transmission.

In [3], a hybrid scheme with 2-time-slot Alamouti STBC followed by 1-time-slot repetition transmission has been proposed. The hybrid transmission scheme can be decoded with linear decoding at the receiver. Unfortunately, it is not a full-diversity scheme due to the 1-time-slot repetition transmission. Recently, a full-rate full-diversity quasi-orthogonal STBC with two transmit antennas and 3 time slots was presented in [4]. However, its maximum-likelihood decoding requires a joint detection of two complex symbols, i.e., non-symbol-wise decoding.

In this paper, we propose a generic construction method for group-decodable (quasi-orthogonal) STBC of arbitrary size. Following the proposed construction, 1) we obtain a 3-time-slot full-rate (code rate 1) STBC for two transmit antennas which can achieve full diversity and symbol-wise decoding complexity; 2) we design a 3-time-slot high-rate (code rate $> 1$) STBC for two transmit antennas, which has the same decoding complexity level but higher code rate than the code in [4]. Simulation results show that the proposed 3-time-slot full-rate STBC has better bit error rate (BER) performances than the codes in [3] and [4] with the same or even lower decoding complexity.

The rest of this paper is organized as follows. In Section II, the signal model is described. The code construction and code examples are proposed in Section III. The performance comparison is presented in Section IV. This paper is concluded in Section V.

In what follows, $j = \sqrt{-1}$; bold upper case and lower case letters denote matrices (sets) and vectors, respectively; $(\cdot)^R$ and $(\cdot)^I$ stand for the real and imaginary parts of a complex element vector and matrix, respectively.

## II. SIGNAL MODEL

We focus on an $N \times M$ MIMO system employing $N$ transmit and $M$ receive antennas over the quasi-static flat fading channel in this paper. The transmitted signal sequences across $N$ transmit antennas over $T$ symbol durations is arranged by the STBC matrix $\mathbf{X}_{T\times N} = \sum_{l=1}^{L} s_l \mathbf{C}_l$ that consists of $\{s_1, s_2, \cdots, s_L\}$ where $s_l$ are real-valued[1] information symbols, $\mathbf{C}_l \in \mathbb{C}^{T\times N}$ are called dispersion matrices. Thus, the code rate is $\frac{L}{2T}$ considering complex symbol transmission. As stated in [5], a $\Gamma$-group-decodable STBC is defined as:

**Definition 1** ( [5]). An STBC is said to be $\Gamma$-group-decodable if

(i) $\mathbf{C}^H_{\gamma_1,p}\mathbf{C}_{\gamma_2,q} = -\mathbf{C}^H_{\gamma_2,q}\mathbf{C}_{\gamma_1,p}$, where $\forall p \in \Theta_{\gamma_1}$, $\forall q \in \Theta_{\gamma_2}, \gamma_1 \neq \gamma_2$;

(ii) $\begin{bmatrix}\mathbf{C}^R_{\gamma,1}\\ \mathbf{C}^I_{\gamma,1}\end{bmatrix}, \cdots \begin{bmatrix}\mathbf{C}^R_{\gamma,k}\\ \mathbf{C}^I_{\gamma,k}\end{bmatrix}, \cdots \begin{bmatrix}\mathbf{C}^R_{\gamma,L_\gamma}\\ \mathbf{C}^I_{\gamma,L_\gamma}\end{bmatrix}$ are linearly independent, where $k = 1, 2, \cdots, L_\gamma \in \Theta_\gamma$, $\Theta_\gamma$ is the set of indexes of symbols in the $\gamma$-th group, $\gamma = \mathrm{I}, \mathrm{II}, \cdots, \Gamma$. And the code matrix $\mathbf{X}$ will be expressed as:

$$\mathbf{X} = \sum_{\gamma=\mathrm{I}}^{\Gamma} \sum_{k=1}^{L_\gamma} s_{\gamma,k}\mathbf{C}_{\gamma,k} \tag{1}$$

In Def. 1, condition (i) guarantees that the symbols $s_{\gamma_1,p}$ and $s_{\gamma_2,q}$ are in different groups, called the quasi-orthogonality constraint (QOC) [6], and condition (ii) guarantees that the decoder of any group is non-rank-deficient [5].

