Dynamic Spectrum Scheduling for Carrier Aggregation: A Game Theoretic Approach

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***Abstract*—In this paper, we investigate the performance of the dynamic allocation of resources between separate cellular networks. We propose a Dynamic Internetworking Carrier Aggregation (DI-CA) framework which involves every network operator releasing some of its exclusive, but excess, spectrum to another network operator for a limited time. We derive the basic condition for which DI-CA can improve the performance for all the operators and then propose a distributed scheduling framework that uses coalition formation with uncertainty, in which each independent operator can decide whether or not DI-CA can improve its performance without having information regarding channel conditions or load experienced by other operators. We propose a distributed Bayesian coalition formation algorithm to approach a neighborhood of the Bayesian Nash equilibrium.**

I. Introduction

Carrier aggregation (CA) is the LTE-Advanced (LTE-A) [1] term that refers to the process of aggregating different blocks of spectrum, called component carriers (CCs), to form larger transmission bandwidths. A mobile network operator (MNO) may aggregate CCs contiguously or non-contiguously within a single frequency band, i.e. intra-band CA, or it may aggregate CCs which are located in separate frequency bands, e.g. a CC in the 1.8GHz band may be aggregated with a CC in the 2.6GHz band. While CA allows for a dynamic allocation of resources within an LTE-A system, we are interested in the dynamic allocation of resources *between* LTE-A systems. Situations may arise in which one network is more lightly loaded than another. In this case it may be economically advantageous for the less loaded network to enable other networks to access its carriers; this is the central premise of much research into dynamic spectrum access systems.

In this paper, we extend the concept of CA by investigating the ability of independent MNOs to dynamically schedule access to portions of each other's spectrum. In our model, each MNO allows a portion of its spectrum, i.e. the range of frequencies exclusively assigned to it under a licence, to be aggregated by other MNOs for a limited time. We refer to this type of dynamic access for the purpose of CA between separate and independent MNOs as Dynamic Inter-network Carrier Aggregation (DI-CA) [2]. Essentially, DI-CA involves one MNO temporarily releasing some of its exclusive, but excess, spectrum to another MNO, which can then aggregate it with its own spectrum.

We propose a generalized model for the DI-CA-based system where an MNO can decide the amount of time for which to grant access to its licensed spectrum to other operators to maximize its payoff. In our model, the payoff of each MNO can be any performance measure, e.g., the uplink or downlink channel capacity, system reliability, etc. We study two fundamental problems for this system: 1) Under which conditions can DI-CA improve performance for all the MNOs? 2) How to achieve distributed scheduling when each MNO does not have any instantaneous information (i.e., payoffs, aggregated bandwidth, etc.) about others?

To study the first problem, we assume that global information is available to every operator and derive the conditions under which DI-CA can improve a network's performance. It is generally infeasible for all MNOs to exchange instantaneous global information, i.e., payoffs, transmit powers, channel gains, etc., with each other before a coalition pair is formed, which makes it difficult for each independent MNO to make timely decisions on whether DI-CA can provide a performance improvement. We hence establish a coalition formation with uncertainty framework in which MNOs do not exchange any instantaneous private information, but can only decide whether DI-CA is likely to be advantageous based on their own private information and their previously observed performance. We propose a simple distributed algorithm to approach a neighborhood of the Bayesian Nash equilibrium (BNE) of the system.

II. Network Model

Consider a mobile cellular environment that consists of two MNOs that control the dynamic aggregation of their component carriers through two entities that we refer to as Mobile Network Aggregators (MNAs), M_1 and M_2. Each MNO operates base stations that are co-located, i.e. the antennas are located at the same site. Let the bandwidth

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under the control of the MNAs M_1 and M_2 be B_1 and B_2, respectively. Let us assume the revenue each MNA obtains from aggregation to be proportional to its transmit bandwidth. We can define the payoff/utility of M_i without DI-CA as $\varpi_i^{noCA} = B_i R_{ii}$ where R_{ii} is the revenue/benefit per Hertz obtained by M_i without DI-CA. In this paper, we consider a general model where the payoff of each MNA can be any performance measure such as system capacity, outage probability, etc. For example, if R_{ii} is the downlink or uplink channel capacity of M_i, then we have $R_{ii} = \mathbb{E}_{n\in\mathcal{K}_i} \log(1 + h_{ii}(n)w_i)$ where w_i is the transmit power of M_i or subscriber n, $\mathcal{K}_i$ is the set of subscribers served by M_i, and $h_{ii}(n)$ is the channel gain between MNA M_i and each mobile subscriber $n \in \mathcal{K}_i$.

