Adaptive Distributed MIMO Radar Waveform Optimization Based on Mutual Information

Y. Nijuure, Y. Chen, C. Yuen, Y. H. Chew, Z. Ding, S. Bouassakta

A novel approach to optimizing the waveforms of an adaptive distributed multiple-input multiple-output (MIMO) radar is developed. The research work aims at improving the target detection and feature extraction performance by maximizing the mutual information (MI) between the target impulse response and the received echoes in the first step, and then minimizing the MI between successive backscatter signals in the second step. These two stages correspond to the design of the ensemble of excitations and the selection of a suitable signal out of the ensemble, respectively. The waveform optimization algorithm is based upon adaptive learning from the radar scene, which is achieved through a feedback loop from the receiver to the transmitter. This feedback includes vital information about the target features derived from the reflected pulses. In this way the transmitter adjusts its probing signals to suit the dynamically changing environment. Simulation results demonstrate better target response extraction using the proposed two-step algorithm as compared with each single-step optimization method. This approach also results in improved target detection probability and delay-Doppler resolution as the number of iterations increases.

I. INTRODUCTION

In recent years, the research on adaptive radar waveform design has received great impetus. Some of the noteworthy works in this area include [1]–[5], where the radar transmission parameters are continuously modified in order to improve the target parameter estimation in a time-varying radar environment. Another related development is the proposal of cognitive radar, which represents an innovative paradigm to describe brain-empowered system architectures that constantly employ information-gathering mechanisms to facilitate intelligent illumination of the dynamic radar scene. For a cognitive radar the perception about the radar environment formed at the receiver is relayed to the transmitter through a continuous feedback loop. Subsequently, the updated information about the environment can be utilized to allocate crucial resources such as transmit power and spectrum in a more efficient manner [6, 7]. Such a constant learning approach allows the development of waveform optimization techniques offering better target resolution capabilities as shown in [8], [9].

Recent results have also shown that a multiple-input multiple-output (MIMO) radar, which employs multiple transmit and receive antennas, can fully exploit waveform and spatial diversity gains by illuminating the target in different directions [10–12]. MIMO radars employ orthogonal signals at distinct transmit antenna elements, which excite different scattering centers on the extended targets, thus enhancing the information content in the received backscatter signal. Since the target returns are strongly dependent on the cross-sectional areas of scatterers in the line-of-sight directions of radar elements, the spatial diversity provided by the distributed MIMO radar improves the target parameter extraction as shown in [13]–[16]. In terms of the pulse optimization, an important school of thought is to apply information theory to radar signal processing [17]. The pioneering work by Bell [18] applied information-theoretic measures for the design of waveforms in order to facilitate improved target detection and classification. Yang and Blum [19] extended the work in [18] by using the mutual information (MI) between the random target response and the reflected signal as a waveform optimization criterion in the MIMO radar configuration. Other existing works [13–15, 20, 21] also utilize similar design criteria.

Following from the above discussions, it is interesting to study the performance of a knowledge-aided MIMO radar that combines the strength of “adaptivity” and “MIMO.” Specifically, we look into the problem of excitation waveform design and present a novel two-stage optimization strategy, which can be summarized as follows.

1) Step 1. Waveform Design: This module involves the design of transmission signals for the distinct MIMO transmit antenna elements. The main objective is to maximize the MI between the backscatter signal and the estimated target response, subject to the transmission power constraint [20]. This approach ensures that the received echoes at each time instant become more statistically dependent on the target features. Once the optimal waveform ensemble is obtained, the next step is to select the appropriate signals for transmission.

2) Step 2. Waveform Selection: This module is based on the principle of minimizing the MI between successive received signals. This selection criterion ensures that we always acquire target echoes that are more statistically independent on each other in time,
with an intention of gaining more knowledge about
the target features at each time instant of reception.

Furthermore, the optimization process is preceded
by channel estimation, wherein an estimate of the
target response and noise characteristics is formed
by the receiver through measurements carried out
in the previous time instant. A feedback loop from
the receiver to the transmitter allows the delivery
of this radar scene information to the transmitter.
Consequently, the excitation signal optimization
process enables the transmitter to dynamically
adjust its operational mode to suit the changing
radar environment. Such architectures are similar to
cognitive radars, where the systems utilize a constant
learning approach by updating the estimates on target
scene parameters through multiple interactions with
the environment.

The contributions of the current work can be
summarized as follows:

1) development of a practical framework for an
adaptive distributed MIMO radar, which benefits from
the principle of cognition as elaborated in [6];
2) design of a novel two-stage algorithm for
waveform optimization in the proposed adaptive
distributed MIMO radar framework;
3) evaluation of the performance of the current
approach in terms of target feature extraction, target
detection, and delay-Doppler resolution; and
4) comparison with other prevalent radar system
configurations through analysis of the target scene
generation and receiver operating characteristics
(ROC) in an interference-prone setup.

The rest of the paper is organized as follows. In
Section II we describe the adaptive distributed MIMO
radar system and formulate the two-step waveform
optimization process. The probing signals are
designed at step 1 of the algorithm and selected based
on the criterion presented in step 2. In Section III
we analyze the delay-Doppler resolution achieved
by the optimized waveform ensemble. In Section IV
we present simulation results for the mean squared
error (MSE) of target response estimation, probability
detection, and ambiguity function (AF) by using
the proposed algorithm, and compare it with other
widely used techniques for a clutter-corrupted target
scene. Finally, some concluding remarks are drawn
in Section V. Throughout this work, we use \( \det(\cdot) \) to
denote the determinant of a matrix, \( (\cdot)^T \) to denote the
transpose, \( (\cdot)^H \) to denote the Hermitian transpose, \( \text{tr}(\cdot) \) to
denote the trace, and \( \mathbb{E}[\cdot] \) to denote the expectation
operator.