## III. CODE CONSTRUCTION

In this section, we propose a method to construct group-decodable STBC with arbitrary code dimensions, and show two 3-time-slot code examples.

Manuscript received August 29, 2011. The associate editor coordinating the review of this letter and approving it for publication was W. Hamouda.

The work of C. Yuen was supported by the International Design Center (Grant No. IDG31100102 & IDD11100101).

T. P. Ren and K. H. Wang are with 63790 troops, Xichang 615000, China (e-mail: tpren@nudt.edu.cn, wangkl2006@126.com).

C. Yuen is with the Singapore University of Technology and Design, Singapore 279623 (e-mail: yuenchau@sutd.edu.sg).

Y. L. Guan is with the School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore 639798 (e-mail: eylguan@ntu.edu.sg).

Digital Object Identifier 10.1109/LCOMM.2011.110711.111783

[1]The in-phase component or the quadrature component of a complex information symbol is real, hence, this signal model is also applicable for complex information symbol transmission.

**Construction**: Suppose that $\{\mathbf{A}_{\mathrm{I},1},\cdots,\mathbf{A}_{\mathrm{I},L_{\mathrm{I}}};\cdots;\mathbf{A}_{\Gamma,1},\cdots,\mathbf{A}_{\Gamma,L_\Gamma}\}$ and $\{\mathbf{B}_{\mathrm{I},1},\cdots,\mathbf{B}_{\mathrm{I},K_{\mathrm{I}}};\cdots;\mathbf{B}_{\Lambda,1},\cdots,\mathbf{B}_{\Lambda,K_\Lambda}\}$ are the dispersion matrices of two original $\Gamma$-group-decodable and $\Lambda$-group-decodable STBC with transmit antenna numbers $N$, time slots $T_1$ and $T_2$, respectively. Without loss of generality, we assume $\Gamma \geq \Lambda$. Then we will construct a $\Gamma-$group-decodable STBC with $N$ transmit antenna and $T = T_1 + T_2$ time slot. The dispersion matrices in the $\gamma$-th group of the newly constructed code can be designed as:

For $\gamma = 1,\cdots,\Lambda$,

$$\{\mathbf{C}_{\gamma,1},\cdots,\mathbf{C}_{\gamma,L_\gamma},\mathbf{C}_{\gamma,L_\gamma+1},\cdots,\mathbf{C}_{\gamma,L_\gamma+K_\gamma}\} = \left\{\begin{bmatrix}\mathbf{A}_{\gamma,1}\\ -\mathbf{B}_{\gamma,1}\end{bmatrix},\right.$$
$$\left.\cdots,\begin{bmatrix}\mathbf{A}_{\gamma,L_\gamma}\\ -\mathbf{B}_{\gamma,1}\end{bmatrix},\begin{bmatrix}\mathbf{A}_{\gamma,L_\gamma}\\ \mathbf{B}_{\gamma,1}\end{bmatrix},\cdots,\begin{bmatrix}\mathbf{A}_{\gamma,L_\gamma}\\ \mathbf{B}_{\gamma,K_\gamma}\end{bmatrix}\right\}; \quad (2a)$$

For $\gamma = \Lambda+1,\cdots,\Gamma$,

$$\{\mathbf{C}_{\gamma,1},\mathbf{C}_{\gamma,2},\cdots,\mathbf{C}_{\gamma,L_\gamma}\} = \left\{\begin{bmatrix}\mathbf{A}_{\gamma,1}\\ \mathbf{0}\end{bmatrix},\begin{bmatrix}\mathbf{A}_{\gamma,2}\\ \mathbf{0}\end{bmatrix},\cdots,\begin{bmatrix}\mathbf{A}_{\gamma,L_\gamma}\\ \mathbf{0}\end{bmatrix}\right\}. \quad (2b)$$

We can derive two propositions on non-rank-deficiency and code rate as follows. The proof of Proposition 1 is straightforward hence omitted.