If an MNA allows a part of its frequency band to be aggregated by others, it will decide the assignment of resource blocks, i.e., the portion of the transmission time and bandwidth that is allowed to be aggregated by others. To ensure a minimum quality of service (QoS) for its own subscribers, each MNA can reserve a certain portion of the total bandwidth exclusively for its own transmission and only allows a portion of its frequency band to be aggregated by others using DI-CA. Let the reserved bandwidth and the dynamically aggregatable bandwidth of the MNA M_i be B_{ii} and B_{i-i}, respectively, where $-i$ denotes the MNA other than M_i, and $B_{ii} + B_{i-i} = B_i$ for $i \in \{1, 2\}$. Note that the value of B_{ii} is used to maintain the basic performance for M_i and hence should be decided by the specific system requirements, i.e, long term average traffic, the minimum QoS, the expected number of subscribers, etc. In addition, it may be impractical to allow each MNA to instantaneously change the aggregatable bandwidth during the communication process. We hence assume B_{ii} and B_{i-i} are fixed during the entire transmission process. We assume the revenue per Hertz of each MNO in different spectrum portions within its licensed bandwidth to be similar and hence can write the revenue of M_i in the reserved bandwidth B_{ii} as $\pi_i^{CA1} = B_{ii} R_{ii}$.

To start aggregating the bandwidth of others, the two MNAs will first exchange information regarding the time scheduling and aggregatable bandwidth of MNAs [3]. Assume each MNA M_i only allows the other MNA to schedule a proportion p_i of its transmission time dynamically aggregating its bandwidth B_{i-i}. Hence, the revenue that M_i obtains from its own subscribers in *non-reserved* frequency band B_{i-i} is given by $\pi_i^{CA2} = (1 - p_i) B_{i-i} R_{ii}$.

Similarly, we can write the revenue that M_i obtains by dynamically aggregating the frequency band of M_{-i} for p_{-i} portion of time as $\pi_i^{CA3} = p_{-i} B_{-ii} R_{i-i}$where R_{i-i} is the revenue per Hertz when M_i aggregates B_{-ii} portion of the bandwidth of M_{-i}. If B_{-ii} and B_i are contiguous to each other, the revenues per Hertz R_{ii} and R_{i-i} may be similar to each other. However, if the frequency bands of M_i and M_{-i} are non-contiguous, R_{ii} and R_{i-i} are likely to be significantly different because of frequency selective fading and Doppler shifts in different frequency bands [4]. It is observed that CA, even of the static kind that does not involve accessing CCs from other networks, introduces more complexity and cost in the system implementation, i.e. CA requires more complex signaling and scheduling, and for non-contiguous CA, extra antenna/RF chains should be employed to allow each MNO to access the spectrum of others. In this paper, we assume the extra cost brought by DI-CA for each MNO M_i is a constant denoted by ζ_i. We hence can write the payoff/utility of M_i when using DI-CA to be[1] $\varpi_i^{CA} = \pi_i^{CA1} + \pi_i^{CA2} + \pi_i^{CA3} - \zeta_i$.

III. Feasible Condition and Fair Scheduling

Both MNAs will independently decide whether or not to use DI-CA. This situation can be modeled using cooperative game theory, where autonomous decision makers (here, the MNAs) decide to cooperate if and only if cooperation brings mutual benefits [7] and the resource allocation is fair. In this section, we first derive the feasible condition for which DI-CA could provide payoff improvement for both MNAs, and then we briefly discuss the fairness criteria for the scheduling between MNAs.

Let us first define the *feasible DI-CA scheduling pairs* for MNAs as follows.

Definition 1: A DI-CA scheduling pair (p_1, p_2) for MNAs is *feasible* if $0 \le p_1, p_2 \le 1$, $\varpi_1^{CA}(p_1, p_2) \ge \varpi_1^{noCA}(p_1, p_2)$ and $\varpi_2^{CA}(p_1, p_2) \ge \varpi_2^{noCA}(p_1, p_2)$.

Following Definition 1, we have the following results about the feasible scheduling pairs for DI-CA.