II. WAVEFORM OPTIMIZATION STRATEGY

A. System Architecture

Figure 1 indicates the general architecture of the
adaptive distributed MIMO radar under consideration.

As discussed extensively in the existing literature,
modern radar applications make use of the pulse
compression techniques such as linear frequency
modulation or phase-coded waveforms employing
Barker codes or Costas codes in order to improve the
target delay-Doppler resolution [22]. In this work we
adopt the idea of utilizing phase-coded waveforms
for generation of sequences required for transmission
over various transmit antennas. We consider the
ultrawideband (UWB) probing signals [7], though
the general methodology is also applicable to any
other type of excitation. The waveform comprises a
sequence of UWB Gaussian pulses in which the phase
of each pulse is modulated in accordance with the
orthogonal sequences corresponding to the column
vectors of a Hadamard matrix [22]. Each normalized
Gaussian pulse takes the following form

\[
u(t) = \frac{1}{\sqrt{2\pi T}} \exp \left( -\frac{t^2}{2T^2} \right)
\]

where \( T \) determines the pulsewidth and is assumed to
be 0.2 ns.

As indicated in Fig. 1, a set of orthogonal
sequences of Gaussian waveforms ready to be sent
over each of the transmit antenna elements are
constructed at the waveform optimization module,
which is discussed in detail in Section II-C. The
optimized waveforms are then transmitted over
the radar channels. The backscatter signals are
gathered by each of the receive antenna elements
and passed on to a matched filter bank, which
 correlates the received signals with each individual
transmit waveform stored in the receiver. Target
response is thus extracted by the target detection
and parameter estimation module, which attempts
to discriminate the target from the surrounding
clutter. The estimated channel response and received
signal characteristics such as noise covariance are
forwarded to the waveform optimization module
through a feedback link. In light of the updated radar
scene, the optimization module designs and selects
suitable sequences for each of the transmit antennas
in order to acquire the best knowledge on the target
in the next time instant. This operation facilitates
adaptive illumination of the radar environment and essentially leads to a cognitive system featuring the following two important properties described in [6]: 1) intelligent signal processing, which builds on real-time learning through continuous interaction of the radar with the surroundings, and 2) feedback from the receiver to the transmitter, which is a facilitator of intelligence.

B. Signal Model

Without loss of generality, we consider the bistatic radar configuration, where the transmitter and receiver are connected but noncollocated. It is assumed that the two direct paths, one between the transmitter and the target and the other between the target and the receiver, have been extracted through some preprocessing steps [23]. For example, the MIMO radar may employ beam-steering [7] and delay windowing [24] to suppress nontarget impulse responses. Thus we focus on the direct path in the following.

Suppose that the distributed MIMO system has \( M \) transmit and \( N \) receive antennas. For simplicity of discussion, it is assumed that \( M = N \). We can express the received signal vector at the \( n \)th \((n = 1, 2, \ldots, N)\) antenna element as [12]

\[
y_n = \sum_{m=1}^{M} h_{m,n} \hat{x}_{m,n} + \eta_n. \tag{2}
\]

In the preceding equation \( y_n \in \mathbb{C}^{K \times 1} \) where \( \mathbb{C} \) indicates the complex number domain. The parameter \( K = K_t + K_d \), where \( K_t \) is the length of the pseudorandom sequence generated and \( K_d \) is the maximum excess delay with respect to the first arrival among all the links. The term \( \hat{x}_{m,n} = [\theta_{1 \times L_{m,n}} \ x_m^T \ 0_{(K_d-L_{m,n})}]^T \), where \( \theta_{1 \times L} \) is a null vector of length \( L \), \( L_{m,n} \) is the propagation delay between the \( m \)th transmit and the \( n \)th receive antennas via the target, and \( x_m \in \mathbb{C}^{K_t \times 1} \) is the probing signal sent by the \( m \)th transmitter. \( h_{m,n} \) represents the channel response between the \( m \)th transmit and the \( n \)th receive antennas. Finally, \( \eta_n \in \mathbb{C}^{K \times 1} \) denotes the noise at the \( n \)th receiver, which characterizes the combination of both additive white Gaussian noise (AWGN) and joint nuisance of the channel estimation and measurement errors. In this work we consider the scenario that the length of the pseudorandom sequence is much larger than the excess delay, which is generally valid for short-range radar applications when the propagation distance is within tens of meters. As a result, \( K \approx K_t \) and \( x_m \) can be used to approximate \( \hat{x}_{m,n} \). A similar condition has also been imposed in other recent works like [20]. We further assume that the minimum transmit/receive antenna spacing is sufficiently larger than half wavelength (distributed MIMO configuration). Hence, the correlation introduced by finite antenna element spacing is low enough that the fades associated with two different antenna elements can be considered independent. Subsequently, the reflection coefficient \( h_{m,n} \) is assumed to be different for various pairs of transmitter and receiver, and its phase is assumed to be uniformly distributed between 0 and \( 2\pi \).

Let \( Y = [y_1, y_2, \ldots, y_N] \in \mathbb{C}^{K \times N} \) be the ensemble of received signals, \( X = [x_1, x_2, \ldots, x_M] \in \mathbb{C}^{K \times M} \) be the set of transmission sequences to be used, \( H = [h_{m,n}]_{M \times N} \in \mathbb{C}^{M \times N} \) be the target response matrix, and \( \Theta = [\eta_1, \eta_2, \ldots, \eta_N] \in \mathbb{C}^{K \times N} \) be the noise matrix. We can conveniently express (2) as

\[
Y = XH + \Theta. \tag{3}
\]

Each \( h_{m,n} \) in the scattering matrix \( H \) is proportional to the target radar cross section (RCS), whose scintillation can vary slowly or rapidly depending on the target size, shape, dynamics, and its relative motion with respect to the radar. The two random matrices \( H \) and \( \Theta \) are assumed to be independent of each other. A detailed description of the construction of \( H \) is presented in Appendix I.