**Proposition 1** (Non-rank-deficiency)**.** If both the original $\Gamma$-group-decodable and $\Lambda$-group-decodable STBC are non-rank-deficiency (i.e., $\begin{bmatrix}\mathbf{A}^R_{\gamma,1}\\ \mathbf{A}^I_{\gamma,1}\end{bmatrix},\begin{bmatrix}\mathbf{A}^R_{\gamma,2}\\ \mathbf{A}^I_{\gamma,2}\end{bmatrix},\cdots,\begin{bmatrix}\mathbf{A}^R_{\gamma,L_\gamma}\\ \mathbf{A}^I_{\gamma,L_\gamma}\end{bmatrix}$ are linearly independent, and $\begin{bmatrix}\mathbf{B}^R_{\lambda,1}\\ \mathbf{B}^I_{\lambda,1}\end{bmatrix},\begin{bmatrix}\mathbf{B}^R_{\lambda,2}\\ \mathbf{B}^I_{\lambda,2}\end{bmatrix},\cdots,\begin{bmatrix}\mathbf{B}^R_{\lambda,L_\lambda}\\ \mathbf{B}^I_{\lambda,L_\lambda}\end{bmatrix}$ are linearly independent [5]), the newly constructed $\Gamma$-group-decodable STBC is non-rank-deficiency (i.e., $\begin{bmatrix}\mathbf{C}^R_{\gamma,1}\\ \mathbf{C}^I_{\gamma,1}\end{bmatrix},\begin{bmatrix}\mathbf{C}^R_{\gamma,2}\\ \mathbf{C}^I_{\gamma,2}\end{bmatrix},\cdots,\begin{bmatrix}\mathbf{C}^R_{\gamma,L_\gamma+K_\gamma}\\ \mathbf{C}^I_{\gamma,L_\gamma+K_\gamma}\end{bmatrix}$ are linearly independent, where $\mathbf{C}_{\gamma,1},\cdots,\mathbf{C}_{\gamma,L_\gamma+K_\gamma}$ are obtained following (2), $\gamma = 1,\cdots,\Gamma$, $\lambda = 1,\cdots,\Lambda$.

Proposition 1 shows that with a code obtained from the proposed code construction, its decoder of any group is non-rank-deficient [5].

In the code construction, denoting $R_1$, $R_2$ and $R$ as the code rates of the two original codes and the resultant code respectively, we have $R_1 = \frac{L}{2T_1}$, $R_2 = \frac{K}{2T_2}$ and $R = \frac{L+K}{2(T_1+T_2)}$, where $L = \sum_{\gamma=1}^{\Gamma} L_\gamma$ and $K = \sum_{\lambda=1}^{\Lambda} K_\lambda$.

**Proposition 2** (Code rate)**.** $R \geq \min(R_1, R_2)$, and $R$ is

(i) equal to $\frac{R_1+R_2}{2}$ if the two original codes have the same time slot or code rate;

(ii) larger than $\frac{R_1+R_2}{2}$ if the original code with longer time slot has higher code rate;

(iii) smaller than $\frac{R_1+R_2}{2}$ if the original code with longer time slot has lower code rate.

*Proof:* Without loss of generality, we assume $R_1 = \frac{L}{2T_1} \geq R_2 = \frac{K}{2T_2}$. Hence,

$$R = \frac{L+K}{2(T_1+T_2)} = \left(\frac{L}{2T_1}+\frac{K}{2T_1}\right)\frac{T_1}{T_1+T_2}$$
$$\geq \left(\frac{K}{2T_2}+\frac{K}{2T_1}\right)\frac{T_1}{T_1+T_2} = \frac{K}{2T_2} = R_2 = \min(R_1,R_2).$$

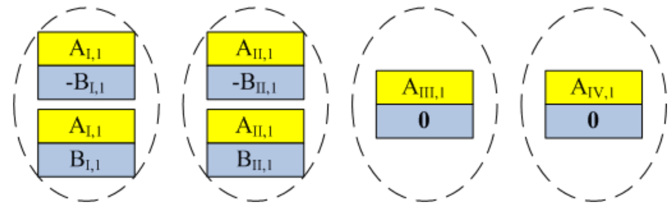


Fig. 1. The dispersion matrices of $\mathbf{X}_{\text{proposed}}$ obtained from $\{\mathbf{A}\}$ in (3) and $\{\mathbf{B}\}$ in (4) are in 4 groups.