Proposition 1: There exists at least one feasible DI-CA scheduling pair (p_1, p_2) if

$$\begin{aligned} \zeta_1 R_{22} + \zeta_2 R_{12} &\le B_{12}(R_{21}R_{12} - R_{11}R_{22}) \text{ and} \\ \zeta_1 R_{21} + \zeta_2 R_{11} &\le B_{21}(R_{21}R_{12} - R_{11}R_{22}) \end{aligned} \quad (1)$$

are satisfied.

Proof: See Appendix A. ■

Note that if the cost of using DI-CA is negligible, i.e., $\zeta_1 = \zeta_2 = 0$, then (1) turns into $R_{21}R_{12} \ge R_{11}R_{22}$. Proposition 1 provides a basic condition for which DI-CA can provide mutual mutual benefits for both MNAs. However, it does not guarantee the spectrum sharing among MNAs to be fair.

Another important problem for DI-CA is that there exists a fundamental tradeoff for each MNA M_i to choose p_i. More specifically, if one MNA M_i wants to reserve a large transmission time for its own transmission, i.e., choose a small p_i, it will also decrease the chance of attracting other MNAs to share their spectrum. On the other hand, if p_i is large, it will decrease the capacity reserved for the MNA's own use. To solve this problem, many fairness criteria such as Nash bargaining solution

[1]Some previously reported works model the spectrum sharing as a virtual market in which a virtual price is introduced for each MNA to charge its spectrum aggregators [2], [5], [6]. These models can be easily integrated into our framework. For example, if the price paid by each spectrum arregator is proportional to its revenue, the price term can be involved in ϖ_i^{CA3}. If the price can be regarded as a constant, the price term can be integrated in ζ_i.

(NBS), Shapley value fairness, and nucleolus fairness have been introduced for different systems. We omit the detailed results for these fairness criteria and assume both MNAs agree to scheduling their transmission according to NBS, denoted as (p_1^{NBS}, p_2^{NBS}), for the remainder of this paper. We will present the numerical results for NBS in Section V. However, it can be observed that most of the fairness criteria require each MNA to have global information, i.e., M_i needs to know R_{-ii} and R_{-i-i}, to calculate (p_1^{NBS}, p_2^{NBS}). This may result in high communication overhead between the MNAs. In addition, (p_1^{NBS}, p_2^{NBS}) changes with the channel gains and hence both MNAs will have to calculate these scheduling pairs whenever the channel condition changes. This also increases the computational complexity of DI-CA. How to minimize the communication overhead and complexity of DI-CA is an important problem for an LTE-A system. In the next section, we propose a model-free Bayesian coalition formation framework which allows the MNAs to avoid exchanging this information when it is obvious that DI-CA cannot improve their payoffs.

IV. DI-CA with Incomplete Information

We assume the revenues of both MNAs change with time and the transmission process can be divided into N time slots, for each of which the revenues can be regarded as constants. We use $[m]$ to denote the transmission in the mth time slot.

Definition 2: Let us define a Bayesian CA scheduling game as $\mathcal{G} = \langle \mathcal{C}, \boldsymbol{a}, \mathcal{T}, \boldsymbol{\pi}, \mathcal{I} \rangle$ where

- $\mathcal{C} = \{M_1, M_2\}$ is the set of *players* (MNAs),
- $\boldsymbol{a} = \{\boldsymbol{a}_1, \boldsymbol{a}_2\}$ where $\boldsymbol{a}_i = [a_i[1], a_i[2], \ldots, a_i[N]]$ and $a_i[m] \in \{0, 1\}$ is the *action* of player i; $a_i[m] = 0$ (or $a_i[m] = 1$) means that M_i does not (or does) use DI-CA,
- $\mathcal{T}$ is the set of types, which includes the instantaneous payoffs a player can achieve through its licensed and aggregated spectrum. Each MNA does not know the type of others,
- π_i is the instantaneous payoff of a player i.

There are generally two forms of uncertainty in a Bayesian coalition formation game: 1) type uncertainty: each player does not know the types of other players, 2) sharing uncertainty: each player cannot know how the resource will be shared among all the members after a coalition has been formed. In this paper, we ignore the sharing uncertainty and assume that if a coalition has been formed, players will exchange enough information and use NBS as the fairness criterion to share the resources.