We consider the target as a point scatterer amidst several clutter sources. Depending upon the motion and clutter characteristics, the radar target has been classified into various Swerling models [25]. In Swerling III the RCS samples measured by the radar are correlated throughout an entire scan, but are uncorrelated from scan to scan (slow fluctuation) and the radar scene comprises a single powerful scattering center and many weak reflectors in its vicinity. This model is applied in the current work, where we assume that the radar scene is dominated by the target and the amplitude returns from nontarget scatterers are lower than those from the target. The random RCS \( \xi = |h_{m,n}|^2 \) is exponentially distributed, which takes the following form

\[
f(\xi) = \frac{1}{\xi_{av}} \exp\left(-\frac{\xi}{\xi_{av}}\right) \tag{4}
\]

where \( \xi > 0 \) represents the variance of RCS fluctuations and \( \xi_{av} \) is the average RCS.

C. Two-Stage Waveform Optimization

The waveform design and selection processes can be formulated as the following two-step algorithm. Note that the subscript \( i \) is used to indicate the parameters for a particular round of radar system adaptation at time \( t \).

Step 1 Maximization of MI between the estimated target response and the received target echoes at time \( t \).

Following the definition of MI,

\[
I(Y_i; H_i | X_i) = \mathcal{H}(Y_i | X_i) - \mathcal{H}(Y_i | H_i, X_i) \\
= \mathcal{H}(Y_i | X_i) - \mathcal{H}(\Theta_i) \tag{5}
\]
where $I(Y_t;H_j | X_t)$ is the MI between two random variables $Y_t$ and $H_j$ given the transmission matrix $X_t$, and $\mathcal{H}(Y_t | X_t)$ represents the conditional entropy or the average information that $Y_t$ conveys about $X_t$.

Our aim is to maximize $I(Y_t;H_j | X_t)$ between $Y_t$ and $H_j$, given $X_t$, i.e., we intend to maximize the MI between the received target echoes and the channel response given the ensemble of transmit waveforms. This implies that the backscatter signals would be more statistically dependent upon the actual radar scene. We can simplify (5) by applying the definition of entropy as follows

$$\mathcal{H}(Y_t | X_t) = -\int p(Y_t | X_t) \ln[p(Y_t | X_t)] dY_t$$

(6)

where $p(Y_t | X_t)$ denotes the conditional probability density function (pdf) of $Y_t$ given $X_t$. The above expression for the entropy can be further simplified by evaluating $p(Y_t | X_t)$ to be

$$p(Y_t | X_t) = \prod_{n=1}^{N} p(y_{tn} | X_t)$$

$$= \prod_{n=1}^{N} \frac{1}{\pi^{\frac{N}{2}} \det(X_t^H R_{H} X_t + R_{\Theta})}$$

$$\times \exp[-\frac{1}{2} \frac{y_{tn}^H (X_t^H R_{H} X_t + R_{\Theta})^{-1} y_{tn}}{\pi^{\frac{N}{2}} \det(X_t^H R_{H} X_t + R_{\Theta})}]$$

$$= \prod_{n=1}^{N} \frac{1}{\pi^{\frac{N}{2}} \det(X_t^H R_{H} X_t + R_{\Theta})}$$

$$\times \exp[-\frac{1}{2} \text{tr}[(X_t^H R_{H} X_t + R_{\Theta})^{-1} y_{tn}^H Y_t]]$$

(7)

where $R_{H} = \mathbb{E}(H_j^H H_j)$ and $R_{\Theta} = \mathbb{E}(\Theta_j^H \Theta_j)$ are the covariance matrices of the target response $H_j$ and the noise $\Theta_j$, respectively. Solving (6) and (7) gives rise to the following result for the entropy [20]

$$\mathcal{H}(Y_t | X_t) = NK \ln(\pi) + \frac{NK}{2} + N \ln[\det(X_t^H R_{H} X_t + R_{\Theta})].$$

(8)

Similarly, we can derive the entropy of the noise as

$$\mathcal{H}(\Theta_j) = NK \ln(\pi) + \frac{NK}{2} + N \ln[\det(R_{\Theta})].$$

(9)

Using (5), (8), and (9), we can compute the MI as

$$I(Y_t;H_j | X_t) = N \ln[\det(X_t^H R_{H} X_t + R_{\Theta})] - N \ln[\det(R_{\Theta})].$$

(10)

Hence, the maximization in step 1 can be simplified as

$$\max_{X_t} \{N \ln[\det(X_t^H R_{H} X_t + R_{\Theta})] - N \ln[\det(R_{\Theta})]\}$$

subject to \ $\text{tr}[X_t^H X_t] \leq P_0$

(11)

with $P_0$ being the total transmission power.

A rigorous solution of (11) has been provided in [20]. We can then find the optimal waveform ensemble $\tilde{\mathcal{X}}_X$ out of the entire set of orthogonal sequences from the Hadamard matrix, and the corresponding power allocation vector over different antenna elements, $\Psi_{\tilde{X}X}$, for each $\tilde{X}_X \in \tilde{\mathcal{X}}_X$. In other words, we start our waveform design with the orthogonal sequences from the Hadamard matrix, and modulate the power of the waveform on an individual pulse level as well as across the transmit antenna elements using the maximization criterion presented in (11).

**Step 2** Minimization of MI between the received target echoes at time $t$ and the estimated target echoes at time $t + 1$.