To prove (i),(ii) and (iii), we can see that

$$\begin{aligned}R-\frac{R_1+R_2}{2} &= \frac{L+K}{2(T_1+T_2)}-\frac{1}{2}\left(\frac{L}{2T_1}+\frac{K}{2T_2}\right)\\ &= \frac{LT_1T_2+KT_1T_2-(LT_2^2+KT_1^2)}{4(T_1+T_2)T_1T_2}\\ &= \frac{1}{2(T_1+T_2)}(T_1-T_2)(R_1-R_2).\end{aligned}$$

When the two original codes have the same time slot or code rate (i.e., $T_1 - T_2 = 0$ or $R_1 - R_2 = 0$), $R - \frac{R_1+R_2}{2} = 0$. Similarly, when the longer original code has higher code rate, $R - \frac{R_1+R_2}{2} > 0$; when the longer original code has lower code rate, $R - \frac{R_1+R_2}{2} < 0$.

Hence, (i), (ii) and (iii) in Proposition 2 are proved. □

### *A. Code Example I: 3×2 rate-1 symbol-wise decodable STBC*

The dispersion matrices of Alamouti code [7] are presented in 4 groups as follows:

$$\begin{aligned}&\{\mathbf{A}_{\mathrm{I},1};\mathbf{A}_{\mathrm{II},1};\mathbf{A}_{\mathrm{III},1};\mathbf{A}_{\mathrm{IV},1}\}\\ =&\left\{\begin{bmatrix}1&0\\0&1\end{bmatrix};\begin{bmatrix}j&0\\0&-j\end{bmatrix};\begin{bmatrix}0&1\\-1&0\end{bmatrix};\begin{bmatrix}0&j\\j&0\end{bmatrix}\right\}.\end{aligned} \quad (3)$$

To construct a 3-time-slot code, we consider a 1-time-slot code with dispersion matrices satisfying QOC as follows:

$$\{\mathbf{B}_{\mathrm{I},1};\mathbf{B}_{\mathrm{II},1}\} = \{\begin{bmatrix}1&1\end{bmatrix};\begin{bmatrix}j&j\end{bmatrix}\}. \quad (4)$$

Based on the $\{\mathbf{A}\}$ in (3) and the $\{\mathbf{B}\}$ in (4), $\mathbf{X}_{\text{proposed}}$ can be obtained using (1), (2a) and (2b), and is as shown in (5) following the code construction illustrated in Fig. 1.

$$\begin{aligned}\mathbf{X}_{\text{proposed}} &= \sum_{\gamma=\mathrm{I}}^{\mathrm{IV}}\sum_{k=1}^{L_\gamma} s_{\gamma,k}\mathbf{C}_{\gamma,k}\\ &= \begin{bmatrix}s_1+s_2+js_3+js_4 & s_5+js_6\\ -s_5+js_6 & s_1+s_2-js_3-js_4\\ -s_1+s_2-js_3+js_4 & -s_1+s_2-js_3+js_4\end{bmatrix}\end{aligned} \quad (5)$$

where

$$\begin{aligned}&\{\mathbf{C}_{\mathrm{I},1},\mathbf{C}_{\mathrm{I},2};\mathbf{C}_{\mathrm{II},1},\mathbf{C}_{\mathrm{II},2};\mathbf{C}_{\mathrm{III},1};\mathbf{C}_{\mathrm{IV},1}\}\\ =&\left\{\begin{bmatrix}1&0\\0&1\\-1&-1\end{bmatrix},\begin{bmatrix}1&0\\0&1\\1&1\end{bmatrix};\begin{bmatrix}j&0\\0&-j\\-j&-j\end{bmatrix},\right.\\ &\left.\begin{bmatrix}j&0\\0&-j\\j&j\end{bmatrix};\begin{bmatrix}0&1\\-1&0\\0&0\end{bmatrix};\begin{bmatrix}0&j\\j&0\\0&0\end{bmatrix}\right\}.\end{aligned}$$

Obviously, the proposed code $\mathbf{X}_{\text{proposed}}$ is full-rate (code rate 1). Due to one complex symbol (i.e., two dispersion matrices) in each group at most, $\mathbf{X}_{\text{proposed}}$ is symbol-wise decodable.