We focus on an infinite horizon, i.e., $N \to \infty$. Each MNA tries to maximize its average payoff,

$$\bar{\varpi}_i(\boldsymbol{a}_1, \boldsymbol{a}_2) = \mathbb{E}_m \varpi_i (R_{ii}[m] | p_i[m], p_{-i}[m], R_{i-i}[m], R_{-ii}[m], R_{-i-i}[m]) . \quad (2)$$

We seek an equilibrium point of the system, called the Bayesian Nash equilibrium (BNE). Note that, in our model, each player i cannot know the instantaneous payoffs of other players for the current time slot. Hence, at the beginning of a time slot m, player i will decide whether or not to form a coalition with others by using its private information and observation history. If the coalition formation request of player i has been rejected by others, $a_i[m] = 0$, player i will not exchange any information with others. However, if a coalition containing player i has been formed, i.e., $a_i[m] = 1$, all the coalition members can know the instantaneous information of each other and use NBS to share their resources.

We can rewrite the payoff of M_i in each time slot m as follows,

$$\varpi_i[m] = B_i R_{ii}[m] + a_i[m] \Delta\varpi_i[m], \quad (3)$$

where $\Delta\varpi_i[m] = p_{-i}^{NBS}[m] B_{-ii} R_{i-i}[m] - p_i^{NBS}[m] B_{i-i} R_{ii}[m] - \zeta_i$. It can be easily shown that the *best response* of M_i in each time slot m is given by

$$a_i[m] = \begin{cases} 1, & \text{If } \Delta\varpi_1[m] + \Delta\varpi_2[m] > 0, \\ 0, & \text{Otherwise.} \end{cases} \quad (4)$$

We assume that, at the beginning of each time slot m, each MNA MNA i can only know $R_{ii}[m]$. Because each MNA M_i cannot know $R_{-i-i}[m]$ and $R_{-ii}[m]$, let us define $\eta_i[m] = B_{i-i} R_{-ii}[m] + \frac{p_{-i}^{NBS}[m] B_{-ii} R_{i-i}[m] - p_{-i}^{NBS}[m] B_{-ii} R_{-ii}[m] - \zeta_i - \zeta_{-i}}{p_i^{NBS}[m]}$. We assume $\eta_i[m]$ is bounded and the condition for using DI-CA (i.e., $a_i[m] = 1$) in (4) can be rewritten as

$$\eta_i[m] > B_{i-i} R_{ii}[m]. \quad (5)$$

The main focus here is to let each MNA M_i estimate whether or not (5) is satisfied by using the known $B_{i-i} R_{ii}[m]$ and an estimated version of $\eta_i[m]$, denoted by $\tilde{\eta}_i[m]$. In this paper, we use the average value as the estimate, i.e., $\tilde{\eta}_i = \mathbb{E}_{l=\{1,2,\ldots,m\}} \eta_i[l]$. More specifically, we assume each MNA can use the stochastic approximation method [8] to estimate the average value from its samples in an online fashion. Then, it is observed from Chebyshev's inequality, if the statistics of all the channels' gains are unchanged, nearly all the sample values are close to a neighborhood of the mean.

Let us present the detailed algorithm below.

Algorithm 1: Distributed Scheduling Algorithm

1) *Initialization:* $m = 0$
   - $a_1[0] = a_2[0] = 1$,
   - Define a set of the time slots $\mathcal{L} \subseteq \{1, 2, \ldots, N\}$ when both MNAs to use DI-CA, i.e., initially $\mathcal{L} = \emptyset$,
   - $1 \le t \ll N$ is a pre-defined integer denoted as the training time duration.
2) *Training:* For $m = 1 : t$,
   a) $a_1[m] = a_2[m] = 1$,
   b) At the end of time slot m, each MNA knows $\pi^{CA}[m]$ and $\pi^{noCA}[m]$ and hence can calculate an estimate of $\eta_i[m]$, i.e., $\tilde{\eta}_i[m] = \frac{1}{m} \sum_{k=1}^{m} \eta_i[k]$. Both MNAs update $\mathcal{L} = \mathcal{L} \cup \{m\}$,

3) *Decision Making:* For $m = l+1 : N$, at the beginning of every time slot m, M_i observes $B_{i-i}R_{ii}[m]$ and knows $\tilde{\eta}_i[m]$,
   a) If $B_{i-i}R_{ii}[m] < \tilde{\eta}_i[m]$, M_i sends a DI-CA request to M_{-i}. If M_{-i} also observes $B_{-ii}R_{-i-i}[m] < \tilde{\eta}_{-i}[m]$, M_{-i} starts to communicate with M_i to start using DI-CA during time slot m and then goes to Step 4). Otherwise, M_{-i} sends a rejection message to M_i and neither MNA can use DI-CA in time slot m. Repeat Step 3).
   b) If $B_{i-i}R_{ii}[m] \geq \tilde{\eta}_i[m]$, M_i does not use DI-CA. Repeat Step 3).