We now proceed to the second module of the waveform optimization process, in which we intend to ensure that successive target echoes are as different from each other as possible. This would ensure that at every instant of reception, we learn something more about the radar scene.

We can express the MI between the received signals in two consecutive times, $t$ and $t + 1$, as

$$I(Y_t,Y_{t+1}) = \mathcal{H}(Y_t | X_t) + \mathcal{H}(Y_{t+1} | X_{t+1}) - \mathcal{H}(Y_t,Y_{t+1} | X_t,X_{t+1}).$$

(12)

In the preceding equation $\mathcal{H}(Y_t | X_t)$ (or $\mathcal{H}(Y_{t+1} | X_{t+1})$) denotes the measure of the uncertainty in the received signal at time $t$ (or $t + 1$) given the knowledge of the transmitted signal $X_t$ (or $X_{t+1}$). Furthermore, $\mathcal{H}(Y_t,Y_{t+1} | X_t,X_{t+1})$ is the entropy of the received signal pair $(Y_t,Y_{t+1})$ given the transmitted signal pair $(X_t,X_{t+1})$. We can simplify (12) in the same way as we did for (6) to obtain the following results

$$\mathcal{H}(Y_t | X_t) = -\int p(Y_t | X_t) \ln[p(Y_t | X_t)] dY_t$$

$$= NK \ln(\pi) + \frac{NK}{2} + N \ln[\det(X_t^H R_{H} X_t + R_{\Theta})]$$

(13)

$$\mathcal{H}(Y_{t+1} | X_{t+1}) = -\int p(Y_{t+1} | X_{t+1}) \ln[p(Y_{t+1} | X_{t+1})] dY_{t+1}$$

$$= NK \ln(\pi) + \frac{NK}{2} + N \ln[\det(X_{t+1}^H R_{H} X_{t+1} + R_{\Theta})]$$

(14)

and

$$\mathcal{H}(Y_t,Y_{t+1} | X_t,X_{t+1})$$

$$= 2NK \ln(\pi) + \frac{NK}{2} + N \ln[\det(X_t^H R_{H} X_t + R_{\Theta})]$$

$$+ N \ln[\det(X_{t+1}^H R_{H} X_{t+1} + R_{\Theta})]$$

$$+ N \ln[\det(I_{M \times M} - [D^{ij+1}]^2)].$$

(15)

In the preceding equation $I_{M \times M}$ is the identity matrix of dimension $M \times M$ and $D^{ij+1}$ is the diagonal matrix obtained by singular value decomposition (SVD) of the covariance matrix $R_{\tilde{X}_X \tilde{X}_X}$, given as the cross-covariance of the whitened expressions for $Y_t$.
and $Y_{t+1}$:

$$R_{Y_t,Y_{t+1}} = E\{Y_t^H Y_{t+1}\} = E\left(\left(\sqrt{R_{Y_t}}\right)^H Y_{t+1} \sqrt{R_{Y_{t+1}}^{-1}}\right)$$

$$= \left(\sqrt{R_{Y_t}}\right)^H R_{Y_t,Y_{t+1}} \sqrt{R_{Y_{t+1}}}$$

(16)

where

$$R_{Y_t} = E\{Y_t^H Y_t\} = X_t^H R_H X_t + R_{\Theta_t}$$

$$R_{Y_{t+1}} = E\{Y_{t+1}^H Y_{t+1}\} = X_{t+1}^H R_H X_{t+1} + R_{\Theta_{t+1}}$$

$$R_{Y_t,Y_{t+1}} = E\{Y_t^H Y_{t+1}\} = X_t^H R_H X_{t+1}.$$  

Note that $R_{Y_t,Y_{t+1}}$ in (17) does not include a noise term as noise at two different time instants is assumed to be uncorrelated. Furthermore, the covariance matrices of $H$ and $\Theta$ estimated at time $t$ are used to approximate the two matrices at time $t+1$ in (14)–(17). Solving (13)–(15), we obtain

$$I(Y_t,Y_{t+1}) = -N \ln \{\det\{I_{M \times M} - [D^{(t+1)}]^2]\}\}
= -N \sum_{m=1}^{M} \ln \{1 - [d_m^{(t+1)}]^2\}$$

(18)

where $d_m^{(t+1)}$ are the diagonal elements of the matrix $D^{(t+1)}$, arranged in the descending order as $d_1^{(t+1)} \geq d_2^{(t+1)} \geq \cdots \geq d_M^{(t+1)}$.

Finally, we can form the minimization problem in step 2 as

$$\min_{X_{t+1} \in \mathcal{S}_X} \left\{-N \sum_{m=1}^{M} \ln \{1 - [d_m^{(t+1)}]^2\}\right\}$$

subject to $\text{tr}[X_{t+1}^H X_{t+1}] \leq B_0$.

Essentially, the minimization criterion presented in (19) is solved by choosing $X_{t+1} \in \mathcal{S}_X$ such that the corresponding singular values of $R_{Y_t,Y_{t+1}}$ (cf. (16) and (17)) minimize the expression in (19).

The waveform ensemble $\mathcal{S}_X$ obtained in step 1 are designed with the purpose of maximizing MI over the spatial domain, and step 2 selects the transmission sequence for each transmit antenna element from $\mathcal{S}_X$ with an objective of minimizing MI over the temporal domain. The proposed waveform optimization algorithm can be summarized as follows.

1) At the initial time $t = 0$, $R_{H_t}$ and $R_{\Theta_0}$ can be estimated through successive measurements with uniform power allocation over the transmit antenna elements by solving equations in (17) simultaneously. This can be achieved through estimation of the target echoes in the next time instant by using (3) and the prospective transmission waveforms from the ensemble $\mathcal{S}_X$.