### *B. Code Example II: 3×2 rate-7/6 group-decodable STBC*

For higher code rate, we consider a 1-time-slot code with dispersion matrices as follows:

$$\{\mathbf{B}_{\mathrm{I},1},\mathbf{B}_{\mathrm{I},2},\mathbf{B}_{\mathrm{I},3}\} = \{\begin{bmatrix} 1 & 0 \end{bmatrix}, \begin{bmatrix} j & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \end{bmatrix}\}. \quad (6)$$

Following the code construction, a new code based on $\{\mathbf{A}\}$ in (3) and $\{\mathbf{B}\}$ in (6) can be obtained as follows:

$$\begin{aligned}&\{\mathbf{C}_{\mathrm{I},1},\mathbf{C}_{\mathrm{I},2},\mathbf{C}_{\mathrm{I},3},\mathbf{C}_{\mathrm{I},4};\mathbf{C}_{\mathrm{II},1};\mathbf{C}_{\mathrm{III},1};\mathbf{C}_{\mathrm{IV},1}\}\\ &=\{\begin{bmatrix} 1 & 0\\ 0 & 1\\ -1 & 0\end{bmatrix},\begin{bmatrix} 1 & 0\\ 0 & 1\\ 1 & 0\end{bmatrix},\begin{bmatrix} 1 & 0\\ 0 & 1\\ j & 0\end{bmatrix},\begin{bmatrix} 1 & 0\\ 0 & 1\\ 0 & 1\end{bmatrix};\\ &\quad\begin{bmatrix} j & 0\\ 0 & -j\\ 0 & 0\end{bmatrix};\begin{bmatrix} 0 & 1\\ -1 & 0\\ 0 & 0\end{bmatrix};\begin{bmatrix} 0 & j\\ j & 0\\ 0 & 0\end{bmatrix}\}.\end{aligned} \quad (7)$$

The resultant code has a code rate of $7/6$, which is higher than 1. Since there are two complex symbols (i.e., four dispersion matrices) in the first group, the decoding complexity level is the same as the code in [4], i.e., $\mathbf{X}_{\mathrm{LYC}}$ in (9).

## IV. Simulation and Discussion

In a 2×1 MIMO system, we compare the proposed full-rate 3-time-slot code $\mathbf{X}_{\mathrm{proposed}}$ in (5) with $\mathbf{X}_{\mathrm{AL}}$ [3] in (8) and $\mathbf{X}_{\mathrm{LYC}}$ [4] in (9), where the MIMO channel is assumed to be quasi-static Rayleigh fading and the channel state information is perfectly known at the receiver. The information symbols in $\mathbf{X}_{\mathrm{proposed}}$, $\mathbf{X}_{\mathrm{AL}}$ and $\mathbf{X}_{\mathrm{LYC}}$ are all modulated by 4QAM and 8QAM, hence their spectral efficiencies are 2 bits per channel use (bpcu) and 3 bpcu, respectively. Alamouti code [7] followed by 1-time-slot blank transmission (i.e., $\mathbf{X}_{\mathrm{AL}}$ with $s_5 = 0$ and $s_6 = 0$) is modulated by 8QAM (hence 2 bpcu) and is used as benchmark.

$$\mathbf{X}_{\mathrm{AL}} = \begin{bmatrix} s_1 + js_2 & s_3 + js_4\\ s_3 - js_4 & -s_1 + js_2\\ s_5 + js_6 & s_5 + js_6 \end{bmatrix} \quad (8)$$

$$\mathbf{X}_{\mathrm{LYC}} = \begin{bmatrix} s_1 + js_2 & \frac{(s_1-js_2)+2e^{j\frac{2\pi}{5}}(s_3-js_4)+2e^{-j\frac{2\pi}{5}}(s_5-js_6)}{3}\\ s_3 + js_4 & \frac{-2e^{j\frac{2\pi}{5}}(s_1-js_2)+e^{-j\frac{\pi}{5}}(s_3-js_4)+2(s_5-js_6)}{3}\\ s_5 + js_6 & \frac{-2e^{-j\frac{2\pi}{5}}(s_1-js_2)+2(s_3-js_4)+e^{j\frac{\pi}{5}}(s_5-js_6)}{3} \end{bmatrix} \quad (9)$$

In order to achieve power balance, the symbol $s_5 + js_6$ in $\mathbf{X}_{\mathrm{proposed}}$ is transmitted with power $\sqrt{2}$. For full diversity, constellation rotation factors for $\mathbf{X}_{\mathrm{proposed}}$ are obtained by computer search[2]. We plot the BER curves of the codes in Fig. 2, which shows that $\mathbf{X}_{\mathrm{proposed}}$ does achieve full diversity, hence the same BER slope as Alamouti code. Even with lower decoding complexity, our $\mathbf{X}_{\mathrm{proposed}}$ has better BER performances than $\mathbf{X}_{\mathrm{LYC}}$.