4) *Belief Update:* If both MNAs use DI-CA in time slot m, each MNA M_i updates its belief about $\eta_i[m]$ by

$$\tilde{\eta}_i[m] = \tilde{\eta}_i[m-1] + |\mathcal{L}|\eta_i[m]/\left(|\mathcal{L}|+1\right), \quad (6)$$

and both MNAs update $\mathcal{L} = \mathcal{L} \cup \{m\}$.

The main idea of the above algorithm is to let each MNA first estimate an initial value of $\tilde{\eta}_i$ using the training step, and then use the estimated $\tilde{\eta}_i$ to decide whether or not the condition in Proposition 1 is satisfied. Each MNA only sends the DI-CA request to the other when it decides that DI-CA can improve the performance of both MNAs. After each iteration, both MNAs update their beliefs about $\tilde{\eta}_i$.

We have the following result about the above algorithm.

Theorem 1: If the expectation and variance of $\eta_i[m]$ are fixed during the transmission process and $\Delta\varpi_i[m]$ and $R_{ii}[m]$ are bounded, i.e., $0 \leq \Delta\varpi_i[m] \leq \Delta\varpi_i^+$ and $R_{ii}^- \leq R_{ii}[m] \leq R_{ii}^+$ $\forall m = 1, 2, \ldots, N$, then Algorithm 1 converges to a BNE within a distance of ϵ (ϵ-BNE) where $\epsilon = \frac{\text{var}(\eta_i)\Delta\varpi_i^+}{\left(B_{i-i}R_{ii}^- - \mathbb{E}\eta_i\right)^2}$.

Proof: See Appendix B. ■

The above result shows that if the difference between $B_{i-i}R_{ii}[m]$ and $\mathbb{E}(\eta_i)$ is much larger than the variance of η_i, Algorithm 1 will converge to the NBE, i.e., $\epsilon \to 0$ when $|B_{i-i}R_{ii}[m] - \mathbb{E}(\eta_i)| \gg \text{var}\left(\eta_i\right)\mathbb{E}\Delta\varpi_i$. In other words, Algorithm 1 will be more useful when the average performance gain brought by DI-CA is large.

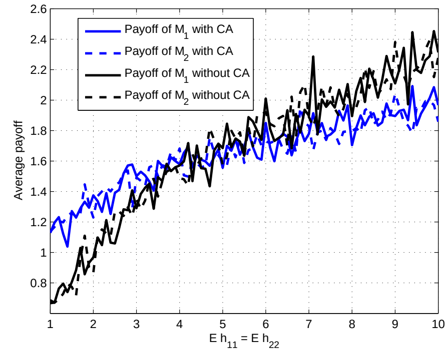

Fig. 1. Average payoff with NBS fairness

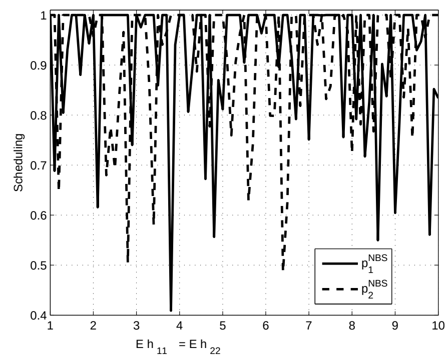

Fig. 2. Scheduling with NBS fairness

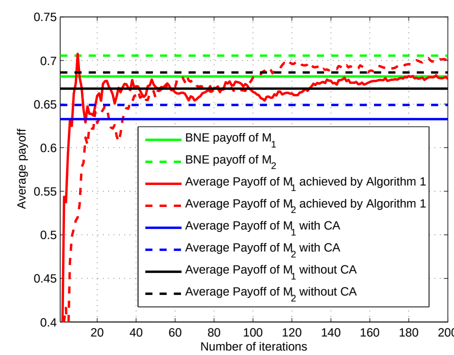

Fig. 3. Convergence rate of Algorithm 1.