2) Solve for the optimum power allocation $\Psi_{X_0}$ and the set of optimal waveforms $\mathcal{S}_X$ as per the maximization criterion stated in (11).

3) Form an estimate of the received signal $Y_t$ at time $t = 1$ based on the current estimate for target impulse response by using (3). This impulse response is estimated by de-convolving the received signal with the transmitted signal. Since it is assumed that the target is the most dominant scatterer in the radar environment, the result of this de-convolution is assumed to be the actual impulse response.

4) Solve for $X_1 \in \mathcal{S}_X$ using the minimization approach as stated in (19).

5) Transmit $X_1$ and process the received signal to obtain the updated $R_{H_t}$ and $R_{\Theta_1}$ at time $t = 1$.

6) Repeat steps 2–5 iteratively.

It is worth emphasizing that cognition is integrated in the above waveform optimization process through the feedback operation implemented in step 5 and multiple interactions with the radar channel in step 6.

III. DELAY-DOPPLER RESOLUTION OF DISTRIBUTED MIMO RADAR

The radar AF represents the time response of a filter matched to a given finite energy signal when the signal is received with a delay $\tau$ and a Doppler shift $v$ relative to the nominal values expected by the filter as described in [22]. Different from the communication systems, the matched filter for a radar receiver is designed to match the transmit waveform but not the channel itself. The radar AF thus explains the ability of the radar receiver to boost the backscatter signal from the target (assumed to be at the origin of the AF plot) in comparison with the backscatter signal from nontarget scatterers [22]. The closer the AF response to unity at the origin the better the delay-Doppler resolution. Ideally the AF plot must be a thumbtack response at the origin as suggested in [22].

The radar AF can be mathematically represented as [25]

$$\chi_0(\tau,v) = \int_{-\infty}^{\infty} u(t) u^*(t+\tau) \exp(j2\pi vt)$$

(20)

where $u$ is the complex envelope of the signal. A positive $v$ implies a target moving toward the radar, whereas a positive $\tau$ implies a target being farther from the radar than the reference position with $\tau = 0$. The radar AF for a single UWB Gaussian pulse as shown in (1) can be represented as

$$\chi_1(\tau,v) = \int_{-\infty}^{\infty} \exp \left( -\frac{t^2}{T^2} \right) \exp \left[ -\frac{(t + \tau)^2}{T^2} \right] \exp(j2\pi vt)$$

(21)
where $T$ determines the Gaussian pulsewidth. For a train of UWB pulses, the radar AF is

$$
\chi_2(\tau, v) = \frac{1}{Q} \sum_{q=-Q}^{Q-1} \left| \chi_1(\tau - qT, v) \frac{\sin(\pi v - |q|T)}{\sin(\pi v T)} \right|^2.
$$

(22)

However, the preceding equations are only applicable to the single-input single-output (SISO) radar architecture. For an MIMO radar the equation needs to be modified and is derived in [23]. The received signal after matched filtering can be expressed as

$$
\chi_3(\tau, v, f) = \left| \int [y(t, \tau', v', f')]^H y(t, \tau, v, f) dt \right|
$$

(23)

where $\tau, v, f$ represent the delay, Doppler shift, and spatial frequency, respectively, and $\tau', v', f'$ are the corresponding parameters used by the matched filter at the receiver. We can match the spatial frequency at each of the receive antenna element by adopting receiver beam-forming. In terms of the transmitted waveform, the above equation can be written as

$$
\chi_3(\tau, v, f) = \left| \sum_{n=1}^{N} \exp[j2\pi(f - f')n] \right|
$$

$$
\times \sum_{m_1=1}^{M} \sum_{m_2=1}^{M} |\chi_{m_1, m_2}(\tau, v)| \exp[j2\pi(f_{m_1} - f_{m_2})\tau] \right|
$$

(24)

where $M$ and $N$ are the number of transmit and receive antenna elements. The cross AF is obtained as

$$
\chi_{m_1, m_2}(\tau, v) = \int u_{m_1}(t - \tau) u_{m_2}^H (t - \tau') \exp[j2\pi(v - v')t] dt.
$$

(25)

As the radar AF is a function of the transmit signal, we can evaluate the performance of the proposed waveform optimization strategy in terms of the delay-Doppler resolution by using (24).

IV. SIMULATION RESULTS

A. Simulation Parameters and Target Impulse Response Extraction

We employ orthogonal sequences of the Gaussian pulse over the transmit antenna elements. The backscatter signals are matched filtered at the receivers and the transmitted signals are later modified by the waveform optimization module as shown in Fig. 1. Figure 2(a) indicates the optimized transmitted sequence at one particular transmit antenna after the two-step optimization process. Figure 2(b) shows the received pulses for the transmission sequence in Fig. 2(a). Figure 2(c) shows the target response extracted from the received target echoes after matched filtering at the end of 20 iterations of the algorithm, where an excellent performance of the target response estimation can be observed. At each iteration, the scattering coefficients for the target and nontarget scatterers in $H$ vary as described by the Swerling III model. This causes the amplitude returns of the backscatter signals to vary at each instance. However, the amplitudes of the echoes from the target are always assumed to be stronger than those from the clutter sources. It is difficult to analytically determine the number of successive measurements required to achieve a sufficiently accurate estimate of the actual target response, which depends upon various target scene parameters such as the distribution of scatterers and their relative motion and orientation. Nevertheless, simulation results show that, on average, 20 iterations of the waveform optimization process are required in order to achieve convergence of the target response estimation for a wide range of radar scenes.