[2]Optimized constellation rotation factors are $e^{j0.0781\pi}$ and $e^{j0.1765\pi}$ for each complex symbol (i.e., $s_1 + js_2$, $s_3 + js_4$ and $s_5 + js_6$) with 4QAM and 8QAM modulation, respectively.

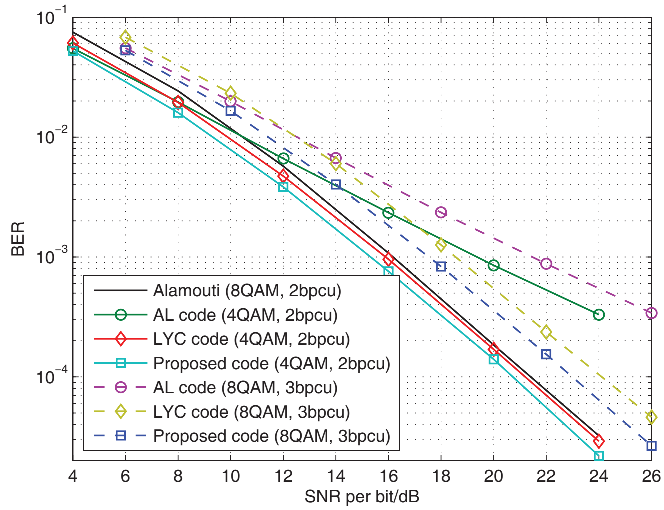


Fig. 2. BER curves of AL code, LYC code and the proposed code (5) benchmarked by Alamouti code.

## V. Conclusion

In this paper, we propose a method for constructing group-decodable STBC with arbitrary code dimensions, including odd time slot. Using the method, 3-time-slot two-transmit-antenna STBCs achieving full or even higher rate, which are suitable for the 3GPP LTE uplink frame structure, are constructed. The proposed full-rate 3-time-slot code can achieve symbol-wise decoding complexity (the lowest decoding complexity for 3×2 full-rate full-diversity code as there is no orthogonal design for such code dimension) and full transmit diversity for all the information symbols. This leads to better performance, and simplifies the modulation and coding scheme selection in the 3GPP LTE system design.

## References

[1] 3rd Generation Partnership Project, Evolved Universal Terrestrial Radio Access (E-UTRA); Physical Channels and Modulation(Release 8), 3GPP TS 36.211, Nov. 2008.
[2] Alcatel Shanghai Bell, Alcatel-Lucent, "STBC-II scheme for uplink transmit diversity in LTE-Advanced," R1-082500, 3GPP TSG RAN WG 1 Meeting #53 bis, June-July 2008.
[3] Alcatel Shanghai Bell, Alcatel-Lucent, "STBC-II scheme with non-paired symbols for LTE-Advanced uplink transmit diversity," R1-090058, 3GPP TSG RAN WG 1 Meeting #55 bis, Jan. 2009.
[4] Z. Lei, C. Yuen, and F. Chin, "Quasi-orthogonal space-time block codes for two transmit antennas and three time slots," *IEEE Trans. Wireless Commun.*, vol. 10, no. 6, pp. 1983–1991, June 2011.
[5] T. P. Ren, Y. L. Guan, C. Yuen, E. Gunawan, and E. Y. Zhang, "Group-decodable space-time block codes with code rate > 1," *IEEE Trans. Commun.*, vol. 59, no. 4, pp. 987–997, Apr. 2011.
[6] C. Yuen, Y. L. Guan, and T. T. Tjhung, "Quasi-orthogonal STBC with minimum decoding complexity," *IEEE Trans. Wireless Commun.*, vol. 4, no. 5, pp. 2089–2094, Sep. 2005.
[7] S. M. Alamouti, "A simple transmitter diversity scheme for wireless communication," *IEEE J. Sel. Areas Commun.*, vol. 16, pp. 1451–1458, Oct. 1998.