V. Discussions and Numerical Results

Let us present numerical results to verify the performance of the spectrum scheduling schemes of DI-CA. We assume the payoff of each MNA is its average downlink channel capacity described in Section II and all the channel gains follow a Rayleigh distribution. In Figure 1, we fix the average value of the channel gains experienced by MNAs in the aggregatable spectrum of each other and consider their performance when the channel gains in their own spectrum change. It is observed that DI-CA cannot always provide performance improvement over the non-CA case. This verifies the observation that DI-CA can only increase the capacity of each MNO if the capacity obtained from the other MNO's spectrum is higher than that obtained in its own spectrum. It is observed that DI-CA can only provide payoff improvement when the MNAs can obtain a higher capacity sum by sending signals in the aggregatable spectrum of each other. An interesting observation is that the high payoff MNA may switch between the two MNAs when the channel gains change. This is because the NBS fairness always forces at least one MNA to allocate all the aggregatable time portion to the other in exchange for a higher chance of using the aggregatable spectrum, and hence the variance of the channel gains may cause the high payoff MNA to change between M_1 and M_2. Similar observations can be found in Figure 2, where we present the values of p_1^{NBS} and p_2^{NBS} under different channel gains. The convergence of Algorithm 1 is illustrated in Figure 3. It is observed that the average payoff achieved by Algorithm 1 may fluctuate when the number of iterations is small. However, as the number of iteration increases, the performance of Algorithm 1 will be higher than either DI-CA or without DI-CA and will eventually approach a BNE.

VI. Conclusion

This paper introduces a DI-CA framework to investigate the dynamic scheduling of spectrum resources between independent cellular networks. We derive the conditions under which DI-CA can improve the performance for all the MNAs. We propose an NBS-based fair spectrum scheduling scheme. To avoid using DI-CA when it cannot provide any performance gain, we devise a Bayesian game-based framework in which each independent operator can decide whether or not DI-CA can improve its performance without knowing the information of others. Finally, we propose a distributed algorithm to approach a neighborhood of the BNE.

Appendix

A. Proof of Proposition 1

Let us assume the condition in (1) is satisfied. In this case, we can claim that there exists at least one pair of (p_1, p_2) for $0 < p_1 \leq 1$ and $0 < p_2 \leq 1$ which satisfies

$$p_1 B_{12}\left(R_{12}R_{21} - R_{11}R_{22}\right) = \zeta_1 R_{22} + R_{12}\zeta_2, \quad (7)$$
$$p_2 B_{21}\left(R_{12}R_{21} - R_{11}R_{22}\right) = \zeta_1 R_{11} + R_{21}\zeta_2. \quad (8)$$

Let us rewrite (7) as follows,

$$\begin{aligned} & p_1 B_{12}\left(R_{11}R_{22} - R_{21}R_{12}\right) + p_2 B_{21} R_{12} R_{22} \\ & -p_2 B_{21} R_{12} R_{22} = \zeta_1 R_{22} + \zeta_2 R_{12} \\ & \Rightarrow R_{12}\left[p_1 B_{12} R_{21} - p_2 B_{21} R_{22} - \zeta_2\right] \\ & +R_{22}\left[p_2 B_{21} R_{12} - p_1 B_{12} R_{11} - \zeta_1\right] = 0 \\ & \Rightarrow R_{12}\Delta\varpi_2 + R_{22}\Delta\varpi_1 = 0. \end{aligned} \quad (9)$$

Similarly, we can rewrite (8) as follows,

$$R_{11}\Delta\varpi_2 + R_{21}\Delta\varpi_1 = 0. \quad (10)$$

Combining (9) and (10) and using the fact that R_{11} (or R_{22}) and R_{12} (or R_{21}) are always positive and different from each other, we can claim that $\Delta\varpi_1 = 0$ and $\Delta\varpi_2 = 0$ must be simultaneously satisfied. In other words, (p_1, p_2) satisfies the condition of feasibility in Proposition 1.

B. Proof of Theorem 1

As mentioned in Section III, if both MNAs can have perfect information, i.e., each MNA knows $\eta_i[m+1]$ at the beginning of time slot $m+1$, it will use (or not use) CA if (5) is satisfied (or not satisfied). In other words, the optimal average payoff of M_i during the first $m+1$ time slots of transmission is achieved by allowing both MNAs to use CA when $\Delta\varpi_i[n] > 0$ and to stop using CA when $\Delta\varpi_i[n] \leq 0$ for all $n \in \{1, 2, ..., m+1\}$.