B. Probability of Target Detection and MSE Performance

Figure 3(a) indicates the MSE achieved by the algorithm with regard to the estimation of target response. This plot demonstrates an improved MSE performance for the two-step optimization as compared with the individual maximization (step 1) and minimization (step 2) modules, particularly for the first few iterations. Figure 3(b) indicates the probability of target detection achieved by the proposed method, which is obtained by averaging over 1000 simulations each at a particular value of the received signal-to-clutter-plus-noise ratio (SCNR). We apply the hypothesis testing method based on the optimal Neyman-Pearson algorithm [22]. Figure 3(b) shows that for a fixed probability of detection, the required SCNR value decreases as the number of iterations increases. An SCNR gain of more than 5.5 dB at a detection probability of 0.8 is observed between iterations 1 and 20. Nevertheless, the system performance does not show any significant improvement beyond 20 iterations.

C. Delay-Doppler Resolution Performance

Figure 4(a) illustrates the normalized $2 \times 2$ distributed MIMO radar AF contours after the 1st iteration of the optimization algorithm. As can be seen from Fig. 4(a), the resolution of the delay and Doppler of the target deteriorates due to the presence of surrounding clutter. This is particularly evident if the nontarget scatterers are positioned in the vicinity along the line joining the target and the antenna element, assuming that the target is located at the origin of the plot. However, as we increase the iteration number, we observe that the target discrimination ability is greatly improved as shown in Fig. 4(b). Specifically, at the end of 20
Fig. 2. (a) Transmission waveform $x_t$ at particular antenna after two-step optimization. (b) Received signal after matched filtering for transmitted signal shown in (a). (c) Target response extraction.

iterations, the clutter interference is suppressed by approximately 2 dB. The delay-Doppler resolution is directly related to the AF of the radar waveforms. It represents the matched filter output of the radar receiver and should ideally be a thumbtack response (with unity at the origin corresponding to the delay-Doppler for the target). The ideal AF response would be observed if we use statistically independent waveforms for transmission with optimum phase shifts and amplitudes for the pulses. With the two-step waveform optimization, we design and select the pulses matched to the approximated target response, which is updated at each iteration. Furthermore, the optimum power allocation ensures that we suppress the clutter interference over the distributed MIMO radar channels and thus improve the SCNR of the received signals. The probability of target detection presented in Fig. 3(b) displays this improvement in SCNR with iteration number in the presence of strong clutter. This result demonstrates the improved capability of the radar system to discriminate the target from its surroundings and resolve it in terms of its range and velocity.

D. Target Discrimination Performance

In Fig. 5 we provide simulation data for the target scene shown in Fig. 1. In this simulation we compare the performance of different radar configurations with respect to their target discrimination capabilities. The radar scene is simulated by placing the target (which is the vehicle in Fig. 1) and nontarget scatterers (which are trees and brick walls) at fixed locations on the two-dimensional (2D) map. The map is spatially discretized in the form of range bins in both the X and Y directions. The boundary cells of all the scatterers fluctuate over subsequent scans as defined by the Swerling III model described in Section II-B and Appendix I. The target scene is illuminated by UWB Gaussian pulses over each of the antenna elements, as described in the earlier sections. The samples of the signals received at each antenna are captured and utilized to perform range estimation for the scatterer boundaries, which is achieved by calculating the delays that maximize the cross-correlation between the received signal samples and the delayed version of the transmission waveform. The range cells corresponding to the boundary of each scatterer are identified by using these estimated delays. This approach to target range estimation is similar to the one adopted in [26]. Subsequently, an approximated map of the target scene can be created. Figure 5(a) shows the target scene image recreated using the conventional synthetic aperture radar (SAR), where the phased antenna arrays are employed at both the transmitter and the receiver. For the SAR we utilize the same Gaussian UWB sequence over each of the antenna elements but with different delays and with the uniform power allocation. As can be seen from Fig. 5(a), the target discrimination offered by this technique is poor. Although the presence of the object can be successfully detected, the SAR fails to discriminate the object from the surrounding
clutter. Furthermore, the target signature extraction is suboptimal as shown in the image. Figure 5(b) represents the approximated target scene by using the $4 \times 4$ distributed MIMO radar and employing the MI maximization approach for designing waveforms as discussed in [20]. The target scene is clearer since the scatterers are better resolved spatially. The enhanced spatial resolution can be attributed to the MIMO radar configuration, which exploits the spatial diversity by illuminating the target from different directions. The waveform design solution employed in this case results in excitation signals better matched to the target response. Hence, the target signature extraction is superior compared with the SAR. Figure 5(c) shows the recreated radar scene by employing the proposed two-step waveform optimization strategy. As can be seen from the image, the target resolution has been significantly improved. Finally, Fig. 5(d) depicts the ROC for four different radar configurations, which are

1) constant false alarm rate (CFAR) detection using the phased antenna arrays at both the transmitter and the receiver as indicated in [27];

2) $4 \times 4$ distributed MIMO radar employing the waveform design solution based on maximization of MI as indicated in [20];

3) $4 \times 4$ distributed MIMO radar employing the proposed two-step waveform optimization approach;

4) conventional SAR architecture employing the phased antenna arrays.

For a probability of false alarm at 0.02, the probability of target detection offered by the proposed scheme is approximately 0.8 as compared with 0.64 offered by the MI maximization technique, 0.42 by the CFAR, and 0.38 by the SAR. The plots for both the MI maximization and two-step waveform optimization algorithms were generated at the end of 20 iterations. No significant improvement was observed after 20 iterations.
E. Detection Variation Performance

The detection constraint optimization has been recently explored in works like [28], where the authors address the problem of radar phase-coded waveform design for extended target recognition in the presence of colored Gaussian disturbance. The objective function in [28] aims to maximize the weighted average Mahalanobis distance or Euclidean distance between the ideal echoes from different target hypotheses. Furthermore, additional practical constraints are considered in [28]. For example, the modulus of the waveform is restricted to be a constant and the detection constraints require that the achievable signal-to-noise ratio for each target hypothesis is larger than a desired threshold. Figure 6 represents the detection variation brought about by the proposed waveform optimization algorithm subject to the detection constraint. Specifically, we consider a radar scene with 7 target scatterers. The backscatter signal from the radar scene is normalized and the radar attempts to discriminate the targets based on a fixed detection threshold. The detection of the multiple targets is improved as the number of iterations of the proposed algorithm increases. This phenomenon also agrees with the probability of detection result in Fig. 3(b). As seen in Fig. 6 the radar is able to discriminate 7 targets successfully at the end of 20 iterations by suppressing the clutter and noise.

This improved detection performance is due to the waveform design step in the proposed method, which ensures that the Euclidean distance between the ideal echoes from different targets is maximized through the optimization process in (11). This methodology is similar to the objective function in [28]. As the number of iterations increases, better estimates of the target responses are generated resulting in improved detection of the targets.
Subsequently, the average RCS of the target is as shown in [12]. We consider an extended target, which consists of scattering centers located at coordinates \( \theta_{T,m} = \{x_m, y_m\} \), \( m = 1, \ldots, M \). The signals scattered by the target are collected by \( N \) receivers placed at arbitrary coordinates \( \theta_{R,n} = \{x_n, y_n\} \), \( n = 1, \ldots, N \). The set of transmitted waveforms in the low-pass equivalent form is \( \sqrt{E/M}x_m(t), m = 1, \ldots, M \), where \( \int_{T_p} |x_m|^2 = 1, E \) is the total transmitted energy, and \( T_p \) is the waveform duration. Normalization by \( M \) makes the total energy independent of the number of transmitters. The low-pass equivalent of the signal observed at the \( n \)th receive antenna due to the probing signal sent from the \( m \)th transmit antenna and reflected from the \( q \)th scatterer (excluding noise) is given by

\[
y_{m,n}^{(q)} = \frac{E}{M} \zeta_q x_m(t - \tau_{T,m}(\theta_{S,q}) - \tau_{R,n}(\theta_{S,q})) \\
\quad \times \exp\{-j2\pi f_c[\tau_{T,m}(\theta_{S,q}) + \tau_{R,n}(\theta_{S,q})]\} \\
\]

where \( \tau_{T,m}(\theta_{S,q}) \) and \( \tau_{R,n}(\theta_{S,q}) \) are the propagation delays between the \( m \)th transmit antenna and the \( q \)th scatterer, and between the \( q \)th scatterer and the \( n \)th receive antenna, respectively. Continuing with the signal model in (26), we can interpret the term

\[
h_{m,n}^{(q)} = \zeta_q \exp\{-j2\pi f_c[\tau_{T,m}(\theta_{S,q}) + \tau_{R,n}(\theta_{S,q})]\} \\
\]

as the equivalent channel for the radar signal path from transmitter \( m \) to receiver \( n \) via scatterer \( q \). Summing over all the scatterers that make up the target, the model in (26) becomes

\[
y_{m,n}(t) = \sqrt{\frac{E}{M}} \sum_{q=1}^{Q} h_{m,n}^{(q)} x_m(t - \tau_{T,m}(\theta_{S,q}) - \tau_{R,n}(\theta_{S,q})). \\
\]

We assume that the target is situated in the far field and the radar elements at the transmitter and receiver sites are noncollocated. The target has an RCS center of gravity at \( \theta_{S,0} \) such that \( x_m(t - \tau_{T,m}(\theta_{S,q}) - \tau_{R,n}(\theta_{S,q})) \approx x_m(t - \tau_{T,m}(\theta_{S,0}) - \tau_{R,n}(\theta_{S,0})) \) for all \( q = 1, \ldots, Q \) [12]. With these conditions, (28) becomes

\[
y_{m,n}(t) = \sqrt{\frac{E}{M}} h_{m,n} x_m(t - \tau_{T,m}(\theta_{S,0}) - \tau_{R,n}(\theta_{S,0})). \\
\]

where \( h_{m,n} = \sum_{q=1}^{Q} h_{m,n}^{(q)} \). The phase of the path gain \( h_{m,n} \) is assumed to be uniformly distributed between 0 and \( 2\pi \), and the RCS \( \xi = |h_{m,n}|^2 \) is exponentially distributed as described in Section II-B. In case of a radar scene with clutter sources present, the model for \( h_{m,n} \) can be extended to accommodate surrounding nontarget scatterers in the form of extended clutter sources in addition to the desired target.

V. CONCLUSION

We have presented a two-stage waveform optimization algorithm for an adaptive distributed MIMO radar, which unifies the signal design and selection procedures. The proposed algorithm is based upon constant learning of the radar environment at the receivers and adaptation of the transmit waveforms to suit the dynamic radar scene. This ensures maximum information extraction from the target of interest. The proposed approach can provide high performance gains in terms of target response extraction, probability of target detection, delay-Doppler resolution, and radar scene generation for real-time practical scenarios as demonstrated through the simulation results. Nevertheless, there is also an increase in the computational load due to the additional step in the waveform optimization. Future works will look into the tradeoff between the performance enhancement provided by the proposed method and the computational complexity involved.