Let us now consider the payoff achieved by Algorithm 1. The main idea of Algorithm 1 is to use the average value as the estimated version of the unknown $\eta_i[m]$. It has already been proved in [8] that the belief update in Step 4) of Algorithm 1 always converges to the average value of η. Let us assume $\tilde{\eta}_i \approx \mathbb{E}_m\left(\eta_i[m]\right)$. We then can focus on the convergence performance of the decision making process in Step 3). Let us consider the following cases in time slot m of Algorithm 1,

$$\begin{aligned} \bar{\varpi}_i[m+1] &= \frac{m\bar{\varpi}_i[m] + \varpi_i[m+1]}{m+1} \\ &\approx \bar{\varpi}_i[m] + \frac{1}{m+1}\left[B_i R_{ii}[m+1]\right. \\ &+ \mathbb{I}_{a_i[m+1]=1}\mathbb{J}_{\eta_i[m+1] > B_{i-i}R_{ii}[m+1]}\Delta\varpi_i[m+1] + \\ &\mathbb{I}_{a_i[m+1]=1}\left(1 - \mathbb{J}_{\eta_i[m+1] > B_{i-i}R_{ii}[m+1]}\right)\Delta\varpi_i[m+1] \\ &+ \left(1 - \mathbb{I}_{a_i[m+1]=1}\right)\mathbb{J}_{\eta_i[m+1] > B_{i-i}R_{ii}[m+1]} 0 \\ &+ \left.\left(1 - \mathbb{I}_{a_i[m+1]=1}\right)\left(1 - \mathbb{J}_{\eta_i[m+1] > B_{i-i}R_{ii}[m+1]}\right) 0\right], \end{aligned} \quad (11)$$

where $\mathbb{I}$ and $\mathbb{J}$ are indicator functions. Note that $\Delta\varpi_i[m+1] < 0$ when $a_i[m+1] = 1$ and $\eta_i[m+1] < B_{i-i}R_{ii}[m+1]$.

Let us now prove that $\lim_{m\to\infty} \bar{\varpi}_i[m+1] \to \bar{\varpi}_i^* \pm \epsilon$. Assuming the probability distribution function of η_i is fixed and using the fact that, in Algorithm 1, $a_i[m+1] = 1$ when $\eta_i[m+1] > B_{i-i}R_{ii}[m+1]$, we can rewrite the above inequality as follows,

$$\begin{aligned} & \lim_{m\to\infty} \frac{1}{m}\sum_{l=1}^{m} \varpi_i[l] - \varpi_i^* \\ &= \lim_{m\to\infty} \frac{1}{m}\sum_{l=1}^{m}\left(\mathbb{I}_{a_i[l]=1}\left(1 - \mathbb{J}_{\eta_i[l] > B_{i-i}R_{ii}[l]}\right)\right. \\ &\quad \left.\Delta\varpi_i[l] + \left(1 - \mathbb{I}_{a_i[l]=1}\right)\mathbb{J}_{\eta_i[l] > B_{i-i}R_{ii}[l]}\Delta\varpi_i[l]\right) \\ &\leq \Pr\left(\eta_i > B_{i-i}R_{ii} \geq \mathbb{E}(\eta_i)\right)\Delta\varpi_i^+ \\ &\quad + \Pr\left(\eta_i \leq B_{i-i}R_{ii} < \mathbb{E}\eta_i\right)\Delta\varpi_i^+ \\ &= \Pr\left(|\mathbb{E}\eta_i - B_{i-i}R_{ii}| < |\eta_i - \mathbb{E}\eta_i|\right)\Delta\varpi_i^+. \end{aligned} \quad (12)$$

From Chebyshev's inequality, we have

$$\Pr\left(|\eta_i - \mathbb{E}\eta_i| > \chi\sqrt{\mathrm{var}(\eta_i)}\right) \leq 1/\chi^2. \quad (13)$$

Substituting $\chi = \left(B_{i-i}R_{ii}[m+1] - \mathbb{E}(\eta_i)\right)/\sqrt{\mathrm{var}(\eta_i)}$ and using (12), we can obtain the results in Theorem 1.

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