APPENDIX I. SCATTERING CHANNEL MODEL

The scattering channel model used in this work is as shown in [12]. We consider an extended target, which consists of \( Q \) scattering centers located at \( \theta_{S,q} = \{x_q, y_q\} \) with reflectivity \( \zeta_q \) \( (q = 1, 2, \ldots, Q) \). The reflectivity of each scattering center is modeled by a zero-mean, independent and identically distributed (IID) complex random variable with variance \( E[|\zeta_q|^2] = 1/Q \). We organize the reflectivity values in a diagonal \( Q \times Q \) matrix, \( \Sigma = \text{diag}(\zeta_1, \ldots, \zeta_Q) \). Subsequently, the average RCS of the target is \( E[|x_m^T(\Sigma\Sigma^H)|] = 1 \), independent of the number of scatterers in the model. If the RCS fluctuations are fixed during an antenna scan but vary independently from scan to scan, our target model represents a classical Swerling case III. Now let the target be illuminated by \( M \) transmitters arbitrarily located at coordinates \( \theta_{T,m} = \{x_m, y_m\} \), \( m = 1, \ldots, M \). The signals scattered by the target are collected by \( N \) receivers placed at arbitrary coordinates \( \theta_{R,n} = \{x_n, y_n\} \), \( n = 1, \ldots, N \). The set of transmitted waveforms in the low-pass equivalent form is \( \sqrt{E/M}x_m(t), m = 1, \ldots, M \), where \( \int_{T_p} |x_m|^2 = 1, E \) is the total transmitted energy, and \( T_p \) is the waveform duration. Normalization by \( M \) makes the total energy independent of the number of transmitters. The low-pass equivalent of the signal observed at the \( n \)th receive antenna due to the probing signal sent from the \( m \)th transmit antenna and reflected from the \( q \)th scatterer (excluding noise) is given by

\[
y_{m,n}^{(q)} = \frac{E}{M} \zeta_q x_m(t - \tau_{T,m}(\theta_{S,q}) - \tau_{R,n}(\theta_{S,q})) \\
\quad \times \exp\{-j2\pi f_c[\tau_{T,m}(\theta_{S,q}) + \tau_{R,n}(\theta_{S,q})]\} \\
\]

where \( \tau_{T,m}(\theta_{S,q}) \) and \( \tau_{R,n}(\theta_{S,q}) \) are the propagation delays between the \( m \)th transmit antenna and the \( q \)th scatterer, and between the \( q \)th scatterer and the \( n \)th receive antenna, respectively. Continuing with the signal model in (26), we can interpret the term

\[
h_{m,n}^{(q)} = \zeta_q \exp\{-j2\pi f_c[\tau_{T,m}(\theta_{S,q}) + \tau_{R,n}(\theta_{S,q})]\} \\
\]

as the equivalent channel for the radar signal path from transmitter \( m \) to receiver \( n \) via scatterer \( q \). Summing over all the scatterers that make up the target, the model in (26) becomes

\[
y_{m,n}(t) = \sqrt{\frac{E}{M}} \sum_{q=1}^{Q} h_{m,n}^{(q)} x_m(t - \tau_{T,m}(\theta_{S,q}) - \tau_{R,n}(\theta_{S,q})). \\
\]

We assume that the target is situated in the far field and the radar elements at the transmitter and receiver sites are noncollocated. The target has an RCS center of gravity at \( \theta_{S,0} \) such that \( x_m(t - \tau_{T,m}(\theta_{S,q}) - \tau_{R,n}(\theta_{S,q})) \approx x_m(t - \tau_{T,m}(\theta_{S,0}) - \tau_{R,n}(\theta_{S,0})) \) for all \( q = 1, \ldots, Q \) [12]. With these conditions, (28) becomes

\[
y_{m,n}(t) = \sqrt{\frac{E}{M}} h_{m,n} x_m(t - \tau_{T,m}(\theta_{S,0}) - \tau_{R,n}(\theta_{S,0})). \\
\]

where \( h_{m,n} = \sum_{q=1}^{Q} h_{m,n}^{(q)} \). The phase of the path gain \( h_{m,n} \) is assumed to be uniformly distributed between 0 and \( 2\pi \), and the RCS \( \xi = |h_{m,n}|^2 \) is exponentially distributed as described in Section II-B. In case of a radar scene with clutter sources present, the model for \( h_{m,n} \) can be extended to accommodate surrounding nontarget scatterers in the form of extended clutter sources in addition to the desired target.
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On the Reversibility of Randomizers and Derandomizers in Aeronautical Telemetry

Data may be recovered when randomization and derandomization are applied in the reverse order as long as the initial register conditions for both the randomizer and derandomizer are identical. In the (very realistic) case where the initial register contents for the derandomizer are not known, simple procedures for identifying the correct initial conditions are described. The procedures leverage the presence of frame synchronization words in the telemetry data.

I. INTRODUCTION

Long sequences of consecutive ones or zeros can cause problems in binary data storage and transmission. Ultimately, these problems can be traced to the absence of transitions in the waveforms used to carry the bits. The approaches used to deal with this problem may be classified in one of two broad categories. The first category comprises solutions based on pulse shapes. Here, a pulse shape that guarantees waveform transitions every bit interval, even in the presence of consecutive ones or zeros, is used. Examples include the return-to-zero (RZ) or Manchester pulse shapes [1]. The popularity of this approach can be limited by practical considerations. In high speed applications (hundreds of Mbits/s to several Gbits/s over fiber), the complexity generating the pulse shapes can be a problem (see [2] and the references cited therein). In wireless applications with severe bandwidth constraints (such as aeronautical telemetry), the bandwidth required by the RZ or Manchester pulse shapes often renders their use impractical.

The second category is based on the application of line coding to the bits prior to waveform mapping. Scrambling (or randomizing) is an important special case of line coding. The goal of scrambling is to randomize the source bits thereby breaking up long sequences of consecutive ones and zeros. At the destination the inverse operation, called descrambling, is applied to the bits. This category comprises three basic approaches [2].

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